Abstract: All over the world, dynamism in electricity markets is raising a number of challenges which has been identified as declining fuel sources, uncertainties in load growth, higher investments required for capacity addition or grid reinforcement and the cost of offsetting green house gases (GHGs) emissions.

The aim of Demand Side Management (DSM) in power industry is to reduce energy consumption and improve overall electricity usage efficiency through the implementation of policies and methods that control electricity consumption pattern.

In this paper, a mathematical model known as steepest ascent method is developed to evaluate the impact of DSM on power system operation and planning in terms of daily peak demand and cost of electricity consumed for a typical residential building in Lagos, Nigeria. The peak clipping approach was evidently showcased whereby at peak period, the DSM technique reduced the power and cost of electricity consumed considerably. Cost is calculated in naira (N)

Keywords: Power demand, Steepest Ascent, Cost of Electricity, Demand Side Management, Optimization.

1. Introduction

Over the years, electric power generated from several conventional and non-conventional energy sources have not been fully utilised, some have been wasted [1]. The depletion of the ozone layer as a result of the release of greenhouse gases (GHGs) emissions by running gas/steam dependent turbines has become so obvious to the point that human medicine researchers have expended a lot of resources by investigating the correlation between good health and global warming; thus identifying the problem this effect has created.

In line with a report from the Nigerian Energy agency (NEA), it has been estimated that lack of the adoption of energy efficient tools in the generation of electricity accounts for 45-53% of the total CO₂ emission which is on the very high side [1, 2].

In Nigeria, electricity demand is growing faster than the country’s population and to sustain its rapid economic and population growth, Nigeria needs to take action to meet the resulting increase in energy needs [3, 4].

China’s GDP is growing at a rapid pace and, consequently, the electricity demand is sharply elevating. A fast demand growth rate is expected to set in and continue if no effective measures are taken to manage the demand. According to [3, 4], the Chinese Electricity Sector is faced with two challenges: insufficient generation capacity due to high demand growth and high fuel costs from insufficient fuel production and policy changes.

Over the past two decades, Vietnam has experienced unprecedented economic growth. Energy demand has grown over 30% faster than the GDP. It has also been reported that the annual demand growth for the past ten
years is between 10-13% and to meet the power needs of the citizens, she requires a corresponding capital investment in generation capacity of around US $18 billion [1, 7]. The Energy demand curve has continued to show an upward trajectory in Canada due to market penetration of more energy-using devices, industrial production growth, increasing housing and commercial building stock [8, 9]. Ethiopia’s Electricity supply system is faced with the problems of inadequate service reliability leading to at times, serious shortage in the face of accelerating demand. Inefficient production and utilization of electricity, incompatible demand to supply is also bearing a negative effect on the environment. Consequently, the required high level and long term investment is beyond the present financial capability of the corporation [10]. It is on record that the state of California, USA, during the power crisis (1999-2000) spent US $20 billion in new power supply contracts and was only able to increase electricity supply by 2 percent [11]. Unless efforts are made to reverse this trend, the activities of governments and private institutions aimed at expanding energy supply will yield few or no dividends, hence the need for a clear understanding of the concept of demand side management.

2. DSM Programmes
Demand Side Management (DSM) was adopted to describe the planning and implementation activities of utilities, with the objective [12]: to influence the load shape, so as to achieve peak load within proper margin, load factor as close as 1.0 and better overall system utilization.

DSM utilizes a range of approach to reduce energy consumption. This includes the application of power saving technologies, tariffs, incentives and government policies to mitigate the peak demand instead of enlarging the generator capacity or reinforcing the network [13, 14].

There are generally six main DSM categories namely: Peak Clipping, Valley Filling, Load Shifting, Strategic Conservation, Load Building and Flexible Load Shape [14] as shown in figure 1 below:

![DSM Load Shape Categories](https://example.com/dsm_categories.png)

Fig. 1: DSM Load Shape Categories [12, 14]

Peak clipping method is adopted when there is a need to decrease the power demand at peak load periods. Meanwhile, these loads can’t be shifted to the off-peak periods. This could be due to lack of installed capacity during these periods. This programme can be achieved by indirectly forcing the consumers to decrease their loads by the use of miniatures (de-energising switches) on their supply points [15, 16].

Valley filling method is used to build-up off-peak loads in order to smooth out the load and improve efficiency of the utility; examples of valley filling is charging electric vehicles at night when the utility is not required to generate as much power as during the day and charging of inverters for serving loads during outage [16].

Strategic conservation programme is used when it is required to decrease the energy consumption all over the load period. This could be achieved by using energy-efficiency components.
3. Methodology

A generalised mathematical model is designed for a residential building. It explains how power demand and cost of electricity consumed can be minimised. From studies on Economic dispatch [1, 16], the following optimization procedure is developed.

Since power demand is a function of demand factor, to obtain the maximum or peak value of power demand in one day, then [16]:

Maximize: $P = f(\delta)$
Subject to: $0 < \delta < 1$

\[
P_g = f(Cx)
\]
\[
P_{min} < P < P_{max}
\]
\[
\xi_{min} < \xi < \xi_{max}
\] (1)

Where $P_{min}$ and $P_{max}$ are the power output of the minimum and maximum operation of the generating unit, $P_g$ is the power generated and $C_x$ is the cost of fuel, $\xi_{min}$ and $\xi_{max}$ are the minimum and maximum operating efficiencies of the generating unit.

For changes in demand factor,

\[
\frac{dP}{d\delta} = f'(\delta)
\] (2)

The maximum value is obtained at $f'(\delta) = 0$. Substituting $\delta^*$ for $\delta$ in equation (1) gives the maximized power demand.

The peak demand can be evaluated before DSM and after DSM. Taking into consideration the lighting appliances before DSM, equation (1) becomes:

Maximize: $P = f(\delta_i, \delta_f, \delta_{fi})$
Subject to: $0 < \delta_i < 1$
\[
0 < \delta_f < 1
\]
\[
0 < \delta_{fi} < 1
\]
\[
P_g = f(Cx)
\]

\[
P_{min} < P < P_{max}
\]
\[
\xi_{min} < \xi < \xi_{max}
\] (3)

Where $\delta_i$ is the demand factor of 60W incandescent lamps, $\delta_f$ is the demand factor of 36W fluorescent lamps and $\delta_{fi}$, the demand factor of 18W fluorescent lamps.

After DSM, the following formulation will be adopted:

Maximize: $P = f(\delta c_1, \delta c_2, \delta c_5)$
Subject to: $0 < \delta c_1 < 1$
\[
0 < \delta c_2 < 1
\]
\[
0 < \delta c_5 < 1
\]
\[
P_g = f(Cx)
\]
\[
P_{min} < P < P_{max}
\]
\[
\xi_{min} < \xi < \xi_{max}
\] (4)

Where $\delta c_1$ is the demand factor of 11W CFL, $\delta c_2$ is the demand factor of 20W CFL and $\delta_{fi}$, the demand factor of 15W CFL.

Also, the cost of electricity consumed is a function of power consumed,

Maximize: $C = f(P)$
Subject to: $0 < \delta < 1$
\[
\xi_{min} < \xi < \xi_{max}
\]
\[
P_{min} < P < P_{max}
\]
\[
P_g = f(Cx)
\] (5)

For changes in power demand,

\[
\frac{dC}{dP} = f'(P)
\] (6)

When $f'(P) = 0$, the maximum cost of electricity consumed can be obtained. Denoting by $P^*$ the value of $P$ at which the maximum of $f(P)$ occurs and substituting $P^*$ for $P$ in equation (5) gives the maximized cost of electricity consumed.

The maximum cost of electricity consumed can also be evaluated before DSM and after DSM. From equation (5), the maximum cost
of electricity consumed before DSM is obtained thus:

Maximize: \( C = f(P_i, P_f, P_{fi}) \)
Subject to: 
\[ 0 < \delta i < 1 \]
\[ 0 < \delta f < 1 \]
\[ 0 < \delta f_i < 1 \]
\[ Pg = f(Cx) \]
\[ P_{min} < P_i + P_f + P_{fi} < P_{max} \]
\[ \xi_{min} < \xi < \xi_{max} \]  
(7)

Where \( P_i \) is the power demand using 60W incandescent lamps, \( P_f \) is the power demand using 36W fluorescent lamps and \( P_{fi} \), the power demand using 18W fluorescent lamps.

After DSM,
Maximize: \( C = f(P_{c1}, P_{c2}, P_{c5}) \)
Subject to: 
\[ 0 < \delta c1 < 1 \]
\[ 0 < \delta c2 < 1 \]
\[ 0 < \delta c5 < 1 \]
\[ Pg = f(Cx) \]
\[ P_{min} < P_{c1} + P_{c2} + P_{c5} < P_{max} \]
\[ \xi_{min} < \xi < \xi_{max} \]  
(8)

Where \( P_{c1} \) is the demand factor of 11W CFL, \( P_{c2} \) is the demand factor of 20W CFL and \( P_{c5} \), the demand factor of 15W CFL.

Multiplying (2) and (6),
\[ \frac{dP}{d\delta} \times \frac{d\delta}{dP} = f'(df) \cdot f'(P) \]  
(9)

So that,
\[ \frac{dC}{d\delta} = f'(\delta) \cdot f'(P) \]  
(10)

Integrating,
\[ C = \int f'(\delta) \cdot f'(P) \, d\delta \]  
(11)

Several methods can be used to maximize the objective function. They include: method of steepest ascent, Newton-Raphson method and Fletcher-Powell Method [1].

The method of steepest ascent is used in this work because it does not require hessian matrix (matrix of second order partial derivatives of the function) and inverse hessian matrix which may not exist.

4. Results and Discussion
The results and discussion of results for peak power demand and cost of electricity consumed using steepest ascent method is presented thus:

4.1 Peak Power Demand
Before DSM,
\[ P_i = 12\delta i \]
\[ P_f = 0.021\delta f^2 + 0.366\delta f + 0.01 \]
Total demand before DSM,
\[ P_b = 12\delta i + 0.021\delta f^2 + 0.366\delta f + 0.01 \]
Considering constraints,
\[ 0 < \delta i < 1 \]
\[ 0 < \delta f < 1 \]
and using a class size of 0.1, we obtain values of \( P_b \).

<table>
<thead>
<tr>
<th>( \delta f )</th>
<th>( \delta i )</th>
<th>( P_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>1.23</td>
</tr>
<tr>
<td>0.2</td>
<td>0.3</td>
<td>2.48</td>
</tr>
<tr>
<td>0.3</td>
<td>0.4</td>
<td>3.72</td>
</tr>
<tr>
<td>0.4</td>
<td>0.5</td>
<td>4.96</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6</td>
<td>6.20</td>
</tr>
<tr>
<td>0.6</td>
<td>0.7</td>
<td>7.44</td>
</tr>
<tr>
<td>0.7</td>
<td>0.8</td>
<td>8.68</td>
</tr>
<tr>
<td>0.8</td>
<td>0.9</td>
<td>9.92</td>
</tr>
<tr>
<td>0.9</td>
<td></td>
<td>11.16</td>
</tr>
</tbody>
</table>

Table 1: \( P_b \) Values for Building

Using initial conditions \( \delta_0 = [0.9 \ 0.9]^T \), the maximum or peak power demand is 11.16kW.

Gradient vector, \( \Delta P_b = \begin{bmatrix} \frac{\partial P_b}{\partial \delta i} \\ \frac{\partial P_b}{\partial \delta f} \end{bmatrix} = \begin{bmatrix} 12 \\ 0.042\delta f + 0.366 \end{bmatrix} \]

For first iteration,
\[ \delta_0 + \lambda \Delta P_b = \begin{bmatrix} 0.9 \\ 0.9 \end{bmatrix} + \lambda \begin{bmatrix} 12 \\ 0.042(0.9) + 0.366 \end{bmatrix} \]
This function of \( \lambda \) assumes a global maximum at \( \lambda o = 0.0005 \), thus:

\[
\delta_o + \lambda \Delta P_{b(\delta o)} = \begin{bmatrix}
0.96 \\
0.90
\end{bmatrix}
\]

\[
P_b (\delta_o + \lambda \Delta P_{b(\delta o)}) = 12(0.906) + 0.021(0.9)^2 + 0.366(0.9) + 0.01 = 11.23kW
\]

Hence, the maximum or peak power demand before DSM = 11.23kW

After DSM,

\[
Pc1 = 2.27\delta c1
\]

\[
Pc2 = 0.39\delta c2^2 + 0.715\delta c2 + 0.19
\]

Total demand after DSM,
\[
P_a = 2.27\delta c1 + 0.39\delta c2^2 + 0.715\delta c2 + 0.19
\]

Considering constraints,

\begin{align*}
0 &< \delta c1 < 1 \\
0 &< \delta c2 < 1
\end{align*}

and using a class size of 0.1, we obtain values of \( P_a \).

Using initial conditions \( \delta_o = [0.9 \ 0.9]^T \), the maximum or peak power demand is 3.19kW.

Gradient vector, \( \Delta P_b = 
\begin{bmatrix}
\frac{\partial P_a}{\partial \delta c1} \\
\frac{\partial P_a}{\partial \delta c2}
\end{bmatrix}
= 
\begin{bmatrix}
2.27 \\
0.78\delta c2 + 0.715
\end{bmatrix}
\]

For first iteration,

\[
\delta_o + \lambda \Delta P_{a(\delta o)} = \begin{bmatrix}
0.9 \\
0.9
\end{bmatrix}
+ \lambda
\begin{bmatrix}
2.27 \\
0.78(0.9) + 0.715
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0.9 + 2.27\lambda \\
0.9 + 1.417\lambda
\end{bmatrix}
\]

This function of \( \lambda \) assumes a global maximum at \( \lambda o = 0.0005 \), thus:

\[
P_a (\delta_o + \lambda \Delta P_{a(\delta o)}) = 2.27(0.9) + 0.39(0.9)^2 + 0.715(0.9) + 0.19
= 3.19kW
\]

Hence, the maximum or peak power demand after DSM = 3.19kW

Values obtained from second and third iterations are relatively close to the value obtained above. Therefore, reducing peak demand = \( P_b - P_a = 11.23 - 3.19 = 8.04kW \)

4.2 Cost of Electricity Consumed

Before DSM,
\[
C_i = 70.788\delta i
\]
\[
Cf = 0.12\delta f^2 + 2.16\delta f + 0.059
\]

Total cost before DSM,
\[
Cb = 70.788\delta i + 0.12\delta f^2 + 2.16\delta f + 0.059
\]

Considering constraints,

\begin{align*}
0 &< \delta i < 1 \\
0 &< \delta f < 1
\end{align*}

and using a class size of 0.1, we obtain values of \( C_b \).

Using initial conditions \( \delta_o = [0.9 \ 0.9]^T \), the maximum or peak power demand is 3.19kW.

Gradient vector, \( \Delta P_b = 
\begin{bmatrix}
\frac{\partial P_a}{\partial \delta c1} \\
\frac{\partial P_a}{\partial \delta c2}
\end{bmatrix}
= 
\begin{bmatrix}
2.27 \\
0.78\delta c2 + 0.715
\end{bmatrix}
\]

For first iteration,
Using initial conditions $\delta_0 = \begin{bmatrix} 0.9 \\ 0.9 \end{bmatrix}^T$, the maximum or peak cost is N65.81.

Gradient vector,

$$\Delta C_b = \begin{bmatrix} \frac{\partial C_b}{\partial \delta_1} \\ \frac{\partial C_b}{\partial \delta_2} \end{bmatrix} = \begin{bmatrix} 70.778 \\ 0.24\delta + 2.16 \end{bmatrix}$$

For first iteration,

$$\delta_0 + \lambda \Delta C_{b(\delta_0)} = \begin{bmatrix} 0.9 \\ 0.9 \end{bmatrix} + \lambda \begin{bmatrix} 70.778 \\ 0.24(0.9) + 2.16 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9 + 70.778\lambda \\ 0.9 + 2.376\lambda \end{bmatrix}$$

This function of $\lambda$ assumes a global maximum at $\lambda_0 = 0.0005$, thus:

$$\delta_o + \lambda \Delta C_{b(\delta_0)} = \begin{bmatrix} 0.94 \\ 0.90 \end{bmatrix}$$

$$C_b (\delta_0 + \lambda \Delta C_{b(\delta_0)}) = 70.778(0.94) + 0.12(0.9)^2 + 2.16(0.9) + 0.059$$

$$= \text{N68.64}$$

After DSM,

$$C_{c1} = 13.39\delta c1$$
$$C_{c2} = 2.38\delta c^2 + 4.22\delta c2 + 1.121$$

Total demand after DSM,

$$C_a = 13.39\delta c1 + 2.38\delta c^2 + 4.22\delta c2 + 1.121$$

Considering constraints,

$$0 < \delta c1 < 1$$
$$0 < \delta c2 < 1$$

and using a class size of 0.1, we obtain values of $C_a$.

Table 4: $C_a$ Values for Building

<table>
<thead>
<tr>
<th>Class</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_a$</td>
<td>6.41</td>
<td>12.76</td>
<td>19.11</td>
<td>25.46</td>
<td>31.81</td>
<td>38.15</td>
<td>44.50</td>
<td>50.84</td>
<td>57.19</td>
</tr>
</tbody>
</table>

Using initial conditions $\delta_0 = \begin{bmatrix} 0.9 \\ 0.9 \end{bmatrix}^T$, the maximum or peak cost is N57.19.

Gradient vector,

$$\Delta C_a = \begin{bmatrix} \frac{\partial C_a}{\partial \delta_1} \\ \frac{\partial C_a}{\partial \delta_2} \end{bmatrix} = \begin{bmatrix} 13.39 \\ 4.6\delta c2 + 4.22 \end{bmatrix}$$

For first iteration,

$$\delta_0 + \lambda \Delta C_{a(\delta_0)} = \begin{bmatrix} 0.9 \\ 0.9 \end{bmatrix} + \lambda \begin{bmatrix} 13.39 \\ 4.6(0.9) + 4.22 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9 + 13.39\lambda \\ 0.9 + 8.36\lambda \end{bmatrix}$$

This function of $\lambda$ assumes a global maximum at $\lambda_0 = 0.0005$, thus:

$$\delta_0 + \lambda \Delta C_{a(\delta_0)} = \begin{bmatrix} 0.9 \\ 0.9 \end{bmatrix}$$

$$C_a (\delta_0 + \lambda \Delta C_{a(\delta_0)}) = 13.39(0.91) + 2.3(0.9)^2 + 4.22(0.9) + 1.121$$

$$= \text{N18.97}$$

Therefore,

$$\text{Peak cost savings} = C_b - C_a$$
$$= 68.64 - 18.97$$
$$= \text{N49.67}$$

From the results obtained, the peak demand reduced from 11.23kW to 3.19kW (a 71.59% decrease) and the cost of electricity consumed...
reduced from N68.64 to N18.97 (a 72.36% decrease).

5. Conclusion and Recommendation
This paper has presented an optimization-based DSM model applied to a residential building for minimizing power demand and cost of electricity consumed. This concept can be used to reduce energy consumed on a daily basis, in other words, this research can be extended to understudy consumption pattern in a group of homes, offices and industries.

References