Analytical Design of a Vernier Variable Reluctance Machine with Smooth Exited Rotor

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Abstract – In the field of energy conversion applications, the tendency of electrical machine design directly coupled is being directed towards those capable of operating at low speed and high torque. The Vernier variable reluctance machines (VVRM) can meet these requirements. In this paper, we introduce the Vernier effect principle and interest on the basis of the air gap energy assessment, and suggest therefore, a detailed sizing method that leads to a prototype of VVRM with smooth excited rotor. A set of results of this prototype are included. In order to validate the adopted analytical design procedure, the prototype results are compared to numerical ones obtained from a finite elements analysis.

Keywords – Electric Machines, Variable reluctance machine (VRM), Design, Vernier machine, direct driven.

I. INTRODUCTION

Since several years, a considerable interest has been focused on direct driven structures, [1-4] to be used for applications requiring low speed and high torque. This operation can be ensured by either permanent magnet synchronous machines with high pair pole number [5] or by variable reluctance machines [6]. The structures can be discoid or cylindrical [7]. Most of the papers that deal with these particular structures do not give the sizing procedure which leads to their prototypes.

The purpose of this paper is to present a design procedure that yields a primary structure. The proposed structure is a Vernier variable reluctance type machine excited by windings supplied by an AC balanced polyphase current or by a DC current.

The electrical machine design is based on two considerations. The first one is the geometric design of the two frames (rotor and stator). The second concerns the design of the electrical circuits (windings) and their power supply.

In general, the overall dimensions of the electrical circuit depend on the m.m.f. required for the magnetic field production, directly linked to the desired power.

The magnetic circuit sizing should take into account its ability to canalize the magnetic field while limiting a decrease of the magnetic potential difference. This design will therefore depend directly on the magnitude of the created field.

In the first part of this paper, we introduce the operation principle of the VVRM to highlight the Vernier effect interest.

The second part is devoted to the conditions to be met to ensure a proper operating of excited smooth rotor Vernier variable reluctance machine. For this aim, an analytical model based on the magnetic energy is introduced and the conditions on the stator teeth and pair pole numbers are deduced.

Then, a procedure to design a prototype of such structure is introduced. The different design stages conducting to a primary prototype are given and explained.

At last, using the sizing procedure, a prototype has been designed. We studied it using 2D-FEM analysis. The results are compared to the ones obtained from the analytical sizing procedure.

II. VERNIER VRM OPERATING PRINCIPLE

Fig 1. Permanent magnet classical structure

Fig 2. Double teeth excited stator structure with Vernier effect.

In order to explain the operating principle of an excited vernier machine and its interest, two structures are introduced. The first one is constituted...
of a polar wheel with \( p \) pairs of poles and a ferromagnetic yoke supporting a winding Fig. 1. When the wheel is driven at a speed \( \Omega \) an \( e.m.f. \) of pulsation \( \omega \) is induced in the coil surrounding the yoke. This pulsation is directly proportional to the product \( p\Omega \). In the case of low speed, the pulsation is low as well. Therefore, in order to obtain an \( e.m.f. \) of significant frequency while rotating at low speed, it is necessary to increase the number of the pole pairs.

The second structure depicted in Fig. 2 is constituted of a ferromagnetic yoke with \( N_s \) teeth and a wheel of \( N_r \) teeth. Furthermore, two separate windings are wound around the yoke. The first one, referred as the excitation circuit, is carrying a DC or AC current \( i_1 \) which ensures the circuit magnetisation. The second one, referred as the armature circuit, will collect the induced \( e.m.f. \). Let \( \theta \) represents the reference axis position of the rotor with respects to the stator reference axis. The total air gap reluctance that depends on rotor position can be expressed as \( R(\theta) \).

Let us consider a generator operating, i.e. the wheel is rotated and the excitation circuit is energized. If \( i_1 \) is a DC current, for the same rotational speed \( \Omega \), the \( e.m.f. \) induced in the armature winding will have a frequency that will be much higher than the case presented in Fig. 1. It will be mainly a function of the air gap reluctance periodicity.

If \( i_1 \) is a sinusoidal current of a frequency \( f_1 \), the frequency \( f_2 \) of the induced \( e.m.f. \) will depend on \( \Omega \) air gap reluctance periodicity and \( f_1 \). It is therefore possible, for a given speed and desired frequency \( f_2 \), to supply a current \( i_1 \) with an adequate frequency \( f_1 \).

The excitation circuit can be moved from the stator in order to be located in the rotor. Thus, we obtain the structure shown in Fig. 3. This operates in the same manner as the structure seen in Fig. 2. with an additional degree of freedom that is the rotor position \( \theta \).

In order to maximize the active surface while keeping the Vernier effect, it is possible to suppress the saliency of one of both surfaces. Hence, in the case of an smooth rotor solution, we yield the structure shown in Fig. 4.

III. ENERGY MODEL AND OPERATING CONDITIONS OF A SMOOTH ROTOR VVRM

To determine the combination between the stator teeth and the polarities of both excitation and armature circuits to reach a synchronous operating, we use the structure presented in Fig. 5 which is a generic cylindrical of a current rotor excited stator teethed reluctance machine.

For the proper functioning of the structure we must combine correctly between the operating parameters of the MRVV: \( N_s, N_r, p, p' \) and \( \Omega \). This point will be the subject of the following section.
1- Energy expression

The expression of the total energy \( W_{em} \) in the air gap of an excited Vernier reluctance machine can be expressed by the following relation:

\[
W_{em} = \frac{1}{2} \int_{\theta_s}^{\theta_e} E_{\phi}^2(\theta, \dot{\theta}) P(\theta, \dot{\theta}) \, d\theta
\]

(1)

where \( E_{\phi} \) is the magneto motive force in the air gap and \( P \) represents the air gap permeance. \( s \) is the position of a given point \( M \) in the air gap with respect to the stator frame and \( \theta \) the angle between the rotor and the stator frames.

2- Air gap permeance

The air gap permeance, per angle unit, depends mainly on the geometric parameters related to the structure. In the case of a smooth rotor VVRM, this permeance can be written, as follows [1],

\[
P(\theta, \dot{\theta}) = P_0 + P_s(\theta)
\]

(2)

where \( P_0 \) represents a constant term related to the average air gap width value and \( P_s(\theta) \) is function of the stator saliency. \( E_{\phi s} \), \( E_s \) and \( E_l \) are geometric coefficients.

3- Difference of air gap magnetic potential

As the permeability of the iron is assumed to be infinite, we consider a radial field distribution in the air gap. In this case, when a winding \( k \) creates an \( m.m.f. \), its expression can be written in a series form as:

\[
e_k = \sum_{i=0}^{\infty} \frac{2 n_k}{\pi} \sin((2i+1)\frac{\pi}{2}) e_i(2i+1) p_i \phi_i(\theta)
\]

(3)

A \( q \) phase circuit, located in the same armature, creates a \( m.m.f. \) that the resulting expression, is obtained by the algebraic sum of the \( m.m.f. \) of each phase. Thus, the armature winding embedded in the stator, gives a total \( m.m.f. \) \( E_s \) expressed as:

\[
e_s = \sum_{i=0}^{\infty} \frac{2}{\pi} \sin((2i+1)\frac{\pi}{2}) e_i(2i+1) p_i \phi_i(\theta)
\]

(4)

The inductor windings, driven by the rotor,

give a total \( m.m.f. \) given by:

\[
\varepsilon' = \sum_{i=0}^{\infty} \frac{2 \pi}{2 \pi} \sin((2i+1)\pi/2) K_i' \cos \left( (t+1) p_i \phi_i \right)
\]

(5)

4- Energy and conditions of conversion

To ensure the conversion of the electrical energy, it is necessary that the structure develops a rotating torque. This implies that the magnetic energy in the air gap must be a function of the rotor position \( \theta \). Now, the expression of this energy (1) involves the air gap permeance which is essentially a function of the stator teeth numbers \( N_s \). It involves also the term that expresses the total \( m.m.f. \) due to the winding supplies. In this term, it the polarities \( p \) and \( p' \) appear.

These conditions are related to the number of teeth of the stator and the polarity of the two circuits: power and excitation supplies. They are based on the fact that the smooth VVRM structure must produce an electromagnetic torque with an average value non-zero [9-11]. Furthermore, the torque has to present minimal ripples. Therefore, the structure will only use the main electromagnetic conversion that corresponds to the interaction between the armature and excitation fields through the variation of the air gap permeance.

In the case of smooth rotor VVRM, these conditions are expressed by the following relations:

\[
\overline{N_s} = \pm p \pm p'
\]

\[
p \neq p'
\]

\[
N_s \neq 2p
\]

\[
N_s \neq 2p'
\]

(6)

Then, the magnitude of the main electromagnetic torque is given by:

\[
T_e = \frac{9}{\pi} p' n n' \frac{\bar{I}}{I} K_B K_{B'} P_s
\]

(7)

This torque is constant when the rotor rotates at the following synchronous speed:

\[
\Omega = \pm \omega \pm \omega'
\]

(8)
IV. SIZING PROCEDURE

The dimensioning of electrical machines requires preliminary data which represent mainly the features of the nominal operating: The output power and the power factor, the supply voltage magnitude and its frequency and the rotating speed.

The design of an electrical machine is based on two considerations. The first one is related to the geometric dimensioning, in other words, the magnetic circuits of the rotor and stator, as well as the slots of both armatures. The second consideration is the design of the electrical circuit, thus the distribution of windings on both armatures and their supply. These two parts are imbricated. Globally, the dimensions of the electrical circuit depend on the ampere-turns required for the production of the magnetic field, directly linked to the desired power.

The dimensions of the magnetic circuit should take into account its ability to canalize the magnetic field by limiting the magnetic potential difference drops. This design will therefore depend directly on the magnitude of the field which should be canalized.

1- Determination of the $D^2L$ magnitude

The useful volume of a structure is defined by the value of the variable $D^2L$, which is expressed, in function of the apparent power $S$, by the following relation:

$$ S = \frac{\pi^2}{60\sqrt{2}} N (K_p / K_f) K_S D^2 L \, B_c A_c $$

(9)

The various parameters of this relation are specified in the nomenclature at the end of the paper.

To separate the internal diameter $D$ from the effective length $L$, one of the two parameter sizes has to be chosen. This choice is made further to a number of constraints. These constraints can affect bulkiness, linear speed or the optimal use of copper.

In order to increase the air gap permeance, and hence the electromagnetic energy, it is advantageous to choose the lowest possible minimum air gap. This choice depends on constraints related to the mechanical implementation. For the studied structure, the minimum air gap is taken equal to 0.3 mm. Actually, this size is easily achievable, for a internal diameter of about 0.5 m [1].

2- Design of VVRM slots and yokes

From a geometric point of view, it remains to determine the seven parameters ($l_{ds}, l_{ds}, l_{es}, P_y, P_y, P_y$ and $P_y$) defined in the Fig. 6.

The first step, in the design of slots, is to choose the stator tooth width $l_{ds}$ according to the tooth step $\lambda_s = l_{ds} + l_{es}$. The electromagnetic torque ripple in a VVRM structure is very sensitive to this tooth width [5]. To limit the torque ripple, the ratio $r_{ds} = l_{ds} / \lambda_s$ should be included in the interval [0.4, 0.5] [1].

The calculation of the ampere-turns in a slot, for the maximal operating mode ($AT_{\text{max/slot}}$), allows us to determine the depth of the stator and the rotor slots. The stator slot depth is given by:

$$ p_x = \frac{AT_{\text{max/slot stator}}}{D_c K_{rem} l_{cs}} $$

(10)

Where $D_c$ is the current density and $K_{rem}$ represents the fill factor. The same formula, with the rotor parameters, can be applied to calculate its slot depth $p_r$.

In order to attenuate the saturation effects, we choose the thickness of the rotor and stator cylindrical yokes, noted $p_y$ and $p_y$, respectively, function of the widths of the stator tooth. These values can be obtained as follows:

$$ p_y \approx p_y $$

(11)
3- Design of the windings

In the studied structure, the excitation circuit is made up of windings. The calculation of the excitation ampere-turns is deduced from the rated operating point. The calculation of the ampere-turns requires the knowledge of the rated armature winding and inductor currents, and the turn number in each winding, such as:

\[ AT_{\text{max/en}} = \sqrt{2} \left( \frac{n}{q_e} I \right) \]  

(12)

Other parameters, which are the number of turns of the armature and the excitation windings, are obtained from two relations. The first one is based on the flux expression per phase and the second one on the equality of the ampere-turns between inductor and armature winding.

The flux linkage in an armature winding phase is expressed as a function of the excitation m.m.f. and the air gap permeance, as given below:

\[ \phi_k = \frac{2\pi}{\theta} \int F_k P(\theta_s, \theta) \, d\theta_s \]  

(13)

In this expression, the term \( F_k \) represents a filter function permitting to isolate the phase \( k \) of the armature winding circuit. This function constitutes the \( k \) winding distribution and can be written, taking into account the only first terms, under the following form:

\[ F_k = \frac{2\pi}{\theta} K_B n \cos \left( p \theta_s - (k - 1) \frac{2\pi}{3} \right) \]  

(14)

Where \( k = 1, 2 \) or \( 3 \) is the order of the considered phase.

Replacing (14) into the relation (13), the flux linkage in an armature winding phase, when the machine is running at the synchronous speed, can be expressed as:

\[ \phi_k = \frac{3}{\pi} K_B K'_B n n' \cos \left( p' t - (k - 1) \frac{2\pi}{3} \right) \]  

(15)

Starting from the desired value of the e.m.f. \( e = -d\phi /dt \) and the choice of excitation current \( I' \), we obtain a value for the product \( n n' \).

In order to optimize the energy conversion of the machine, the ampere-turns of the armature winding and the inductor circuits have to be equal [1]. In such circumstances, we can write:

\[ pK_B n I = p'K'_B n' I' \]  

(16)

This last expression combined with the product value of \( n n' \) permits to determine the number of turns of both stator and rotor circuits.

4- Expressions of self and mutual inductances

The self inductance of phase \( k \) can be obtained by calculating the flux linkage through this phase when it is fed by a unit DC current. The mutual inductances are determined in the same manner, by calculating the flux throughout the other phases.

Consequently, the self and mutual inductances can be expressed by:

\[ L_{l_m(m = l)} = \int \frac{2\pi}{\theta} \int F_n P(\theta_s, \theta) \, d\theta_s \]  

(17)

\[ M_{l_m(m = l)} = \int \frac{2\pi}{\theta} \int F_n P(\theta_s, \theta) \, d\theta_s \]  

(18)

Where the index \( l \) is the phase winding in which the unit current is imposed. The index \( m \) is related to the phase in which the flux linkage is calculated. The m.m.f. \( E_1 \), is obtained from the general expression of the m.m.f. given by the relation (3). These expressions give:

\[ L_{l1} = \frac{4}{\pi} n^2 K_B^2 P_0 \]  

(19)

\[ L_{44} = \frac{4}{\pi} n'^2 K_B^2 P_0 \]  

\[ M_{14} = \frac{2}{\pi} p l_s K_B K'_B n n' \cos (p' \theta) \]  

(20)

V. DESIGNED PROTOTYPE

A prototype has been designed using the sizing procedure developed above. Its specifications are given in the following table:

<table>
<thead>
<tr>
<th>Table 1: Prototype specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active power</td>
</tr>
<tr>
<td>P=5 KW</td>
</tr>
</tbody>
</table>
In Fig. 7, we give the geometrical characteristics of the designed prototype.

![Fig. 7: Geometrical parameters of the designed prototype](image)

Table 2: Characteristics of the designed prototype

<table>
<thead>
<tr>
<th>VVRM-Prototype</th>
<th>p=2</th>
<th>p′=10</th>
<th>Ns=12</th>
<th>$\Omega$ =5.23 rad/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>n=130</td>
<td>n′=82</td>
<td>$q_e=1$</td>
<td>$q_e=1$</td>
<td>$f'$=58 Hz</td>
</tr>
<tr>
<td>I=9A</td>
<td>I′=2.8A</td>
<td>$q=3$</td>
<td>$q'=3$</td>
<td>$f=50$ Hz</td>
</tr>
</tbody>
</table>

In the sake of validating the operating principle and the sizing procedure developed previously, we used the finite element method to perform accurate analysis. A comparison of results such as self inductances, mutual inductances and the e.m.f. at no load between the analytical and finite element models is presented. The finite element calculations, taking into account the movement, is fulfilled with the calculation code developed in the laboratory L2EP of Lille (France).

In Fig. 8, we present the 2D mesh in a cutting plane of the prototype. It is constituted of 39500 elements and 19900 nodes.

![Fig. 8: Mesh in a cutting plane of the prototype](image)

The field distribution in a cutting plane of the structure, at no load, is presented hereafter. One can clearly notice the excitation poles modulated by the permeance air gap. Only two poles seem to be energized on half part of the structure. This corresponds to the difference between the armature and the excitation pole pair numbers.

![Fig. 9: Field distribution, at no load, in a cutting plane of the structure](image)

Calculations have been carried out at no load. In the case of the self inductances presented in the Fig. 10 and 11, numerical computations show slight ripples. This does not appear on the analytical calculation because it does not take into account the small slots in the rotor.

![Fig. 10: The self inductance of an armature phase winding](image)
Finally, we show, in Fig. 14, the no load torque of the designed prototype obtained from numerical calculations. The ripples of this torque are relatively high related to the rated torque. Indeed, the rotor slots induced an additional variable reluctance that leads to an electromagnetic conversion whose synchronous speed is different from the one of the main conversion.

In order to decrease the magnitude of these ripples, it is possible to skew the slots of the rotor by an angle equal to the stator tooth step. Using the slice technique coupled to 2D-FEM, we carried out calculations to simulate the no load operating with skewed rotor slots.

In Fig. 15, we show the results obtained with 10 slides. We can notice that the magnitude of the torque ripples is divided by 2.

Calculations have also been performed at load using 2D-FEM. In this case, the excitation circuit is supplied by its rated currents while the armature windings are supplied by half the rated magnitude. Besides, the rotor is moved at its rated synchronous speed. In the following table, we give the average values of the electromagnetic torque. We compare the analytical value to the ones obtained by the numerical model at linear and non linear iron characteristic.
CONCLUSION

In the first part of this work, we introduced an energy model based on the air gap permeance and m.m.f., for rotor excited vernier variable reluctance machines with a smooth rotor. This model has allowed us to determine the conditions on the teeth and the polarities of windings to ensure a synchronous operating of these machines.

In the second part, we established a sizing procedure, which allows leading to a detailed plan of an initial machine structure.

To validate the proposed sizing procedure, a comparison between the results of the proposed model and a 2D-FEM analysis is presented. The obtained results, in the hypothesis of an infinite magnetic permeability of the iron, correspond to the expected ones and are very similar for both models. These validate the principle of the studied structure and the proposed pre-sizing procedure.

X. NOMENCLATURE

$N_s$: Stator teeth Number
$P$: Number of pairs of poles of the armature winding
$P'$: Number of pairs of poles of the inductor coils
$B_e$: Air gap magnetic induction
$\theta$: Rotor position with respect to stator axis
$\theta_e$: Air gap position with respect to stator axis
$L$: Machine effective length
$R_s$: Internal radius
$K_{B k}$: Coefficient of windings for the $k$th harmonic
$K_B$: Fill factor of the armature winding
$K'_{B}$: Fill factor of the inductor winding
$i_k$: Supply current of the winding $k$
$I_{\text{max}}$: Supply maximum current

$I'_{\text{max}}$: Maximum excitation current
$I$: Supply r.m.s Current food
$I'$: Excitation r.m.s Current
$\omega$: Pulsation of the armature winding currents
$\omega'$: Pulsation of the inductor currents
$n$: Number of turns of the armature winding
$n'$: Number of turns of the inductor winding
$q$: Number of phases of the armature winding
$q'$: Number of phases of the inductor winding
$q_e$: Number of slots by sector and by armature winding phase
$q_{e'}$: Number of slots by sector and by phase of the inductor

$A_c$: Linear current density
$K_f$: Coefficient of flux waveform
$K_S$: Coefficient of active surface in the air gap
$N$: The rotation speed in rpm

XI. REFERENCES


