SLIDING MODE CONTROL APPLICATION TO THE DOUBLY FED INDUCTION MACHINE SUPPLIED BY CURRENT SOURCES
Youcef HARBOUCHE, Laïd KHETTACHE and Rachid ABDESSEMED
LEB Research Laboratory, Department of Electrical Engineering,
University of Batna – Algeria
Youcef_harbouche@hotmail.com

Abstract - This study deals with the application of sliding mode control theory to wound rotor induction motor with its rotor fed by current sources in which the system operate in stator field oriented control. After determining the model of the machine, a set of simple surfaces have been applied to a cascade structure and the associated control laws have been synthesised. Furthermore, in order to reduce chattering phenomenon, smooth control functions with appropriate threshold have been chosen. Simulation study based on idealized motor is conducted to show the effectiveness of the proposed method.

Keywords Double fed induction motor, vector control, sliding mode control, current sources.

1. Introduction
In the area of the control of the electric machines, the research works are oriented more and more towards the application of the modern control techniques. These techniques involve in a vertiginous way with the evolution of the computers and power electronics. This allows to lead to the industrial processes of high performances. These techniques are the fuzzy control, the adaptive control, the sliding mode control etc. The recent interest accorded to the latter is due primarily to the availability of the high frequency commutation switches and of the increasingly powerful microprocessors.
It is regarded as one of the simplest approaches for the control of both non-linear systems and inaccurate model systems.
A considerable attention was concentrated on the control of the uncertain dynamic non-linear systems which are subject to disturbances and variations of the external parameters.
The sliding mode control concept consists of moving the state trajectory of the system towards the sliding surface and to maintain it around with the appropriate logic commutation. This latter gives birth to a specific behaviour of the state trajectory in the neighbourhood of the sliding surfaces known as sliding regimes.
In this study, we suggest a control scheme to achieve the goal of speed regulation with stator field oriented DFIM drive and sliding mode control [4,5].

2. System description and machine modelling
Using the frequently adopted assumptions, like assuming sinusoidally distributed air gap, flux density distribution and linear magnetic conditions, in the referential axis linked to rotating field, the following electrical equations are deducted [1,2,3]:

\[
\begin{align*}
V_s &= V_s e^{j\phi} = R_s I_s + j\omega_s \Phi_s + \frac{d\Phi_s}{dt} \\
V_r &= V_r e^{j\phi} = R_r I_r + j\omega_r \Phi_r + \frac{d\Phi_r}{dt} \\
\Phi_s &= \Psi_s e^{j\phi} = L_s I_s + M \Phi_r \\
\Phi_r &= \Psi_r e^{j\phi} = M I_s + L_r I_r
\end{align*}
\]

The torque is:
\[
T_e = \frac{2}{3} p M I_m \left( \overline{I_s} e^{j\theta_s} \overline{I_r} e^{-j(\theta + \theta)} \right)
\]

Note that \( \theta_s = \theta_r + \theta \), by introducing the latter into the equation (1), the torque can be expressed by:

\[
T_e = \frac{2}{3} p M I_m \overline{I_s} \overline{I_r}^*
\]

By taking into account the following currents:
\[
\begin{align*}
\overline{I_s} &= I_s e^{j\phi_i} \\
\overline{I_r} &= I_r e^{j\phi_r}
\end{align*}
\]
The torque becomes:

\[ T_e = \frac{2}{3} p M L_s I_r \sin \varphi \]

(4)

where \( \varphi \) : phase angle between \( I_s \) and \( I_r \).

This model expressed in the synchronous reference frame coordinates, the parameters are only function of the module and the position of the vector associated with the considered parameter.

The variables which appear in this model are the currents \( I_s, I_r \), the voltages \( V_r, V_s \), the fluxes \( \varphi_r, \varphi_s \) and pulsations \( (\omega_s, \omega_r, \omega) \).

Current \( I_r \) and flux \( \varphi_s \) are determined by using to the state variables \( I_r, \varphi_s \):

\[
\begin{align*}
\overline{I_s} &= -\frac{M}{L_s} I_r + \frac{\varphi_s}{L_s} \\
\overline{\varphi_r} &= \sigma L_r I_r + \frac{M}{L_s} \varphi_s
\end{align*}
\]

(5)

Introducing equations (5) into the voltage equations of the general mathematical model (1), we obtain:

\[
\begin{align*}
\overline{V_s} &= -\frac{M}{L_s} R_s I_r + \left[ \frac{R_s}{L_s} + j \omega \right] \varphi_s + \frac{d}{dt} \varphi_s \\
\overline{V_r} &= (R_r + j \sigma L_r \omega_r) I_r + \sigma L_r \frac{d}{dt} I_r + \frac{M}{L_s} \left[ j \omega \varphi_s + \frac{d}{dt} \varphi_s \right]
\end{align*}
\]

(6)

The choice of \( I_r \) and \( \varphi_s \) as state variables allows a simple representation of the machine.

The decomposition of the state equations for the rotor currents gives:

\[
\begin{align*}
\frac{d I_r}{dt} &= -\frac{1}{\sigma T_s} I_r + \frac{1 - \sigma}{\sigma M T_s} \varphi_s + \frac{1}{\sigma T_s} \varphi_r + \frac{1 - \sigma}{\sigma M} \varphi_s + \frac{1}{\sigma M} \varphi_r \\
\frac{d I_q}{dt} &= -\frac{1}{\sigma T_s} I_q - \frac{1 - \sigma}{\sigma M} \varphi_s + \frac{1}{\sigma T_s} \varphi_r + \frac{1 - \sigma}{\sigma M} \varphi_r
\end{align*}
\]

(7)

with:

\[ \frac{1}{T_s'} = \frac{1}{T_r} + \frac{1 - \sigma}{T_s} \]

The flux derivatives are:

\[
\begin{align*}
\frac{d\varphi_s}{dt} &= \frac{M}{L_s} I_r - \frac{1}{T_r} \varphi_s + \omega_s \varphi_r + V_s \\
\frac{d\varphi_r}{dt} &= \frac{M}{T_s} I_q - \frac{1}{T_r} \varphi_s - \omega_s \varphi_r + V_r
\end{align*}
\]

(8)

The torque equation is given by:

\[ T_e = \frac{2}{3} p M L_s (\varphi_s I_r - \varphi_r I_q) \]

(9)

The position of the chosen reference frame is obtained from the following law:

\[ \omega_s = \left[ \frac{M}{T_s} I_r q - \frac{1}{T_r} \varphi_s + V_s - \frac{d}{dt} \varphi_s \right] \frac{1}{\varphi_s} \]

(10)

The equations of the stator flux are determined from equations (8):

\[
\begin{align*}
\varphi_s &= \left[ M I_r q - \frac{1}{T_s} \varphi_s + V_s - \frac{d}{dt} \varphi_s \right] \frac{1}{\varphi_s} \\
\varphi_r &= \left[ M I_q r - \frac{1}{T_r} \varphi_s + V_r - \frac{d}{dt} \varphi_r \right] \frac{1}{\varphi_r}
\end{align*}
\]

(11)

In this study, the stator is supplied by a voltage of fixed amplitude and frequency voltage, and the rotor by a three-phase current source.

This machine is controlled by acting on the rotor parameters. The control laws are as follows:

\[
\begin{align*}
\varphi_{sd} &= \left[ M I_{rd} + \omega_s T_s \varphi_{sq} + V_{sd} T_s \right] \frac{1}{T_s T_r + 1} \\
\varphi_{sq} &= \left[ M I_{rq} - \omega_s T_s \varphi_{sd} + V_{sq} T_s \right] \frac{1}{T_s T_r + 1} \\
\Omega &= \left( \Gamma_e - \Gamma_r \right) \frac{1}{J_s} \\
\omega_r &= \omega_s - \omega \\
\theta_r &= \int (\omega_s - \omega) dt
\end{align*}
\]

(12)
The projection of $\vec{V}_S$ on the d and q axes is given:

$$
\begin{align*}
V_{sd} &= \frac{3\sqrt{2}}{2} V_s \cos \varphi_{Vs} \\
V_{sq} &= \frac{3\sqrt{2}}{2} V_s \sin \varphi_{Vs}
\end{align*}
$$

(13)

In this research, we consider the orientation of stator flux:

$$
\begin{align*}
\Psi_{sd} &= \Psi \\
\text{and} \\
\Psi_{sq} &= 0
\end{align*}
$$

(14)

3. Sliding mode control

3.1. General concept

The variable structure and its associated sliding regimes are characterised by a discontinuous nature of the control action with which a desired dynamic of the system is obtained by choosing appropriate sliding surfaces. The control actions provide the switching between subsystems which give a desired behaviour of the closed loop system [6-16]. Figure 1 illustrates a sliding mode phenomenon, which consists of an infinite switchings of the control action within the neighbourhood of the sliding surface.

![Fig.1 State trajectory in sliding mode regime](image)

Assuming that the system is controllable and observable, the sliding mode control objectives consist of the following steps:

- Design of the switching surface $S_x$ so that the state trajectories of the plant restricted to the equilibrium surface have a desired behaviour such as tracking, regulation and stability.
- Determine a switching control strategy, $U_x$ to drive the state trajectory into the equilibrium surface and maintain it on the surface.

This strategy has the form:

$$
U = \begin{cases} 
U_{\text{max}} & \text{if } S(x) > 0 \\
U_{\text{min}} & \text{if } S(x) < 0
\end{cases}
$$

(15)

where $S_x$ is the switching manifold; reduce the chattering phenomenon due to discontinuous nature of the control.

A well known surface chosen to obtain a sliding mode regime which guarantees the convergence of the state $x$ to its reference $x_{\text{ref}}$ is given as follows:

$$
S(x) = \left( \frac{d}{dt} + \lambda \right)^{r-1} (x_{\text{ref}} - x)
$$

(16)

Where $r$ is the degree of the sliding surface.

Two parts have to be distinguished in the control design procedure. The first one concerns the attractivity of the state trajectory to the sliding surface and the second represents the dynamic response of the representative point in the sliding mode. This latter is very important in terms of application of non-linear control techniques. Because it eliminates the uncertain effect of the model and the external disturbance. Among the strategies of the sliding mode control available in the literature, we can choose for the controller the following expression:

$$
U_c = U_{\text{eq}} + U_n
$$

(17)

Where $U_{\text{eq}}$ is the control function defined by Utkin, and noted equivalent control, for which the trajectory response remains on the sliding surface [3-4]. In this case, the condition of invariance is expressed as:

$$
\begin{align*}
\dot{S}(x) &= 0 \\
\bullet \quad S(x) &= 0
\end{align*}
$$

(18)

The equivalent control can be interpreted as the average value of control switching representing the successive commutation in the range $[U_{\text{min}}, U_{\text{max}}]$, [1-2].

Let us consider the system described by equation (7), when the sliding mode regime arise, the dynamic of
the system in sliding mode is subject to the following equation \( S(x) = 0 \) thus for the ideal sliding mode, 
\[
\dot{S}(x) = 0
\]
we have also :
\[
\dot{S}(x) = \frac{\partial S}{\partial x} \frac{dx}{dt} = \frac{\partial S}{\partial x} \left[ f(x) + g(x)U_{eq} \right] + \frac{\partial S}{\partial x} g(x)U_n
\]  
(19)

when \( U_n = 0 \) ( on \( S(x) = 0 \) , we obtain:
\[
\dot{S}(x) = 0
\]
\[
U_{eq} = \left[ \frac{\partial S}{\partial x} g(x) \right]^{-1} \left[ \frac{\partial S}{\partial x} f(x) \right]
\]  
(20)

by replacing \( U_{eq} \) , in equation (19) we obtain:
\[
\dot{S}(x) = \frac{\partial S}{\partial x} g(x)U_n
\]  
(21)
The term \( U_n \) is added to the global function of the controller in order to guarantee the attractiveness of the chosen sliding surface. This latter is achieved by the condition:
\[
S(x) \dot{S}(x) = S(x) \frac{\partial S}{\partial x} g(x)U_n < 0
\]  
(22)

A simple form of the control action using sliding mode theory is a relay function (fig.2). However, this latter produces a drawback in the performances of a control system, which is known as a chattering phenomenon, 
\[
S(x) = K \cdot \text{sgn} \ S(x)
\]  
(23)

Replacing \( U_n \) , we obtain:
\[
\dot{S}(x)S(x) = \frac{\partial S}{\partial x} g(x)K |S(x)| < 0
\]  
(24)
The term \( \left( \frac{\partial S}{\partial x} \right) g(x) \) is negative for the class of the system considered, whereas the gain \( K \) is chosen positive to satisfy attractivity and stability conditions. In this context, we can verify the stability of the sliding surface by using Lyapunov theorem. Let’s choose the following positive function \( (V(x)>0) \) such us:
\[
V(x) = \frac{1}{2} S^2(x)
\]  
(25)
Its derivative is given by:
\[
\dot{V}(x) = S(x) \dot{S}(x)
\]  
(26)

We must decrease of the Lyapunov function to zero. For this purpose it is sufficient to assure that its derivative is negative.

In order to reduce the chattering phenomenon due to the discontinuous nature of the controller, a smooth function is defined in some neighbourhood of the sliding surface with a threshold (fig.3). If a representative point of the state trajectory moves within this interval, a smooth function replaces the discontinuous part of the control action. Thus, the controller becomes:
\[
U_n = \begin{cases} 
\frac{K}{\varepsilon} S(x) & \text{if } |S(x)| < \varepsilon \neq 0 \\
K \cdot \text{sgn} S(x) & \text{if } |S(x)| > \varepsilon 
\end{cases}
\]  
(27)
where \( K \) takes an admissible value.
### 3.2 Application to the DFIM

The surface of speed regulation has the DFIM following form:

\[ S(\omega) = \omega^* - \omega \]  
\[ (28) \]

The derivative of the surface is:

\[ \dot{S}(\omega) = \dot{\omega} - \omega \]  
\[ (29) \]

By taking into account the expression of \( \dot{\omega} \) given by equation (12), and knowing that \( \Omega = \frac{\omega}{P} \), equation (28) becomes:

\[ \dot{S}(\omega) = \dot{\omega} - \left[ \frac{2P^2 M}{3jLs} \Psi_{sd} \dot{I}_{rq} - \frac{P}{j} \frac{Cr}{M} \right] \]  
\[ (30) \]

By replacing the \( I_{rq} \) with the control current \( I_{rq}^* \) such as \( I_{rq}^* = I_{rq}^{eq} + I_{rq_n} \), equation (29) can be written as follows:

\[ \dot{S}(x) = \dot{\omega} - \left[ \frac{2P^2 M}{3jLs} \Psi_{sd} I_{rq}^{eq} + \frac{2P^2 M}{3jLs} \Psi_{sd} I_{rq_n} - \frac{P}{j} \frac{Cr}{M} \right] \]  
\[ (31) \]

During the sliding mode and steady state, we have \( S(\omega) = 0 \) consequently \( \dot{S}(\omega) = 0 \) and \( I_{rq_n} = 0 \), so we obtain the equivalent control formula for \( I_{rq}^{eq} \):

\[ I_{rq}^{eq} = \frac{1}{\Psi_{sd}} \frac{3jLs}{2P^2 M} \left( \dot{\omega} + \frac{P}{j} \frac{Cr}{M} \right) \]  
\[ (32) \]

During the convergence mode, the condition \( S(\omega)S(\omega) < 0 \) must be checked. By replacing \( I_{rq}^{eq} \) formula into equation (31), we obtain:

\[ \dot{S}(\omega) = - \frac{2P^2 M}{3jLs} \Psi_{sd} I_{rq_n} \]  
\[ (33) \]

Figure 4 shows the block diagram of the sliding mode control of the DFIM. From the choice of the smoothed control we can write:

\[ I_{rq}^* = \begin{cases} \frac{K}{\varepsilon} S(\omega) & \text{if } |S(\omega)| < \varepsilon \\ K \text{sign} S(\omega) & \text{if } |S(\omega)| \geq \varepsilon \end{cases} \]  
\[ (34) \]

For attenuating all the overshoot of the current \( I_{rq} \), we bound the reference current \( I_{rq}^* \). The bounded current \( I_{rq}^{lim} \) has the following expression:

\[ I_{rq}^{lim} = I_{rq}^{max} \text{sign} (I_{rq}^*) \]  
\[ (35) \]

From these equations we can simulate the sliding mode control; note we are in the case of setting by the sliding mode with a non linear surface, only one surface is sufficient for setting the speed with direct bounding of the rotoric current in quadrature.

### 4. Results and discussions

A sliding mode control of the stator flux oriented control has been simulated using the parameters:

\[ (220 / 380) V; (3.8 / 2.2) A; 0.8 kW; 1500 \text{rpm}; \]
\[ p = 2; L_s = 0.0605 \text{H}; L_r = 0.0736 \text{H}; \]
\[ R_s = 11.98 \Omega; R_r = 9.08 \Omega; M = 0.209 \text{H}. \]

Thus, the speed regulation is obtained using such a controller in spite of the presence of stern disturbances such as reference speed variation and step changing of the load torque.

Figure 5 shows the dynamic responses of the speed and the electromagnetic torque when a load torque perturbation is imposed in the system at \( t=2s \) and at \( t=3s \) an application of speed reference change with step changing load torque. It is clearly shown from the results that the input reference is perfectly tracked by the speed and the introduced disturbance is immediately rejected by the control system.

The control by the sliding mode of the DFI.M gives high dynamic and static performances. It offers a good pursuit and a rejection of disturbances.
5. Conclusion

In this study, a sliding mode control of the stator field oriented doubly fed induction machine is proposed. Satisfying results are obtained. In order to reduce the chattering phenomenon, a smooth function has been applied. The robustness quality of the proposed controllers appears clearly in the test results by changing machine operations and especially the load torque variation.
Fig. 5 Simulation results.

References


