ON THE APPLICATION OF VARIOUS META-HEURISTIC METHODS TO THE OPTIMAL REACTIVE POWER FLOW PROBLEM FOR PRACTICAL POWER NETWORKS

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Abstract: An efficient distribution of the reactive power in an electric network aims generally to maintain voltages in acceptable limits and control transmission losses. This paper presents a comparative study of solving the optimal reactive power flow (ORPF) problem using several meta-heuristic techniques such as simulated annealing, genetic algorithms, evolutionary strategy, particle swarm optimization and Tabu search. Results application to a large electric network that is representative of the 114 bus Algerian electric system are reported. To show the contribution of these techniques, the results of simulation are compared with previous studies using the reduced gradient method technique.

Key words: Optimal Reactive Power Flow (ORPF), Meta-Heuristic methods, Optimization, Non Linear Programming.

1. Introduction

Nowadays electric utilities are paying more and more attention to the voltage profile optimization both in daily scheduling and in VAR planning. Voltage scheduling and VAR planning are inherent to tightly correlated objectives even if they are relevant to different time horizons. In daily scheduling the optimization of voltage profiles is performed several hours in advance on loads forecast values by minimizing the real losses.

This problem known as optimal reactive power dispatch (ORPD) is a particular case of the optimal power flow in which the means of active power control are fixed while those of reactive power are adjustable. The optimal power flow (OPF), nonlinear problem of optimization, was proposed by Carpentier at the beginning of the sixties based on the economical dispatch of the power [1].

Thus, ORPF is a complex combinatorial optimization problem involving non-linear functions having multiple local minima and non-linear discontinuous constraints. Conventional optimization methods used to solve this problem involved linear programming [2], nonlinear programming [3], decomposition method [4] and other techniques. However, these conventional methods can only be lead to a local minimum and most of them cannot deal with integer problem. In recent years, some artificial intelligence methods such as expert systems, neural networks [5], genetic algorithms [6], evolutionary strategies [6,7], simulated annealing [8], particle swarm optimization [9] and tabu search [10] have been used to solve reactive power optimization problem.

The goal of this work is to give a synthesis of our modest experience on the use of several meta-heuristic techniques to the optimal reactive power flow [11-15]. A qualitative comparison between various meta-heuristic techniques and a conventional optimization using steepest gradient method is made. First, a mathematical problem formulation of the ORPF is given. Then, a short description of the basic concept of all meta-heuristics studied is presented. Simulations were first run on several networks of different size, then application on the Algerian network that contains 114 nodes was made and results reported. The assumptions of the elaborated programs as well as the adjustment of the meta-heuristics control parameters will be summarized. Conclusions and comments are also presented.
2. Problem Formulation

The main objectives of ORPF address three important aspects:

- keeping the voltage profiles in an acceptable range,
- minimizing the total transmission energy loss, and
- avoiding excessive adjustment of transformer tap settings and discrete var sources switching.

In a general form, the problem of the ORPF can be formulated as:

\[
\begin{align*}
\min \ f (X, U) \\
G(X, U) &= 0 \\
H(X, U) &\leq 0
\end{align*}
\]

subject to

\[
X_{\min} \leq X \leq X_{\max}
\]

\[
U_{\min} \leq U \leq U_{\max}
\]

with \( X, U \) set of state and control variables.

The set of equality constraints represent the power flow equations. Upper and lower voltage limits and capacity restrictions in various reactive power sources, generators, shunt capacitor banks and transformer taps constitute the inequality constraints.

In an explicit form, the problem is written as [7, 11]:

\[
\begin{align*}
\min & \quad P_t = \sum_{i=1}^{N_t} P_{i,t} - \sum_{j=1}^{N_t} P_{j,t} \\
\text{s.t.} & \quad P_{i,t} - P_{i,n} - V_i \sum_{j=1}^{N} \left( V_j (G_{ij} \cos \Theta_{ij} + B_{ij} \sin \Theta_{ij}) \right) = 0 \\
& \quad Q_{i,t} - Q_{i,n} - V_i \sum_{j=1}^{N} \left( V_j (G_{ij} \cos \Theta_{ij} - B_{ij} \sin \Theta_{ij}) \right) = 0
\end{align*}
\]

subject to equality constraints:

\[
P_{i,n} - P_{i,n} - V_i \sum_{j=1}^{N} \left( V_j (G_{ij} \cos \Theta_{ij} + B_{ij} \sin \Theta_{ij}) \right) = 0
\]

and a set of inequality constraints:

\[
Q_{i,n} - Q_{i,n} - V_i \sum_{j=1}^{N} \left( V_j (G_{ij} \cos \Theta_{ij} - B_{ij} \sin \Theta_{ij}) \right) = 0
\]

\[
T_{i,n} - T_{i,n} - V_i \sum_{j=1}^{N} \left( V_j (G_{ij} \cos \Theta_{ij} + B_{ij} \sin \Theta_{ij}) \right) = 0
\]

and

\[
Q_{i,n} - Q_{i,n} - V_i \sum_{j=1}^{N} \left( V_j (G_{ij} \cos \Theta_{ij} - B_{ij} \sin \Theta_{ij}) \right) = 0
\]

where

- \( N_1, N_{cap} \) number of transformers and shunt capacitors,
- \( V_i, \Theta_i \) voltage magnitude and angle at \( i \)-th bus,
- \( \Theta_{ij} = \Theta_i - \Theta_j \)
- \( P_{G,i}, Q_{G,i} \) real and reactive power generation at \( i \)-th bus,
- \( P_{D,i}, Q_{D,i} \) real and reactive power load at \( i \)-th bus,
- \( G_{ij}, B_{ij} \) mutual conductance and susceptance between \( i \) and \( j \) buses,
- \( Q_{G_{\min}, i}, Q_{G_{\max}, i} \) reactive power limits of \( i \)-th generator,
- \( Q_{C_{\min}, i}, Q_{C_{\max}, i} \) reactive power limits of \( i \)-th shunt capacitors,
- \( V_{\text{min}, i}, V_{\text{max}, i} \) limits on voltage at \( i \)-th bus
- \( T_{\text{min}, i}, T_{\text{max}, i} \) limits on \( i \)-th transformer taps.

For the functional inequalities constraints, in our case the voltage limits at the load buses, we have used a penalty method in which the objective function is increased by penalties of violation on these constraints.

Thus, the objective function \( f \) must be replaced by:

\[
F(X, U) = f(X, U) + \sum \omega_j
\]

where the penalties factors \( \omega_j \) are introduced for each functional constraint violation. The used penalties functions are as follows:

\[
\omega_j = \begin{cases} 
S_j (x_j - x_j^M)^2 & \text{if } x_j > x_j^M \\
S_j (x_j - x_j^m)^2 & \text{if } x_j < x_j^m
\end{cases}
\]

\( S_j \) is a scalar to be chosen correctly.

3. Basic concepts of meta-heuristics

Meta-heuristics means search algorithms for combinatorial optimization obtained by repeating a simple search process with some heuristics. They are inspired by biological information process, artificial life, physical process,...,etc. The meta-heuristic approaches aim at evaluating a globally optimal solution rather than locally optimal one.

3.1 Genetic Algorithms [16, 17]

Genetic algorithms search for an optimal solution using the principles of evolution and heredity. They operate on populations which consist of a number of individuals, each representing a particular selection of the values of the variables coded in binary form. The
initial population of binary strings is randomly generated. Each individual is evaluated to obtain a measure of its fitness in terms of the objective function to be optimized; then a new population is formed by selecting the fitter individuals. Some members of the new population undergo transformations by genetic operators to form new solutions. Such operators include “crossover” and “mutation.” Crossover creates new individuals by combining substrings from the parent individuals and takes place according to a given probability value. Mutation creates a new individual by changing a randomly selected bit in its coding. The flow chart (Fig.1) shows the representation of a simple genetic algorithm.

A genetic algorithm is governed by some parameters which can be summarized thus:
- $N_{gen}$ maximum number of generations,
- $P_c, P_m$ crossover and mutation rates,
- $T_{pop}$ population size,
- $T_{cros}$ crossover type.

![Fig.1 Organization of a simple genetic algorithm.](image)

### 3.2 Evolutionary Strategy [17, 18]

In Evolutionary Strategies ES, the components of a trial solution are viewed as behavioral features of an individual, not as genes along a chromosome. It is supposed a genetic source for these traits, but the nature of the linkage is not detailed. Thus, an individual is represented as a pair of float-valued vector i.e. $a = (x, \sigma)$, were $x$ represents a point in the search space. The second is the vector of standard deviations and provides instructions on how to mutate $a$ and is itself subject to mutation. In other words, both components, $x$ and $\sigma$, are submitted to evolution process by application of operators of mutation and also recombination. Thus, a suitable adjustment and diversity of parameter mutations should be provided under arbitrary circumstances.

The approach used in our case is denoted by $(\mu+\lambda)$-ES. In the former, $\mu$ parents generate $\lambda$ offspring and all solutions compete for survival with the best $\mu$ individuals being selected as parents of the next generation. The following pseudocode algorithm summarizes the components of a simple $(\mu+\lambda)$-Evolutionary Strategy (SES).

```
Input : $\mu$, $\lambda$, $\Theta_o$, $\Theta_m$, $\Theta_r$
$t = 0$ ;
$P(t) = initialization (\mu)$ ;
while termination criterion not fulfilled do
    $P'(t) = \text{Recombination} (P(t), \Theta_o)$ ;
    $P''(t) = \text{Mutation} (P'(t), \Theta_m)$ ;
    $F(t) = \text{Evaluation} (P(t),P''(t), \mu, \lambda)$ ;
    $P(t+1) = \text{Selection} (P(t),P''(t), F(t), \mu, \Theta_o)$ ;
    $t = t + 1$ ;
end do
```

The following list represents the most significant parameters to define in a $(\mu+\lambda)$-ES algorithm:
- $N_{gen}$ maximum number of generations,
- $\lambda, \mu$ number of offspring and parents in one generation,
- $mut$ mutation operator (with $\Theta_m$ probability),
- $rec$ recombination operator (with $\Theta_r$ probability).

### 3.3 Particle Swarm Optimization [17, 19, 20]

The particle Swarm Optimization (PSO) is an optimization algorithm where, one must have, at a given iteration, a set of solutions or alternatives called “particles”. From one iteration to the following, each particle $X_i$ moves according to a rule that depends on three factors as follows. In order to understand this rule, one must also keep record of the best point $b_i$ found by the particle in its past life and the current global best point $b_g$ find by the swarm of particles in their past life.

The movement rule states that

$$X_i^{t+1} = X_i^t + V_i^{t+1}$$

(7)
where $V_i$ is called the $i$th particle velocity and is defined by:

$$V_i^{new} = w_i V_i + Rnd_i w_1 (b_i - X_i) + Rnd_i w_2 (b_g - X_i)$$  \hspace{1cm} (8)$$

where the first term of the summation represents inertia or habit (the particle keeps moving in the direction it had previously moved), the second represents memory (the particle is attracted to the best point in its trajectory) and the third represents cooperation or information exchange (the particle is attracted to the best point found by all particles).

The parameters $w_1$ and $w_2$ are weights fixed in the beginning of the process. $Rnd_i$ are random numbers sampled from a uniform distribution in $[0,1]$. The following weighting function is usually used in determining $w_{0i}$:

$$w_{0i} = w_{max} - \frac{w_{max} - w_{min}}{iter_{max}} \times iter$$  \hspace{1cm} (9)$$

where $w_{max}$, $w_{min}$ initial and final weights, $iter_{max}$ maximum number of iterations, $iter$ current iteration number.

This function supports the speed of the first iterations.

In addition to $w_{max}$, $w_{min}$, $w_1$, $w_2$ and $iter_{max}$, the according parameters must be fixed:

- $N$ number of variables in the function to be optimized,
- $T_{pop}$ population size.

3.4 Simulated Annealing [17, 21]

The simulated annealing (SA) is a powerful general purpose approach employed for solving combinatorial optimization problems. It is a randomisation algorithm and can asymptotically search for global optimal solution with probability of one. This approach exploits the resemblance between a minimization process and cooling of a molten metal. The energy of a metal becomes minimal when the annealing process is finished. An equivalence is then obtained, i.e. the cost is minimal as the SA approach is applied. A parameter $T$ called temperature is defined in this algorithm. The algorithm of SA can be described as below:

**Step 1** Randomly choose an initial condition (solution).

**Step 2** Generate a feasible point neighbour of the current point from the solution space.

**Step 3** Evaluate the increase in the cost $\Delta C$.

**Step 4** If $\Delta C \leq 0$ then accept the new solution point and go to step 6.

**Step 5** A random number $r$ uniformly distributed in the interval $[0,1]$ is chosen. If $\exp(-\Delta C/T) > r$ then accept a new point, otherwise the new point is discarded.

**Step 6** If the moves are not finished, go to step 2.

**Step 7** Cooling down Temperature, $T = R_f T$.

**Step 8** If $T > T_{min}$, go to step 2.

**Step 9** Output global optimal solution.

Parameters characterizing the simulated annealing program can be summarized below:

- $N$ Number of variables in the function to be optimized,
- $T_0$ Initial temperature,
- $R_f$ Temperature reduction factor,
- $EPS$ Error tolerance for termination,
- $N_c$ Number of cycles. After $N_c * N$ function evaluations, each element of $V_m$ (step length vector) is adjusted so that approximately half of all function evaluations are accepted,
- $N_f$ Number of iterations before temperature reduction. After $N_f * N_c * N$ function evaluations, temperature $T$ is changed by the factor $R_f$,
- $NEPS$ Number of final function values used to decide upon termination.
3.5 Tabu Search [17, 22]

Tabu search (TS) is based on the use of prohibition-based techniques and basic heuristic algorithms like local search. Therefore the main advantage of TS with respect to conventional GA and SA lies in the intelligent use of the past history of the search to affect its future search procedures. Since the method utilizes a tabu list for storing the past history of the search, the efficient structure of the tabu list is important for fast computation. The procedure of TS can be expressed as follows:

- **Step 1) Initialization:**
  Give the initial state, searching point and put the current state into the tabu list.

- **Step 2) Generation and evaluation of neighbouring states:**
  Generate all of possible neighbouring states and check whether the states neighbours are tabu or not.

- **Step 3) Generation of the next state:**
  Move the current state to the next state that is not tabu and have the lowest objective function value. Repeat step 2 and 3 until the convergence criterion is satisfied.

The basic parameters characterizing the algorithm of the tabu search are, namely:

- $N_{div}$ maximum number of diversifications,
- $L$ tabu list size,
- $I_{max}$ maximum number of iterations,
- $M$ number of moves allowed in neighbouring region.

### 4. Simulations and results

#### 4.1 Programs assumptions

For all above outlined procedures, FORTRAN 90 standard codes have been entirely elaborated to run with Power Station MSDEV FORTRAN compiler. All programs have been successfully applied to IEEE 57 bus network [11] and several other networks of different sizes. In this paper, only the simulation results to the main 220/60 kV Algerian Transmission Network are presented.

A fast decoupled load flow (FDL) was used, with tolerances 0.0001 per unit for both real and reactive power mismatches. Any divergence is detectable by the programs themselves. Upper and lower voltage limits at all buses must be given and also limits on all transformers taps. After many tests, it was noticed that the best values of $S_j$, weighting factor of functional constraint violations, range between 5 and 10. In this study, a choice of $S_j = 10$ was made.

#### 4.2 Meta-heuristics control parameters

In each of the above meta-heuristic methods, several parameters must be adjusted in order to find the optimal solution. For a good analysis of the simulation results, we carried out multiple tests while varying through the execution on simple functions, the basic control parameters of the algorithms. As a consequence, choosing suitably control parameters, led to obtain the best optimization results.

For the case of the Algerian network, the control parameters of all methods are summarized in Table 1. These parameters were obtained with a suitable adjustment and after several tests. In the SGA, each gene (in a chromosome) is presented by 10 bits in binary code.

<table>
<thead>
<tr>
<th>Meta-heuristics</th>
<th>Control Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>SGA</td>
<td>$N_{pop}=300, P_c = 0.5, P_m = 0.02$</td>
</tr>
<tr>
<td></td>
<td>$T_{pop} = 30, T_{gen} : uniform$</td>
</tr>
<tr>
<td>SES($\mu + \lambda$)</td>
<td>$N_{pop}=100, \lambda = 100, \mu = 20$</td>
</tr>
<tr>
<td></td>
<td>mut : self adaption, rec : intermediary</td>
</tr>
<tr>
<td>PSO</td>
<td>$T_{pop} = 150, w_{max} = 0.9, w_{min} = 0.4$</td>
</tr>
<tr>
<td></td>
<td>$w_{1} / w_{2} = 1.5, \text{iter}_{\text{max}} = 150$</td>
</tr>
<tr>
<td>SA</td>
<td>$T_0 = 0.4, R_1 = 0.5, EPS = 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>$N = 2, N_f = 5, NEPS = 4$</td>
</tr>
<tr>
<td>TS</td>
<td>$N_{div} = 11, L = 10$</td>
</tr>
<tr>
<td></td>
<td>$I_{max} = 20, M = 10$</td>
</tr>
</tbody>
</table>

#### 4.3 Simulation on the Algerian Electrical Power System

The 220/60 kV Algerian electric system studied contains respectively 114 buses, 159 lines, 16 tap ratio transformers, 15 generation buses (Fig.3).
The size of optimization problem is characterized by 31 control variables, with their upper and lower variables, and 226 equality constraints on the dependent variables. Tests are made for the following limits on voltages (in p.u) and tap transformers:

\[ 0.9 \leq V_L \leq 1.1; \quad 0.9 \leq V_G \leq 1.1; \quad 0.9 \leq T \leq 1.1 \]

The results of simulation were compared with those in previous works [23] obtained by a traditional method (Reduced Gradient).

Figures 4, 5 and 6 (identical color legend) represent simultaneously active losses, voltages magnitudes at controlled buses and transformers ratios before and after optimization for all techniques.

As a first observation, we can say that all meta-heuristics give clearly better results than traditional techniques (such as reduced gradient) especially that the latter are not able to escape from a local minimum.
The second observation is that the simulated annealing and tabu search techniques attain the best results but with a larger computing time for the tabu search (see figure 8). It should be noted that the tabu search method is better familiarized with integer optimization problems, this is why when it is applied to a continuous optimization problem the computation time is much greater.

The obtained results show that the comparison between the meta-heuristic methods and the conventional methods (reduced gradient) should be rather based on the aspect of losses minimization with a full respect of the acceptable limits of the dependent variables, which is clearly demonstrated.

However in terms of computing time, it is obvious that the conventional methods are much faster. But our problem is not involved in real time computation. This is explained by the frequency of execution of the ORPF program at the dispatching centres. This frequency can vary from several minutes to hours. This fact depends on some important factors, such as the load profile variation, the constraint violations, the importance of power loss reduction and/or maintaining an appropriate voltage profile, and the exploitation philosophy dictated by the utility company. It is also possible that different control variables can be adjusted at different frequencies.

In this work, we must also note that the comparative synthesis study applied to the ORPF was limited to the basic meta-heuristic methods. It is still possible to improve all the found results through a better prospection of each one of the above cited techniques, through other alternatives, combinations or hybridization methods.

5. Conclusion

This paper has presented a comparative study of the use of various meta-heuristic techniques to the optimal reactive power flow (ORPF). The elaborated programs were first validated and applied on several networks (such as IEEE 57 bus system and other networks of different sizes). The presented results concern the application simulations on the main Algerian Transmission Network (114 buses).

The analysis of the results show that meta-heuristics give quantitatively satisfactory results compared to those obtained with classical methods (reduced gradient) in terms of active power losses minimization and a good respect of the dependent variables (load buses voltages) to the allowed limits. It is remarked that from all presented techniques, the simulated annealing technique shows to be an excellent method because it is not only simple to use but it obtained the best results in minimizing power systems active losses with a comparatively acceptable time consuming.
References