An Extended Kalman Filter For Sensorless Direct Torque and Field Controlled PMSM Speed Drive Using SVM Approach

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Abstract— In this paper a speed sensorless Direct Torque and Flux Control (DTFC) method based on space vector modulation (SVM) and extended Kalman Filter (EKF) is proposed. This method is based on the PMSM models in the coordinate of stator flux linkage, flux and torque are controlled through stator voltage components in stator flux linkage coordinate axes and space vector modulation is used to control inverters. Simulation results verify that this proposed sensorless DTFC scheme could effectively decrease flux and torque ripples, and the speed estimation method could accurately track the motor speed and has good dynamic and static performance.

Index Terms—Permanent magnet synchronous motor, direct torque control, space vector pulse width modulation, Kalman filter.

I. INTRODUCTION

The first and most popular vector control (VC) method was field oriented control (FOC) [1]. This vector control technique is one of the most important closed loop techniques for AC machines in variable speed applications. Using this control technique, the torque and flux can be decoupled so each can be controlled separately. In recent years mid 80’s a direct torque control (DTC) has become an alternative to the well known vector control. It was introduced in Japan by Takahashe (1984) and also in Germany by Depenbrock (1985) [2] [3]. The main feature of DTC is the high performance achieved with a simpler structure and control diagram. But it presents a problem of field linkage and torque ripple. In order to solve this problem the conventional DTC is combined with space vector pulse width modulation (SVPWM). This control theory has achieved great success in the control of PMSM. That has becoming a hotspot for resolving... Both methods, DTC, achieve decoupled control of torque and flux and it was first implemented in the control of induction motor drives [4][5][6]. It was recently also applied to different machines such as PMSM drives and Brushless DC motors [7-11].

In this paper, a DTC design for PMSM speed control is proposed. The EKF is adopted to estimate the speed and the torque. That is robust to the parameter uncertainties and load torque disturbance. The rest of this paper is organized as follows. Section 2 reviews the PMSM modeling. Section 3 shows the principle of the DTC and SVM-DTC. Section 4 resumes the EKF theory. Section 5 gives some simulation results. Finally, the conclusion is drawn in section 6.

II. MATHEMATICAL MODEL OF THE PMSM

The PMSM stator field of is obtained from the following equation:

\[ \vec{V}_s = R_s \vec{I}_s + \frac{d\vec{\phi}_s}{dt} \]  

We obtain

\[ \vec{\phi}_s = \vec{\phi}_{so} + \int (\vec{V}_s - R_s \vec{I}_s) \ dt \]  

The voltage drop due to the stator resistance can be neglected (for high speeds) we find then:

\[ \vec{\phi}_s \approx \vec{\phi}_{so} + \int \vec{V}_s \ dt \]  

For one sampling period, the voltage vector applied to the PMSM remains constant, we can write then:

\[ \vec{\phi}_s(K + 1) \approx \vec{\phi}_{so}(K) + \vec{V}_s T_e \]  

Where still:

\[ \Delta \vec{\phi}_s \approx \vec{V}_s T_e \]  

With:

• \[ \vec{\phi}_s(K) \]: the field stator vector of the current sampling
• \[ \vec{\phi}_{so}(K + 1) \] the field stator vector of following sampling
• \[ \Delta \vec{\phi}_s \] The variation of the stator field vector
• \[ (\vec{\phi}_s(K + 1) - \vec{\phi}_s(K)) \]
• \[ T_e \]: The sampling period

For one constant sampling period \[ \Delta \vec{\phi}_s \] is proportional to stator applied voltage vector of the MSAP [12-13]. Figure 1 shows the evolution of the stator field vector in the \((\alpha, \beta)\) plan.

![Fig.1 Stator field vector evolution in the (\(\alpha, \beta\)) plan](image)

The electromagnetic torque is proportional to the vector product between the vectors of stator field and rotor one according to the follow expression [7]:

\[ T_e = k(\vec{\phi}_s \times \vec{\phi}_r) = \frac{r}{\tau_q} |\vec{\phi}_s| |\vec{\phi}_r| \sin \delta \]  

Such as:

• \[ \vec{\phi}_s \]: The stator field vector
• \[ \vec{\phi}_r \]: The rotor field vector brought back to the stator;
• \[ \delta \]: The angle between the stator field vectors and rotor one.
### III. DTC AND DTC - SVPWM DEVELOPMENT

The basic concept of DTC is to control directly both the stator flux and electromagnetic torque of a machine simultaneously by the selection of optimum inverter switching modes. The use of a switching table for voltage vector selection provides fast torque response, low inverter switching frequency and low harmonic losses without the complex field orientation by restricting the flux and torque errors within respective flux and torque hysteresis bands with the optimum selection being made[14]. The DTC controller consists of two hysteresis comparator (flux and torque) to select the switching voltage vector in order to maintain flux and torque between upper and lower limit as show in figure 2.

![Fig. 2 block diagramm of DTC for PMSM](image)

The six different directions of $V_s$ noted as $V_i$ the combination of switches status of the inverter are given shown in figure 3 represents the choice of this vector selection.

![Fig. 3 Inverter output voltage vectors](image)

The develop torque control of inverter fed PMSM is carried out by hysteresis control of magnitude stator flux and torque that selects one of the voltage vectors. The selection is made in order to maintain torque and flux error inside the hysteresis band in which the errors are indicated by $\Delta T_e$ and $\Delta \phi$ respectively.

- $\Delta T_e$: Torque error given by $T_{ref} - T_e$
- $\Delta \phi$: Field error given by $\phi_{ref} - \phi$

Then we can write the different rules for $\Delta \phi$ and $\Delta T_e$.

$$
\text{if } \Delta \phi > \varepsilon \phi \Rightarrow C_1 = 1 \\
\text{if } \Delta \phi > \varepsilon \phi \text{ and } \frac{d \Delta \phi}{dt} > 0 \Rightarrow C_2 = 0 \\
\text{if } \Delta \phi > \varepsilon \phi \text{ and } \frac{d \Delta \phi}{dt} < 0 \Rightarrow C_1 = 1 \\
\text{if } \Delta \phi > \varepsilon \phi \Rightarrow C_1 = 0
$$

(7)

If the field error is negative, one defines an angle $\delta$ between the vector $V_s$ and the vector $\phi$, equal to $2\pi/3$. So the angle $\delta$ is given as: $\delta = \theta_s + \delta_\phi$

### Table I

<table>
<thead>
<tr>
<th>Increase</th>
<th>Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{s1}$, $V_{s2}$ and $V_{s3}$</td>
<td>$V_{s1}$ and $V_{s2}$</td>
</tr>
<tr>
<td>$T_{e1}$</td>
<td>$T_{e1}$</td>
</tr>
</tbody>
</table>

While basing on the generalized table I, one can draw up the table II of the sequences below summarizing the vector PWM, proposed by Takahashi, to control stator field and the torque of the PMSM.

### Table II

<table>
<thead>
<tr>
<th>$n$</th>
<th>$C_1$</th>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$C_1 = 1$</td>
<td>$V_1$</td>
<td>$V_2$</td>
<td>$V_3$</td>
</tr>
<tr>
<td>2</td>
<td>$C_1 = 0$</td>
<td>$V_1$</td>
<td>$V_2$</td>
<td>$V_3$</td>
</tr>
<tr>
<td>3</td>
<td>$C_1 = 1$</td>
<td>$V_2$</td>
<td>$V_1$</td>
<td>$V_3$</td>
</tr>
<tr>
<td>4</td>
<td>$C_1 = 1$</td>
<td>$V_2$</td>
<td>$V_1$</td>
<td>$V_3$</td>
</tr>
<tr>
<td>5</td>
<td>$C_1 = 1$</td>
<td>$V_2$</td>
<td>$V_1$</td>
<td>$V_3$</td>
</tr>
<tr>
<td>6</td>
<td>$C_1 = 1$</td>
<td>$V_2$</td>
<td>$V_1$</td>
<td>$V_3$</td>
</tr>
</tbody>
</table>

For such high power industrial applications, if in the DTC the hysteresis bands of the controllers become relatively wide, with the lower inverter switching frequency, the resulting motor torque pulsations become high up to an undesired level. To overcome the above problems, some researchers have suggested, the DTC scheme using the space vector modulation (SVM) techniques [16-17]. In [5] and [18] control method has been discussed that allows constant switching frequency operation and uses two PI controller in order to generate the inverter reference voltage in the PMSM stator flux reference frame. In this control scheme, a PI speed controller is also used to produce the torque reference signal.

![Fig.4 Angle and voltage vectors $V_s$, determination](image)
If the field error is positive, $\delta_d$ between the vector $\vec{V}_s$ and the vector $\phi_s$ equal to $\pi/3$. So the angle $\delta$ is given as:

$$
\delta = \delta_d + \delta_a
$$

The table III resume the different choice of $\delta$

<table>
<thead>
<tr>
<th>Field</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>Angle $\delta$</td>
<td>$\delta_d$</td>
<td>$\delta_d + \pi$</td>
</tr>
</tbody>
</table>

IV. EXTENDED KALMAN FILTER

Now the extended Kalman filter observer will be applied to estimate the rotor speed which is feedback controlled by PI regulator. The EKF observer is based on the error of the stator currents generated from their measured and estimated values which must be converged toward zero via defined design.

The EKF has been described in many papers and is summarized in this section [19][20].

State equations for PMSM can be written as (31).

$$
\hat{x} = g(x,u) + w \\
y = c.x + v
$$

Here

$$
x = [I_s, I_d, \omega, \theta]^T, u = [u_d, u_s] \\
c = [1, 0, 0, 0]^T, y = [I_s, I_d]^T
$$

$w$ and $v$ are random disturbances. In fact $w$ is the process noise which stands for the errors of the parameters; $v$ is the measurement noise which stands for the errors in the measurement and sample. The noise covariance matrices are defined as follows:

$$
Q = \text{cov}(w) = E \{ww^T\} \\
R = \text{cov}(v) = E \{vv^T\}
$$

Kalman filter can be built by this follow derivation:

$$
x(k+1) = f = \begin{bmatrix}
I_s(k) + \frac{U}{L_s} \left( \frac{R}{L_s} + \frac{L_d}{L_s} \right) I_d & \frac{1}{L_s} \\
\frac{1}{L_d} & 0
\end{bmatrix} T
$$

Define matrix $F$ and $H$ as:

$$
F = \begin{bmatrix}
\frac{1}{L_s} & \frac{1}{L_d} & \frac{U}{L_s} & 0 \\
0 & 0 & \frac{L_d}{L_s} & \frac{R}{L_s}
\end{bmatrix}
$$

$$
h = \begin{bmatrix}
I_s \\
I_d
\end{bmatrix}, \quad H = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
$$

$$
\tau_s = \frac{L_s}{R}, \tau_d = \frac{L_d}{R}
$$

are stator constants.

Extended Kalman filter can be realized by iteration as follows:

After deciding how to initialize the covariance matrices, the next step is prediction of the state vector at sampling time $(k+1)$ from the input $u(k)$, state vector at previous sampling time, $x(k)$:

$$
\hat{x}_{k+1} = \hat{x}_{k} + \hat{K}(y - \hat{y})
$$

The notation $x[k]$ means that it is a predicted value at the $(k+1)$th instant, and it is based on measurements up to $k$-th instant. In the following step of the recursive EKF computation, covariance matrix of prediction is computed.

$$
P_{k+1} = F P_k F^T + Q
$$

In the second stage which is the filtering stage, the next estimated states $\hat{x}_{k+1}$, are obtained from the predicted estimates $x_{k+1}$ by adding a correction term $K(y - \hat{y})$ to the predicted value.

The predicted state-vector is added to the innovation term multiplied by Kalman gain to compute state-estimation vector. The state-vector estimation (filtering) at time (k) is determined as:

$$
\hat{x}_{k+1} = x_{k+1} + K (y - \hat{y})
$$

Where

$$
\hat{x}_{k+1} = C \hat{x}_{k+1}
$$

Here we define the estimation covariance computation as:

$$
P_{k+1} = \left[ I - K_{k} C \right] R
$$

This observer can be constituted from the PMSM model. The choice of the initial values for the matrices $P$, $R$ and $Q$ is very important for the EKF. Generally, $P_{d0}$ determines the initial transient characteristics of the filter, but has little influence on the initial tuning procedure of the EKF. Since the algorithm does not require initial rotor position information and the motor is assumed to start from the standstill, the initial state vector $x_{d0}$ is considered to be a null vector. Based on the discussion given by [21] after the trial-and-error procedure, initial values for the states and matrices $P$, $R$ and $Q$ were selected as follow:

$$
\hat{x}_{0} = \begin{bmatrix}
I_s \\
I_d \\
\omega \\
\theta
\end{bmatrix}, \quad P_{x0} = \begin{bmatrix}
0.1 & 0 & 0 & 0 \\
0.1 & 0 & 0 & 0 \\
0.1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.1
\end{bmatrix}
$$

$$
Q = \begin{bmatrix}
100 & 0 & 0 & 0 \\
0 & 1000 & 0 & 0 \\
0 & 0 & 1000 & 0 \\
0 & 0 & 0 & 0.01
\end{bmatrix}, \quad R = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
$$

Based on the EKF and the DTC, the proposed sensorless speed control of a PMSM is shown in Fig1. As depicted the proposed PI regulator compares the reference $\omega^*$ with speed $\omega_{meas}$ calculated with the estimated speed given by Kalman filter and it is delivers as output the torque reference $T_{meas}$. The flux is controlled to follow its reference.

V. SIMULATION RESULTS

In order to prove the effectiveness and the feasibility of the proposed SVM-DTC associated to EKF, the simulation
module is built in MATLAB/SIMULINK®. The specifications for the used PMSM are listed in table IV.

Figure 5 shows the torque dynamic performance, the locus of the stator flux and the stator current response.

Figure 6 gives the comparison between the two stator flux locus with classical DTC and SVM-DTC.

Figure 7 shows the speed tracking controller operated in a critical situation of benchmark commands rapidly changes as $50 - 100 - 0$ (rad/s). The observed speed converges with reference speed in very short time with a negligible overshoot and no steady state. It can be observed with the Zoomed response.

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**Fig.5** Simulation results without speed sensor

**Fig.6** Comparison of the stator flux locus with DTC (right) and SVM-DTC (left)

**Fig.7** Speed simulation results under SVM-DTC - EKF
VI. CONCLUSION

To solve the problems of large flux and torque ripples and inconstant switching frequency in direct torque control for permanent magnet synchronous motors and to eliminate speed sensor since it deteriorates system reliability and increases the cost of system, a novel speed sensorless DTC method based on space vector modulation and EKF is proposed in this paper.

The torque ripple for this SVM-DTC is significantly improved and switching frequency is maintained constant. This study has successfully demonstrated the design of the EKF control for the speed control of a PMSM. The control laws were derived based on the motor model. Numerical simulations have been carried out showing the advantages of the SVM-DTC method with respect to the conventional DTC. The effectiveness and robustness at tracking a reference speed under critical situation of benchmark commands rapidly changes.

Table IV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated voltage</td>
<td>208V</td>
</tr>
<tr>
<td>Pole pair number</td>
<td>3</td>
</tr>
<tr>
<td>d-axis inductance, L_d</td>
<td>66 mH</td>
</tr>
<tr>
<td>q-axis inductance, L_q</td>
<td>58 mH</td>
</tr>
<tr>
<td>Stator resistance, R_s</td>
<td>1.4Ω</td>
</tr>
<tr>
<td>Motor inertia, J</td>
<td>0.00176 kgm²</td>
</tr>
<tr>
<td>Friction coefficient, B</td>
<td>3.88. 10⁻¹ Nm/rad/s</td>
</tr>
<tr>
<td>Magnetic flux constant, φ_m</td>
<td>0.1546 Wb</td>
</tr>
</tbody>
</table>

VII. REFERENCES