MPC-SVPWM based Controller for PMSM Drives

Thangarajan, K*, Soundarajan, A**
*RVS College of Engineering and Technology, Coimbatore, India. Email:thangajansk@gmail.com
** PSG College of Technology, Coimbatore, India

Abstract- In this paper, a model predictive control based drive for the permanent magnet synchronous motor is proposed. This proposed method is suggested to achieve high dynamic torque in the PMSM. The control variable’s long run behavior is predicted by the MPC within the time domain of the control system and the selection of the optimization cost function. This method combines the advantages of the MPC and SVPWM technique for PMSM drive to overcome the variations in the electromagnetic variables and also supplies fixed frequency for the inverter. The effectiveness of the proposed method is analyzed and demonstrated in the simulations. The experimental setup is done with a frequency inverter that controls the PMSM motor. The dead-time for the inverter is fixed to prevent a shoot-through fault. The experimental results are presented which shows the PMSM performance.

Keywords- Model Predictive Controller (MPC), Space Vector Pulse Width Modulation SVPWM, Permanent Magnet Synchronous Motor (PMSM).

I. INTRODUCTION

PMSM found wide application where variable torque and speed control is required. PMSM has high dynamic performance, torque to weight ratio and high power density which needs precise dynamic control methods. In general Direct Torque Control (DTC) and Field Oriented Control (FOC) are the two famous speed control methods for the PMSM drives.

The principle of DTC control is based on the values of flux, torque and rotor position estimation. This method does not need any sensors to measure the mechanical position of rotors. DTC has the advantages over the other methods are no need of current controllers position sensors and transformation networks. It has the drawbacks of high current ripples, torque ripples variable switching frequency and speed control in low speed is difficult [6-7][10].

FOC Technique is to control torque and flux by controlling its corresponding current components which are derived from stationary reference frame to rotating reference frame. The advantages of FOC is independent flux and torque control can be achieved. It has fast dynamic torque comparatively with DTC and better steady and transient state response [12]. High torque and low current during starting of the motor can be achieved in this method.

MPC is more suitable in parameter sensitivity evaluation than other methods. MPC combines the advantage of DTC and classical FOC. The cost function design strategy in MPC will be chosen based on the power rating. In high power rating, the switching losses is to be reduced by reducing the switching frequency and in low power application is to achieve high dynamics [8-9]. MPC control for PMSM are presented in [1-5]. The main focus of this paper is designing and implementing a high performance MPC-SVPWM control for PMSM and the results are compared with classical FOC.

II. MODELLING OF PMSM

The model of PMSM drive covers the current and voltage relations at equilibrium and individual phase voltage distribution [11]. The model has no of parameters and thus notations are as follows.

\[ v_d \] - $d$-axis stator voltages
\[ v_q \] - $q$-axis stator voltages
\[ i_d \] - $d$-axis stator currents
\[ i_q \] - $q$-axis stator currents
\[ \phi_d \] - $d$-axis flux linkages
\[ \phi_q \] - $q$-axis flux linkages
\[ r_s \] - Stator resistance
\[ \theta \] - Rotor angle
\[ L_d \] - $d$-axis synchronous inductances
\[ L_q \] - $q$-axis synchronous inductances
\[ \phi_f \] - Permanent magnetic flux
\[ \omega_r \] - Rotor speed
\[ T_e \] - Electromagnetic torque
\[ T_l \] - Load torque
\[ \omega_t \] - Mechanical speed of rotor
\[ J \] - Moment of inertia
\[ B \] - Shaft friction coefficient
\[ T_{e}^* \] - Reference torque
\[ \phi_{e}^* \] - Reference flux
\[ T_{e}^{k+1} \] - Predicted torque
\[ \phi_{e}^{k+1} \] - Predicted flux
\[ k_i \] - Weighting factor
\[ v_{d}^{k}, v_{q}^{k} \] - Stator voltages at instant $k$
\( i_d^k, i_q^k \) - Stator currents at instant \( k \)
\( \omega_e^k \) - Rotor speed at instant \( k \)
\( T_s \) - Sampling time
\( T_{e}^{k+2} \) - Electromagnetic torque at instant \( k + 2 \)
\( \phi_e^{k+2} \) - Stator flux at instant \( k + 2 \)
\( V_{a}, V_{b}, V_{c} \) - Voltage vectors
\( T_{4}, T_{6}, T_{0} \) - Time periods
\( V_{dc} \) - reference DC voltage
\( V_{s\text{ref}}, V_{d\text{ref}} \) - components of reference voltage

The PMSM is described by the rotating d-q reference frame axis voltage equations

\[
\begin{align*}
    v_d &= r_i i_d + \frac{d\phi_d}{dt} + \phi_d \frac{d\theta}{dt} \\
    v_q &= r_i i_q + \frac{d\phi_q}{dt} + \phi_d \frac{d\theta}{dt}
\end{align*}
\]

\( (1) \) and \( (2) \)

The d-axis and q-axis fluxes are linkages given by

\[
\begin{align*}
    \phi_d &= L_d i_d + \phi_f \\
    \phi_q &= L_q i_q
\end{align*}
\]

\( (3) \) and \( (4) \)

The equations \( (1) \) and \( (2) \) can be written as

\[
\begin{align*}
    v_d &= r_i i_d + L_d \frac{di_d}{dt} - \omega_e L_q i_q \\
    v_q &= r_i i_q + L_q \frac{di_q}{dt} + \omega_e L_d i_d + \omega_e \phi_f
\end{align*}
\]

\( (5) \) and \( (6) \)

The mechanical torque of the model can be

\[
T_e = T_i + J \frac{d\omega_e}{dt} + B\omega_e
\]

\( (7) \)

The electromagnetic torque is derived as

\[
T_e = \frac{3}{4} p(\phi_d i_q + \phi_q i_d)
\]

\( (8) \)

Substituting \( (3) \) and \( (4) \) to \( (8) \), \( T_e \) can be rewritten as

\[
T_e = \frac{3}{4} p(\phi_d i_q + (L_d - L_q) i_d i_q)
\]

\( (9) \)

The electromagnetic torque is the combination of permanent magnet torque and reluctance torque. The synchronous inductance of the PMSM equals the \( d \) and \( q \) axis inductance. Neglecting the reluctance torque, electromagnetic torque can be rewritten as

\[
T_e = \frac{3}{4} p(\phi_d i_q)
\]

\( (10) \)

III. MPC-SVPWM CONTROL OF PMSM

The proposed MPC-SVPWM method fed PMSM drive shown in Fig. 1. The comparison is made between reference and actual speed. The error in speed is given to PI controller and the torque reference is given to MPC. According to the MTPA principle stator flex reference is calculated and fed to MPC controller. The voltage vector selected by SVPWM reduces the difference between the predicted and reference values.

![Fig. 1. Proposed MPC-SVPWM fed PMSM drive](image)

The factors considered as cost function of MPC are torque and the stator flux \([12-14]\). These cost functions should made to approach reference values at the end of the control period. The cost function \( G \) is given by

\[
\min G = \left| T_e' - T_e^{k+1} \right| + k_1 \left| \phi_e' - \phi_e^{k+1} \right|
\]

\( (11) \)

The stator currents in the reference frame are given by

\[
\begin{align*}
    \frac{di_d}{dt} &= -r_i i_d + \omega_e L_q i_q + v_d \\
    \frac{di_q}{dt} &= -r_i i_q + \omega_e L_d i_d + v_q - \omega_e \phi_f
\end{align*}
\]

\( (12) \) and \( (13) \)

At instance \( k+1 \) the currents are derived as

\[
\begin{align*}
    i_d^{k+1} &= i_d^k + \left( \frac{r_i i_d^k + \omega_e^k L_q i_q^k + v_d^k}{L_d} \right) T_s \\
    i_q^{k+1} &= i_q^k + \left( \frac{-r_i i_q^k - \omega_e^k L_d i_d^k + v_q^k - \omega_e^k \phi_f}{L_q} \right) T_s
\end{align*}
\]

\( (14) \) and \( (15) \)

At instance \( k+1 \) the electromagnetic torque flux linkages are derived as

\[
\begin{align*}
    \phi_d^{k+1} &= L_d i_d^{k+1} + \phi_f \\
    \phi_q^{k+1} &= L_q i_q^{k+1}
\end{align*}
\]

\( (16) \) and \( (17) \)

\[
T_e^{k+1} = \frac{3}{4} p(\phi_d^{k+1} i_q^{k+1} + \phi_q^{k+1} i_d^{k+1})
\]

\( (18) \)

To eliminate the digital implementation step delay, the cost function can be changed

\[
\min G = \left| T_e' - T_e^{k+1} \right| + k_1 \left| \phi_e' - \phi_e^{k+1} \right|
\]

\( (19) \)

A two-step prediction is required to obtain this minimum cost function.
IV. VOLTAGE SPACE VECTOR MODULATION

The phase voltages for the PMSM are generated using SVPWM and are applied to the stator phases. The space vector plane is divided by six sectors. From these sectors, the voltage vectors are generated as shown in Fig. 2.

![Voltage Vectors](image)

Consider the reference voltage vector $V_{sref}$ in the $3^{rd}$ sector of the space vector plane. All the sectors are given the application time which is derived as in equation (20). The reference voltage vector $V_{sref}$ can be given as in (21).

\[ T = T_a + T_b + T_0 \]  
\[ V_{sref} = \frac{T}{3} V_a + \frac{T}{3} V_b V_c \]  

The application time of the adjacent vector is given as

\[ T_a = \frac{T}{2V_{dc}} (3V_{sref} - \sqrt{3} V_{sref}) \]  
\[ T_b = \sqrt{3} \frac{T}{V_{dc}} V_{sref} \]

The variable associated with vector application time is expressed as

\[ X = \sqrt{3} \frac{T}{V_{dc}} V_{sref} \]  
\[ Y = \frac{\sqrt{3} T}{2V_{dc}} V_{sref} + \frac{3 T}{2V_{dc}} V_{sref} \]  
\[ Z = \frac{\sqrt{3} T}{2V_{dc}} V_{sref} - \frac{3 T}{2V_{dc}} V_{sref} \]

With the knowledge of each sector, the voltage vectors are defined by $V_{sref}$ and $V_{sref}$. The boundary vector durations are tabulated in the Table 1.

<table>
<thead>
<tr>
<th>Sector</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
<th>v</th>
<th>vi</th>
</tr>
</thead>
<tbody>
<tr>
<td>t₁</td>
<td>z</td>
<td>y</td>
<td>-z</td>
<td>x</td>
<td>y</td>
<td>-y</td>
</tr>
<tr>
<td>t₂</td>
<td>y</td>
<td>-x</td>
<td>x</td>
<td>z</td>
<td>-y</td>
<td>-z</td>
</tr>
</tbody>
</table>

The calculated duty cycles are

\[ t_{A_{ON}} = \frac{T - T_a - T_b}{2} \]  
\[ t_{B_{ON}} = t_{A_{ON}} + T_a \]  
\[ t_{C_{ON}} = t_{B_{ON}} + T_b \]  

Table. 2 represents the duty cycle according to the sector for the motor phases.

<table>
<thead>
<tr>
<th>Sector / Phase</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
<th>v</th>
<th>vi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sₐ</td>
<td>tₐ,ON</td>
<td>tₐ,ON</td>
<td>tₐ,ON</td>
<td>tₐ,ON</td>
<td>tₐ,ON</td>
<td>tₐ,ON</td>
</tr>
<tr>
<td>Sₙ</td>
<td>tₙ,ON</td>
<td>tₙ,ON</td>
<td>tₙ,ON</td>
<td>tₙ,ON</td>
<td>tₙ,ON</td>
<td>tₙ,ON</td>
</tr>
<tr>
<td>Sₙ</td>
<td>tₙ,ON</td>
<td>tₙ,ON</td>
<td>tₙ,ON</td>
<td>tₙ,ON</td>
<td>tₙ,ON</td>
<td>tₙ,ON</td>
</tr>
</tbody>
</table>

V. SIMULATION OF THE PROPOSED SYSTEM

The proposed method of MPC-SVPWM for the PMSM is modelled using MATLAB/Simulink. The simulation is done with 20 kHz of sampling frequency. A VSI with IGBT switches is fed from a 380V, 50Hz supply in modelled. The PMSM model using d-q reference frame is fed from the inverter. The Table 3 shows the motor parameters used in the simulation. The DC link capacitance across the inverter is fixed as 1F and the speed is set to 3600 rpm. The difference between actual and reference speed is given the PI controller which generates the reference torque $T_{r}$ and stator reference flux $\phi_{s}$. The three phase voltage $V_{a}$, $V_{b}$, and $V_{c}$ are obtained using park transformation a constant value of torque is given. The two phase current $i_{d}$ and $i_{q}$ are obtained from rotor speed, electromagnetic torque $T_{e}$, and the rotor position using equation (5), (6), (7) and (9). By using inverse park transformation the two phone current are converted as three phase currents $i_{a}$, $i_{b}$ and $i_{c}$.

From the equation (14) and (15) the current prediction values are derived. Similarly for the voltages the current values $i_{d}$, $i_{q}$ are replaced by $i_{d}^{{k+1}}$, $i_{q}^{{k+1}}$. The second step prediction is also done by replacing $i_{a}^{k}$, $i_{b}^{k}$ at instant $k+2$ by $i_{a}^{k+1}$, $i_{b}^{k+1}$ in the equation (14) and (15).
The torque and flux prediction can also be done by equations (16), (17), and (18). The cost function optimization has been obtained from (19). The equation (24), (25), and (26) gives the commutation durations of the switches. The boundary vectors duration is calculated as per the Table 2 and also the switching sequences are computed using (27), (28), and (29).

The Fig.3, Fig.4, Fig.5, and Fig.6 show stator currents, switching pulses, speed, and torque wave forms.

For surface PMSM, the relationship between $T_e^*$ and $\phi_s^*$ is

$$|\phi_s^*| = \sqrt{\phi_f^2 + \left(\frac{L_s T_e^*}{3 P\phi_f}\right)^2} \quad (30)$$

\[\text{Fig. 3. Simulation response of Phase currents the PMSM drive with MPC-SVPWM}\]

\[\text{Fig. 4. Switching Pulses to the Inverter}\]

\[\text{Fig. 5. Simulative response of Torque}\]

\[\text{Fig. 6. Simulation response of Speed of the PMSM with MPC-SVPWM}\]

\[\text{VI. EXPERIMENTAL RESULTS}\]

The experimental setup for the proposed method is shown in Fig.13. The steady state and transient state performance is investigated and compared with classical FOC. To implement the control algorithm ARM cortex M3 micro controller which operates at a CPU frequency of 100MHz. The advantages of this microcontroller are computation speed, separate local instruction and peripheral bus. For the speed measurement an incremental optical encoder is used and 100 micro second sample time is set for all the experiments. The experimental results are plotted as shown in Fig.7, Fig.8, and Fig.9.
It can be said that the predictive controller implementation is relatively easy compared with the classic FOC scheme as shown in Fig.10, Fig.11 and Fig.12. The predictive controller is based on the model of the motor drive in order to predict the future value. Therefore, the main difficulty comes in the precision of the model parameters. According to the experimental results, although in steady state FOC is still better, MPC has an acceptable steady state performance with a low current ripple and an improved transient response. MPC-SVPWM is proved as a good alternative to FOC with an improved torque and dynamic performance.

### Table 3. Motor Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>KW</td>
<td>2.2 kw</td>
</tr>
<tr>
<td>Rated torque</td>
<td>5.8 N-m</td>
</tr>
<tr>
<td>Rated Current</td>
<td>4.5 Amps</td>
</tr>
<tr>
<td>Speed</td>
<td>3600 rpm</td>
</tr>
<tr>
<td>B \text{EMF}</td>
<td>283 V</td>
</tr>
<tr>
<td>Frequency</td>
<td>180 Hz</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>13.3 mH</td>
</tr>
<tr>
<td>Stator Resistance</td>
<td>2.8 ohm</td>
</tr>
</tbody>
</table>

### VII. CONCLUSION

MPC SVPWM is chosen in this paper among many control schemes available in PMSM drives, because of its simplicity in the implementation and flexibility. The MPC-SVPWM and classical FOC methods are considered for the comparison. Both the controller were modelled in MATLAB / Simulink and the results are compared. The designs are also implemented in laboratory. Based on the experimental results the MPC – SVPWM method found as the better replacement for the classical FOC.
REFERENCES


