MATLAB-Simulink Model of a Stand-Alone Induction Generator

Gheorghe Scutaru, Constantin Apostoaia

1 Transilvania University of Brasov - Romania, Faculty of Electrical Engineering and Computer Science, Department of Electrotechnics, 500039 Brasov - Romania, Tel., Fax: 00 40-268-474718, e-mail: scutaru@unitbv.ro
2 Member, IEEE, Purdue University Calumet-USA, Department of Electrical and Computer Engineering, Tel. 001 219-989-2472, Fax: 001 219-989-2898, e-mail: capostoaia@yahoo.com

Abstract — This paper presents modeling, simulation and optimization of an induction generator. The induction machine is represented with a saturation adaptive induction machine dynamic model having state variables expressed in terms of fluxes and reactances. Transients of machine self-excitation under a three-phase load application are simulated using a Simulink block diagram. The magnetization reactance and generated frequency are computed by constrained minimization method for generator operating mode by using the function fmincon from Matlab Optimization Toolbox.

Index Terms — Induction generator, state-space model, constrained minimization method.

I. INTRODUCTION

There is no doubt that so called the problem of “exhaustion of planet classical energy resources” is now very well known. Some practical realizations and numerous scientific papers insisted on the key solution of increasing the efficiency of so called renewable energies of biomass, wind, seas and oceans, earth heat, solar energy, small rivers, etc. Wind turbine based power generation systems are the fastest growing source of renewable energy. During a recent year, the market for small wind systems in the U.S., those with less than 100kW of generating capacity, grew more than 35%. American Wind Energy Association (AWEA) expects continued growth in this market [1].

In the field of electric generators driven by wind turbines the research and development of new control techniques can increase the system efficiency based on variable speed operation mode. An alternative for operation of energetic groups at variable speed may be the use of the induction generator. In numerous studies the induction machines are frequently modeled by means of the space phasors theory applied on the d – q axis models [2] – [8].

Self excited induction generators are today frequently considered as the most economical solution for powering costumers isolated from the utility grid. At the present time self excited induction generator technology is replacing conventional generators based on permanent magnet technology because of their low unit cost, robustness, ease of operation and maintenance. Various dynamic operation regimes of the stand-alone induction generator (i.e., the self excitation process) require advanced new models. Main flux saturation adaptive models have been a subject of great interest in recent years [3], [7], [9] – [13].

In many scientific papers models of the induction machine based on parameters identification algorithms and estimation techniques were reported [13], [14].

Based on adaptive models to parameters variation, different studies of the transients of the induction machine were performed [3], [4], [7], [11], [15] – [18].

The analysis of the steady state regime of the induction generator and the optimization of the machine design were the subject of some studies [19] – [22].

The progress in power electronics reduces the cost of implementing induction generator control, a past disadvantage associated with the use of an induction machine. Secondarily, the electronic power converters can ease integration of wind turbines working in groups.

In the last two or three decades, we have seen extensive research and development efforts for variable-frequency, variable-speed ac machine drive technology. In most cases, new applications use ac drives replacing progressively variable-speed dc drives. Various strategies for induction generator control of power, voltage, frequency, or current, were proposed and some DSP-based systems have been implemented, [4], [6], [7], [11], [12], [18], [23] – [29].

Since the induction generator operating frequency and output voltage are determined by the speed of the generator, the load, and the value of the excitation capacitor, fixed capacitor values can result in unstable power output under changing load conditions. An isolated induction generator can operate in a stable manner with an appropriate control strategy and the
use of solid state power switching devices to regulate the amount of self excitation.

Among all types of ac machines (induction machines, synchronous machines, and variable reluctance machines), the induction machine, particularly the cage type, is very economical, reliable, and is available in the ranges of fractional horse power to multi-megawatt capacity. Low-power machines are available in single-phase, but three-phase machines are used most often in variable-speed drives.

We are interested in this paper to study the generator operating mode of the three-phase induction machine. We perform modeling and simulation studies of a self excited induction generator, the most economical solution for powering costumers isolated from the utility grid [1].

Finally, the aim of this paper is to optimize the design of the induction generator and to describe the simulation results in Matlab-Simulink environment.

II. STAND-ALONE INDUCTION GENERATOR MODEL

A. Induction machine dynamic model

The dynamic d-q model of the induction machine derived in this paper is based on some assumptions: cylinder type rotor, constant air gap, three-phase symmetrical stator and rotor windings (on the cage type rotors, the squirrel cage equivalent to a three-phase symmetrical coil), sinusoidal distribution of the air gap magnetic field (space harmonics are neglected). Rotor variables and parameters are referred to the stator winding, and core losses are neglected.

In d-q synchronously rotating frame, the stator and rotor circuit equations expressed in terms of space phasors are the following [2]:

\[
\begin{align*}
\mathbf{u}_s &= R_s \mathbf{i}_s + \frac{d}{dt} \mathbf{w}_s + j \omega_s \mathbf{w}_s, \\
\mathbf{u}_r &= R_r \mathbf{i}_r + \frac{d}{dt} \mathbf{w}_r + j (\omega_s - \omega) \mathbf{w}_r,
\end{align*}
\]

(1)

(2)

where all the variables and parameters are referred to the stator windings.

The model includes the equation of rotational motion

\[
J \frac{d}{dt} \omega_s + B \omega_r + T_L = T_{em}
\]

(3)

In (3), \( \omega_s = \omega z_p \) is the mechanical angular speed of the rotor, \( z_p \) number of machine pole pairs, \( J \) is the rotor inertia, \( T_L \) is the load torque (negative value for generating operation), and \( T_{em} \) the electromagnetic torque. Other symbols used are explained in the List of Symbols.

In this paper, the dynamic model of induction machine is derived by choosing computation quantities expressed in terms of stator and rotor flux d-q axis components, as state variables.

The state variables expressed in terms of d-q axis flux linkages are defined as [4], [6]:

\[
\begin{align*}
x_1 &= \omega_b y_{sd} ; \\
x_2 &= \omega_b y_{sq} ; \\
x_3 &= \omega_b y_{rd} ; \\
x_4 &= \omega_b y_{rq} ;
\end{align*}
\]

(4)

where \( \omega_b \) is the base angular frequency of the machine.

The d-q axis voltage equations can be expressed by expanding the space phasors, in (1) and (2), in their components [2], and taking into account (4), as follows:

\[
\begin{align*}
\mathbf{u}_s &= R_s \mathbf{i}_s + \frac{1}{\omega_b} \frac{d}{dt} (\mathbf{w}_s) + \frac{\omega_s}{\omega_b} \mathbf{w}_s, \\
\mathbf{u}_r &= R_r \mathbf{i}_r + \frac{1}{\omega_b} \frac{d}{dt} (\mathbf{w}_r) + \frac{\omega_s}{\omega_b} \mathbf{w}_r,
\end{align*}
\]

(5)

(6)

(7)

(8)

For a cage type induction machine it is assumed that \( u_{sd}=u_{sq}=0 \). Because the machine parameters are frequently given in ohms, or per unit of base impedance [6], it is often convenient to express the flux linkage equations in terms of reactances rather then inductances. Therefore, the expressions of state variables are given with

\[
\begin{align*}
x_1 &= X_{ls} i_{sd} + x_6 , \\
x_2 &= X_{ls} i_{sq} + x_7 , \\
x_3 &= X_{lr} i_{rd} + x_6 , \\
x_4 &= X_{lr} i_{rq} + x_7 ,
\end{align*}
\]

(9)

(10)

(11)

(12)

where

\[
\begin{align*}
X_{ls} &= \omega_b L_{ls} ; \\
X_{lr} &= \omega_b L_{lr} ; \\
X_m &= \omega_b L_m ,
\end{align*}
\]

and

\[
\begin{align*}
x_6 &= X_m (i_{sd} + i_{rd}) , \\
x_7 &= X_m (i_{sq} + i_{rq}).
\end{align*}
\]

(13)

(14)

The currents can be expressed in terms of the state variables as

\[
\begin{align*}
i_{sd} &= \frac{x_1 - x_6}{X_{ls}} ; \\
i_{sq} &= \frac{x_2 - x_7}{X_{ls}} ;
\end{align*}
\]

(15)
The expressions of $x_6$ and $x_7$ have to be written in terms of chosen state variables, and therefore, are derived, by substituting (15)-(16), in (13)-(14), as

$$x_6 = \frac{X_{m1}}{X_{lr}} x_1 + \frac{X_{m1}}{X_{lr}} x_3,$$

$$x_7 = \frac{X_{m1}}{X_{lr}} x_2 + \frac{X_{m1}}{X_{lr}} x_4,$$

where the following notation is used [4]:

$$X_{m1} = \left( \frac{1}{X_m} + \frac{1}{X_{ls}} + \frac{1}{X_{lr}} \right)^{-1}.$$ 

The $d$-$q$ axis voltage equations of the induction machine are derived in state-space form, by substituting the current equations (15), (16) into the voltage equations (5) – (8), as

$$\frac{dx_1}{dt} = \omega_b \left[ -\frac{R_s}{X_{ls}} x_1 + \frac{\omega_b}{X_{ls}} x_2 + \frac{R_r}{X_{lr}} x_6 + u_{sd} \right],$$

$$\frac{dx_2}{dt} = \omega_b \left[ -\frac{\omega_b}{X_{ls}} x_1 - \frac{R_s}{X_{ls}} x_2 + \frac{R_r}{X_{lr}} x_7 + u_{sq} \right],$$

$$\frac{dx_3}{dt} = \omega_b \left[ -\frac{R_r}{X_{lr}} x_3 + \frac{(\omega_b - \omega)}{X_{lr}} x_4 + \frac{R_r}{X_{lr}} x_6 \right],$$

$$\frac{dx_4}{dt} = \omega_b \left[ -\frac{R_r}{X_{lr}} x_3 - \frac{(\omega_b - \omega)}{X_{lr}} x_4 + \frac{R_r}{X_{lr}} x_7 \right].$$

The expressions of $u_{sd}$ and $u_{sq}$ are presented in the load model (25), (26), as additional state variables.

The electromagnetic torque expression [7], is given, taking into account Eqs. (4), as follows

$$T_{em} = \frac{3}{2} z_p \frac{L_m}{L_{lr}} \frac{1}{\omega_b} \left( i_{sq} x_3 - i_{sd} x_4 \right)$$

Thus, the equation governing the rotational motion can be expressed by (24), in which the viscous frictional torque was neglected, the fifth state variable $x_5$ represents the electrical rotor angular speed $\omega$, and the current expressions are given in (15).

$$\frac{dx_5}{dt} = \frac{3}{2} \frac{z_p^2}{J} \frac{X_m}{X_{lr}} \frac{1}{\omega_b} \left( i_{sq} x_3 - i_{sd} x_4 \right) - \frac{z_p}{J} T_L,$$

The fifth order dynamic model of the induction machine, having the state variables defined in (4) and written in terms of reactances, is formed by (19) – (22), and (24).

B. Excitation capacitor and load impedance model

An induction generator may be self-excited by providing the magnetizing reactive power by a capacitor bank [30], as is shown in Fig.1 where a stand-alone induction generator under a series resistive-inductive load is drawn.

The capacitor and load equations of the induction generator are represented in the same reference frame as the generator [12]. These expressions form a new set of state-equations and can be derived by the generator stator currents, $i_s$, capacitor currents, and load currents $i_L$, as follows

$$\frac{di_L}{dt} = \frac{1}{L_L} \left( i_s - i_L \right),$$

$$\frac{di_S}{dt} = \frac{1}{L_L} \left( i_s - i_L \right).$$

Fig. 1. Equivalent single-phase $R$-$L$ load circuit of a three-phase induction generator

![Diagram](image-url)
\[
\frac{du_{sd}}{dt} = -\frac{1}{C}i_{sd} + \frac{1}{C}i_{Ld}, \quad (25 \text{a})
\]
\[
\frac{du_{sq}}{dt} = -\frac{1}{C}i_{sq} + \frac{1}{C}i_{Lq}, \quad (25 \text{b})
\]
\[
\frac{di_{Ld}}{dt} = \frac{1}{L}(u_{sd} - Ri_{Ld}) + \omega_L i_{Lq}; \quad (26 \text{a})
\]
\[
\frac{di_{Lq}}{dt} = \frac{1}{L}(u_{sq} - Ri_{Lq}) - \omega_L i_{Ld}. \quad (26 \text{b})
\]

Finally a ninth order dynamic model of the induction generator and load system is described by the system of equations (19) – (26), Fig. 2 shows a Simulink block diagram of this model.

An user-definable S-Function block labeled “Induction Generator” is used in Fig.2 to model the system of self-excited induction generator. The numerical values of induction machine parameters (given in Appendix), \( R_m, X_m, X_s, R_s, X_d, \) and \( J \) are specified in ‘S-Function parameters’ field.

A Matlab M-file S-Function was created to model the state-space representation of the induction generator system. The S-Function is built by using the template sfunmpl.m available in the standard Simulink installation [31]. The induction generator system in Fig. 2 consists of one input, that is the shaft torque \( T_s \) provided by a prime mover (i.e. wind turbine), and nine state variables, \( x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9 \) and \( i_{Lq} \).

**C. Induction generator steady-state model optimization**

The well known steady state model of the induction machine can be derived by substituting the time derivative terms to zero in the dynamic model equations. Therefore we derive the steady state model by substituting the space phasors by the corresponding rms phasors in (1) and (2) as

\[
U_s = R_s I_s + j\omega_L \Psi_s, \quad (27)
\]
\[
U_{r} \bigg|_{t=0} = R_r I_r + j(\omega_L - \omega)\Psi_r. \quad (28)
\]

In (28) we substitute the slip frequency, \( \omega_{sl} = \omega_L - \omega \), using the definition of per unit slip \( s \) as \( s = \omega_{sl} / \omega_L \), and rewrite as

\[
s = \frac{R_r}{R_m} I_r + j \omega_L \Psi_r. \quad (29)
\]

The model expressed by (27) and (29) satisfy the well known steady state equivalent circuit of an induction machine, if the parameter \( R_m \) of equivalent resistance for core loss is neglected.

We consider a three-phase self-excited induction generator to which a prime mover is coupled mechanically and operating in parallel with a three-phase capacitor bank capable of supplying the necessary magnetizing current. The generator operating frequency and voltage are determined by the speed of the generator, its load, and the capacitor rating. As for the dc shunt generator, for the induction generator to self-excite, its rotor must have sufficient remnant flux. Since the machine is driven at variable speed, and thus operates at a variable stator frequency \( f = \omega_s / 2\pi \), it is convenient to use in analysis the machine equivalent circuit where all the parameters are referred to base rated frequency, assuming that all inductive reactances are proportional to the frequency. The per-phase steady-state equivalent circuit of a self-excited induction generator supplying a balanced R-L load is shown in Fig. 3.

Defining the base impedance in ohms, \( Z_b = V_b / I_b \), all resistances and reactances of the equivalent circuit in Fig. 3 are expressed in per units (e.g. stator resistance, \( R_s \) (p.u.) = \( R_s / Z_b \)), and \( V_o \), \( V_r \) are the air gap and output voltages respectively.

The symbols \( F \) and \( u \) in Fig. 3, have the following significance:

\( F \) is the ratio of the generated frequency to base frequency, \( F = f / f_b \);

\( u \) is the ratio of the actual rotor speed to the synchronous speed corresponding to base frequency, \( u = \omega_r / \omega_s \).

If \( n \) is the actual rotor speed in rpm, the slip \( s \) can be expressed as \( s = (f - n/60) / f = (F - u) / F \).

For the machine to operate, \( X_m \) must have a value in the saturated region of the magnetization characteristic. That is the bound \( 0 < X_m < X_s \), and \( X_s \) is the (p.u.) unsaturated magnetizing reactance.

Because the value of slip is negative for generator action this gives the bound \( 0 < F < u \), on the range of generated frequency.

**D. Analysis by Constrained Minimization Method**

The steady state analysis of three phase self-excited induction generator can be formulated as a numerical multidimensional optimization problem, where no detailed derivation of analytical equations is needed [19]. An efficient method, using several-variable constrained minimization, solves the problem directly.

This method returns the parameters of interest by minimizing a cost function.
In Fig. 3 all the circuit parameters are assumed to be constant and are independent of saturation except the magnetizing reactance \( X_m \). Core loss and effect of harmonics in the machine are neglected.

Under steady state conditions, KVL applied in Fig.3 gives \( I_r Z_{total} = 0 \).

The total impedance \( Z_{total} \) is computed considering the following impedance definitions

\[
Z_r = \frac{R_r}{F} + jX_r; \quad Z_m = jX_m;
\]

\[
Z_s = \frac{R_s}{F} + jX_s;
\]

\[
Z_L = \frac{R_L}{F} + jX_L; \quad Z_C = \frac{jX_C}{F^2};
\]

\[
Z_{eq} = \frac{Z_r Z_m}{(Z_r + Z_m)};
\]

\[
Z_{eq}^\prime = Z_s + Z_{eq}^\prime; \quad Z_{eq}^\prime = \frac{Z_r Z_C}{(Z_L + Z_C)};
\]

\[
Z_{total} = Z_{eq}^\prime + Z_{eq}^\prime
\]

Since \( I_s \neq 0 \), it implies that \( Z_{total} = 0 \) in equation \( I_r Z_{total} = 0 \), or

\[
\text{Re}(Z_{total}) = 0, \quad \text{Im}(Z_{total}) = 0.
\]

As a consequence, the total impedance will be considered as an objective function and the equations \( \text{Re}(Z_{total}) = 0 \) and \( \text{Im}(Z_{total}) = 0 \) can be solved simultaneously for two unknowns[19].

In this paper the constrained minimization method is applied to find simultaneously the values of the frequency \( F \), and of the magnetization reactance \( X_m \) by minimizing the total impedance, \( Z_{total} \). Gradient optimizers, such as those built in the Optimization Toolbox from Matlab, can be used for this purpose. In our analysis, the function \textit{fmincon} available in Matlab 6.x versions is used.
III. STAND-ALONE INDUCTION GENERATOR
SIMULATION

Fig. 4 shows the simulation results of the stator voltage build-up and the stator current transients of the induction generator described by the system of equations (19) - (26) and the machine data given in Appendix. The torque independent input is of -2.5 Nm, the excitation capacitance C = 180 µF, the load R=7000Ω, and L=10H.

![Stator phase voltage build-up and stator phase current transients](image)

There are three external elements of self-excited induction generator that can be controlled; these are the speed, excitation capacitance, and load. Changing any one of these elements will change magnetizing reactance \( X_m \) and frequency \( F \).

Fig. 5 and Fig 6 show the variations of \( X_m \) and \( F \) versus motor speed \( n \), based on constrained minimization method presented in section II.C.

Fig. 5 gives the speed bounds \( v_l, v_u \) where below or above these values \( X_m > X_0 \), the machine is not saturated and consequently will not operate. As can be seen from Fig. 5, \( X_m \) changes in a convex manner in a speed range \( v_l < v < v_u \). The output voltage and power will be zero outside of this speed range, and between these two extremes a maximum for both the output voltage and power exists.

Once the values of frequency \( F \) and magnetization reactance \( X_m \) (or capacitive reactance \( X_C \) ) are computed simultaneously by minimizing the objective function of total impedance \( Z_{total} \), the equivalent circuit in Fig. 3 is to be solved to determine the generator performance [21]. This can be obtained from the magnetization characteristic of the machine. Typically the function \( \Psi_m = f(i_m) \) has the general shape shown in Fig. 7 which was obtained by cubic spline experimental data interpolation considering the induction machine in Appendix.

![Variation of \( X_m \) (p.u) versus speed](image)

![Variation of frequency (p.u.) \( F \) versus speed](image)
IV. CONCLUSION

In this paper, two approaches were used in order to achieve the proposed goal of the study, that is, to add a contribution to the design optimization of the induction generator. On one hand, a flexible dynamic model is designed for transients simulation purposes. On the other hand, a numerical constraint minimization method is applied with the aim of machine’s parameter design optimization. For the dynamic model, the contribution of this study is the creation of the induction generator subsystem in the Simulink block diagram (Fig. 2). This subsystem is based on an original Matlab source code written to build an m.file S-Function which models the state space model of the induction generator. The transients of the stator phase voltages build-up and stator phase currents were simulated and presented in Fig. 4. The subsystem block of the induction generator is flexible and easy to be inserted in future control systems of the induction machine.

In order to determine optimum values of some parameters an objective function, based on the steady state model shown in Fig. 3, is minimized. In this study, a new Matlab script file was created by using the function fmincon available in the Optimization Toolbox of Matlab 6.x versions.

The results of this study will be used in future research to design a prototype model of an isolated induction generator, to design a vector control system of a stand-alone induction generator system, and to model and simulate the overall energy conversion system containing a cluster of machines “turbine-induction generator-controller-grid”.

APPENDIX

A. List of Symbols

$s, r$ : subscripts, denoting stator and rotor quantities

$u, i, \psi$ : transient voltages, currents, and fluxes

$\psi_{sd}, \psi_{sq}, \psi_{rd}, \psi_{rq}$ : $d$-$q$ components of stator and rotor transient flux linkages

$L_{sl}, L_{sr}$ : stator (rotor) leakage inductances

$L_{mq}, L_{mr}$ : magnetization inductance

$X_{sl}, X_{sr}$ : stator (rotor) leakage reactances

$X_{mq}, X_{mr}$ : magnetization reactance

$u_{sd}, u_{sq}, u_{rd}, u_{rq}$ : $d$-$q$ components of stator and rotor transient voltages

$i_{sd}, i_{sq}, i_{rd}, i_{rq}$ : $d$-$q$ components of stator and rotor transient currents

$l_{sd}, l_{sq}$ : $d$-$q$ components of transient load currents
B. Induction machine data

\[ P_N = 1.1 \text{ kW}; \quad V_b = 220 \text{ V}; \quad I_b = 2.2 \text{ A}; \quad z_p = 2; \quad \text{star}; \quad T_{emN} = 5 \text{ Nm}; \quad n_b = 1500 \text{ rpm}; \quad Z_b = 100 \Omega; \quad J = 0.0106 \text{ kgm}^2; \quad R = 7.78 \Omega; \quad R_s = 7.10 \Omega; \quad X_m = 10.62 \Omega; \quad X_b = 6.28 \Omega; \quad X_m = 142 \Omega. \]

REFERENCES