A GENERALIZED MATHEMATICAL MODEL OF THE TWO-LEVEL CASCADED INVERTERS FEEDING THE OPEN-END STATOR WINDING INDUCTION MACHINE

A. NAYLI  S. GUIZANI
University of Tunis, ESSTT, Tunisia. University of El Manar, IPEIEM, Tunisia.
Email: n.ayli@hotmail.fr, Guizani_sami@yahoo.fr,

F. BEN AMMAR
University of Carthage, MMA Laboratory, INSAT, Tunisia
Email: Faouzi.Benamar@insat.rnu.tn

Abstract: In this paper the authors propose the supply of the open-end stator winding asynchronous machine by voltage converter to each input, these converters constituted by two or several 2-level cascaded inverters. A generalized mathematical model of the « n » levels inverters feeding the open-end stator winding induction machine is presented. An extended generalized PWM strategy is carried out to control the converter.

Key words: Open-end winding induction machine, two-level cascaded inverter, generalized mathematical model of cascaded inverters.

1. Introduction

The power segmentation, the improvement of reliability and availability become priority in the industrial drives at variable speeds such as railways applications, aeronautics, electrical propulsion of ships and electrical vehicles… [1].

The drive system is primarily based on the three-phase synchronous or asynchronous machines associated with three-phase 2-level inverters but this association is not without disadvantage. To improve reliability and consequently the availability of this association inverter machine, Several researches have also been developed in inverter structures :
- H-Bridge inverters [2-4].
- Two-level cascaded inverters [5-11].
- Different cascaded inverters [12-13].
- Multilevel inverters with flying-capacitor [14].

Or in machine structures:
- Multi-phase asynchronous or synchronous machine, where each phase is fed by its own voltage inverter. [15-17].
- Multi-star asynchronous or synchronous machine, where each star is fed by three-phase voltage inverter. [18-19].
- The open-end stator winding asynchronous or synchronous machine, where each windings extremity is fed by three-phase voltage inverter. [3-14] and [20].

This last type of machine, associated with the three-phase 2-level cascaded inverters will be the subject of this paper.

In the first part, the modelling of the open-end winding induction machine is successively presented in Clark (α,β) and Park (d,q) reference frames.

The second part is devoted to the implementation of a simulation model of the machine. The model will be validated in the environment of «Matlab Simulink ».

In the third part, the authors devote this paper to the supply of the open-end stator winding asynchronous machine by multilevel structures with 2-level cascaded inverters. The type PWM strategy is used phase disposition PWM. A THD analysis is presented for the feeding the machine by 2-level cascaded inverters.

In the fourth part, this article gives the mathematical model for the supply of the open-end stator winding asynchronous machine by 2-level cascaded inverters and an extended generalized PWM strategy is carried out to control the converter.

Finally, the authors present the advantages of the association open-end winding induction machine – 2-level cascaded inverters.
2. Simulation model of open-end winding induction machine for voltage supply

The supply of the open-end stator winding induction machine is illustrated in the figure 1.
The voltages of the machine are defined as:

\[ V_{ss1} = [V_{s11}, V_{s12}, V_{s13}]^T \] Voltage vector inverter 1.
\[ V_{ss2} = [V_{s21}, V_{s22}, V_{s23}]^T \] Voltage vector inverter 2.
\[ V_s = [V_{s11} - V_{s21}, V_{s12} - V_{s22}, V_{s13} - V_{s23}]^T \] Voltage vector of a stator winding of the machine.

![Functional diagram of the open-end stator winding asynchronous machine](image)

The output vector:
\[ V_{ss} = [V_{s1}, V_{s2}, V_{s3}]^T \]

The functional diagram is given by figure 2:

![Functional diagram of the open-end winding machine](image)

The mathematical flux model is written in (d,q) reference frame, and described by the following state equations representation:

\[
\frac{dX(t)}{dt} = \left[ A(\omega(\omega_{dq})) \right] X(t) + [B]\ U(t)
\]

Where:
\[ X(t) = [\Phi] = [\Phi_{sd} \Phi_{sq} \Phi_{id} \Phi_{iq}]^T \] : The State vector
\[ U(t) = U_1(t) - U_2(t) = [V_{sd1} - V_{sd2}, V_{sq1} - V_{sq2}]^T \] : The Control vector
\[ Y(t) = [I] = [I_{sd} I_{sq} I_{id} I_{iq}]^T \] : The output vector

The functional diagram is given by figure 2:

\[
\begin{bmatrix}
U_1 \\
U_2
\end{bmatrix} \times \begin{bmatrix}
[B] \\
[A]
\end{bmatrix} \times \begin{bmatrix}
X \\
Y
\end{bmatrix} \times \begin{bmatrix}
[C]
\end{bmatrix}
\]

The drive mechanical equation is given as follows:

\[
T_{em} - T_r = j \frac{d\omega}{dt} + f \omega
\]

\[
T_{em} = \frac{3}{2} p(\Phi_{sa} s\beta - \Phi_{sb} s\alpha)
\]

3. Feeding the open-end stator winding induction machine by three phase inverters in cascade.

To circumvent the balancing problems of floating sources and congestion encountered in the inverters,

\[
\tau_s = \frac{L_s}{R_s} \quad \text{Constant of time for the stator}
\]

\[
\tau_r = \frac{L_r}{R_r} \quad \text{Constant of time for the rotor}
\]

\[
\sigma = 1 - \frac{M_{sr}^2}{L_s L_r} \quad \text{Coefficient of dispersion of Blondel}
\]

\[
R_r: \text{Rotor resistance} \\
R_s: \text{Stator resistance} \\
L_s: \text{Stator cyclic Inductance} \\
L_r: \text{Rotor cyclic Inductance} \\
M_{sr}: \text{Mutual Maximal cyclic inductance between stator and rotor.}
\]

The equation of current vector is:

\[
[I] = [L]^{-1}[\Phi]
\]

we obtained for the matrices A,B et [C] are:

\[
[A] = \begin{bmatrix}
-\frac{1}{\sigma L_s} & \frac{\omega_{dq}}{\sigma \tau_s} & 0 \\
-\frac{\omega_{dq}}{\sigma L_s} & \frac{1}{\sigma \tau_s} & 0 \\
0 & -\frac{1}{\sigma \tau_s} & -\frac{\omega_{dq} - \omega}{\sigma \tau_s}
\end{bmatrix}
\]

\[
[B] = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
[C] = [L]^{-1} = \begin{bmatrix}
\frac{1}{\sigma L_s} & 0 & -\frac{M_{sr}}{\sigma L_s \sigma L_r} & 0 \\
0 & \frac{1}{\sigma L_s} & 0 & -\frac{M_{sr}}{\sigma L_s \sigma L_r} \\
-\frac{M_{sr}}{\sigma L_s} & 0 & \frac{1}{\sigma L_r} & 0 \\
0 & -\frac{M_{sr}}{\sigma L_s \sigma L_r} & 0 & \frac{1}{\sigma L_r}
\end{bmatrix}
\]
with flying-capacitor, then we will use the three phase cascaded inverters because they are modular inverters which therefore allows to reduce congestion.

3.1. Feeding the open-end stator winding machine by two 2-level cascaded inverters

The open-end stator winding induction machine is fed by two 2-levels inverters in cascade based on V/f law, is shown in figure 3.

To control the two three phase cascaded inverters, the three reference signals of frequency \( f_m \) and amplitude \( A_m \) are compared with two triangular carriers of frequency \( f_p \) and amplitude \( \frac{A_p}{2} \), as shown by figure 4.

Figure 4 shows two triangular carriers and a signal reference.

The simulation results of the open-end winding induction machine where each entry is supplied by two 2-level cascaded inverters as shown in the figure 6. The load torque is the type \( k_n^2 \).

Figure 7 shows the voltage \( (V_{S11}-V_{S12}), (V_{S21}-V_{S22}) \) and phase-to-phase machine voltage \( U_1 \) which is 5 levels to supply with two three-level inverters. With:

- \( V_{S11}, V_{S12} \) simple voltage of inverter 1
- \( V_{S11}-V_{S12} \) pole voltage of inverter 1
- \( V_{S21}, V_{S22} \) simple voltage of inverter 2
- \( V_{S21}-V_{S22} \) pole voltage of inverter 2
- \( U_1 = (V_{S11}-V_{S12}) - (V_{S21}-V_{S22}) \) pole voltage of the machine.
Fig. 7. Pole voltage inverter and phase-to-phase machine Voltage.

Figure 8 shows the waveform of the stator and rotor flux.

Figure 9 shows space vector voltage of the two 2-level cascaded inverters and machine.

Figure 10 shows the waveform of the phase-to-phase machine voltage and the harmonic content of the voltage.

Figure 11 shows the waveform of the stator current and the harmonic content of the current.

It is clear that the use of the 2-level cascaded inverters improves THD voltage, THD current, torque undulations, and especially increases the degrees of liberty in degraded mode, while keeping the advantage of the power segmentation offered by the association of these structure inverters with the open-end winding induction machine, indeed any number of the inverters put in cascaded, they will be of the dimension inferior or equal to the power half of the machine.

At the end of this article, we show of the tables that summarize the dimensioning of the inverters and their advantages for the different number of the inverters put in cascaded.

3.2. Feeding the open-end stator winding machine by 6 2-level cascaded inverters

The structure of supply of the open-end stator winding induction machine by six 2-level cascaded inverters is shown figure 12.
Figure 13 shows three triangular carriers and signals references for control the six 2-level cascaded inverters.

Figure 14 shows the waveform of the phase-to-phase machine voltage and the harmonic content of the voltage. The simulation results of the voltage (Vs1 - Vs2), (Vs1 - Vs2) and phase-to-phase machine voltage U1 which is 13 levels to supply with six 2-level cascaded inverters of voltage is shown by figure 14.

Figure 15 shows the waveform and harmonic ration of voltage.

Figure 16 shows the waveform of the current machine and the harmonic content of the current.

Table I summarizes the results for different levels of the phase to phase converter 1, 2 and phase to phase machine voltage and harmonic ratio of machine voltage for different structure to supply by the 2-level cascaded inverters with phase disposition PWM strategy. It is possible to observe the increase level of machine voltage and lower THD.
In the phase disposition PWM strategy, the reference signals modulating of frequency $f_m$ and amplitude $A_m$ are compared with $p$ triangular carriers, with the same frequency $f_p$ and amplitude $A_p = \frac{A_m}{p}$ as shown in the figure 18.

### Table I. Different levels of voltage and THD voltage.

<table>
<thead>
<tr>
<th>Inverter</th>
<th>Inverter</th>
<th>Machine</th>
<th>THD Current ( %)</th>
<th>THD Voltage ( %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>N</td>
<td>0.39</td>
<td>24.99</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>13</td>
<td>0.18</td>
<td>15.07</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>13</td>
<td>0.09</td>
<td>7.04</td>
</tr>
</tbody>
</table>

4. **Generalized phase-disposition PWM strategy for control p-inverters in cascade**

In figure 17, we represent the feeding of the open-end winding induction machine by $p_1$ inverters in cascade at entry 1 and $p_2$ inverters in cascade at entry 2. For the DC bus, the entry of two converters becomes $E_1 = E_2 = E$, with the input of the $p$ cascadedinverters are $\frac{E}{p}$.

![Fig. 17. Open-end stator winding induction machine supplied by «p» 2-level cascaded inverters.](image1)

![Fig. 18. General principle of the phase-disposition PWM strategy for inverter by «p» cascaded inverters.](image2)

5. **Generalized mathematical model of 2-level cascaded inverters**

We presented the generalized mathematical model of 2-level cascaded inverters. That the DC bus for each inverter is $E$.

Figure 19 shows the structure of the two 2-level cascaded inverters.

![Fig. 19. Asynchronous machine supplied by two 2-level cascaded inverters.](image3)
Notations:

n : fictitious neutral.
i: index indicating the number of the phase i, i = {1,2,3}
j: index indicating the number of the switch j, j = [1...p]
T(i,j), T'(i,j) : power switches.
S(i,j), S'(i,j) : the corresponding switching signal of the T(i,j)
S'(i,j) : the corresponding switching signal of the T'(i,j).
The S(i,j) and S'(i,j) are complementary signals.

From the scheme of Figure 19, we determine the following equations:

\[ V_{A_nO_1} = -\frac{E}{2}S_{11} + S_{11}(V_{A_2O_2} + V_{O_1O_2}) \]
\[ = -\frac{E}{2}(1-S_{11}) + S_{11}[2S_{12} - 1]\frac{E}{2} + E \]  \hspace{1cm} (8)
\[ = -\frac{E}{2} + S_{11}E + S_{11}S_{12}E \]

We assume the DC bus for each inverter is E and it is written by the following relation:

For inverter 12:

\[ V_{12}' = \frac{E}{2} + \frac{E}{2}V_{A_2O_2} = 2S_{12} = E \]  \hspace{1cm} (9)

For inverter 11:

\[ V_{11}' = \frac{3E}{2} + \frac{E}{2}V_{A_2O_2} + (2S_{12} - 1)\frac{E}{2} + 2E \]  \hspace{1cm} (10)

In this case, the open-end winding induction machine of power P supplied to each entry by three phase cascaded inverters, there will be two inverters Inv\textsubscript{11} and Inv\textsubscript{12} dimensioned for a power \( \frac{P}{2} \) also the two other inverters Inv\textsubscript{12} and Inv\textsubscript{22} dimensioned for a power \( \frac{P}{4} \).

To put three cascaded inverters is the first \( \frac{P}{2} \) power, the second is \( \frac{P}{3} \) power, the third is \( \frac{P}{6} \) power and so on.

Thereafter figure 20 shows the structure of the three 2-level cascaded inverters, we obtained the following equations:

\[ V_{A_1O_1} = -\frac{E}{2}S_{11} + S_{11}(V_{A_2O_2} + V_{O_1O_2}) \]  \hspace{1cm} (11)
\[ V_{A_2O_2} = \frac{E}{2}S_{11} + S_{11}(V_{A_3O_3} + E) \]  \hspace{1cm} (12)

That is to say:

\[ V_{A_1O_1} = -\frac{E}{2} + S_{11}E + S_{11}S_{12}E + S_{11}S_{12}S_{13}E \]  \hspace{1cm} (13)

From the general case of \( \{ p \} \) cascaded inverters, we obtained the general equation:

\[ V_{A_1O_1} = -\frac{E}{2} + \sum_{j=1}^{p} E \frac{P}{1 - S_{1j}} \]  \hspace{1cm} (14)

The three voltages are obtained directly by the following expressions:

\[ V_{A_1n} = V_{A_1O_1} + V_{O_1n} \]
\[ V_{B_1n} = V_{B_1O_1} + V_{O_1n} \]
\[ V_{C_1n} = V_{C_1O_1} + V_{O_1n} \]  \hspace{1cm} (15)

\[ V_{A_1n} + V_{B_1n} + V_{C_1n} = 0 \]  \hspace{1cm} (16)

\[ V_{O_1n} = -\frac{1}{3}(V_{A_1O_1} + V_{B_1O_1} + V_{C_1O_1}) \]  \hspace{1cm} (17)

That is to say:

\[ V_{A_1n} = \frac{2}{3}(V_{A_1O_1} - \frac{1}{3}V_{B_1O_1} - \frac{1}{3}V_{C_1O_1}) \]  \hspace{1cm} (18)

The three voltages are described by the following matrix:

\[ \begin{bmatrix} V_{A_1n} \\ V_{B_1n} \\ V_{C_1n} \end{bmatrix} = \begin{bmatrix} \frac{p}{L} & L & \quad \frac{p}{L} \\ \frac{p}{L} & L & \quad \frac{p}{L} \\ \frac{p}{L} & L & \quad \frac{p}{L} \end{bmatrix} \begin{bmatrix} S_{1j} \\ S_{2j} \\ S_{3j} \end{bmatrix} \]  \hspace{1cm} (19)
For feeding the open-end winding induction machine by cascaded inverters, the three voltages are written by the following matrix.

For converter 1:

\[
\begin{bmatrix}
V_{A_{11n}} \\
V_{B_{11n}} \\
V_{C_{11n}}
\end{bmatrix} = \frac{E}{3} \begin{bmatrix} 2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix} \begin{bmatrix} \sum_{L=1}^{p} \frac{L}{L_{1j}} S_{11j} \\
\sum_{L=1}^{p} \frac{L}{L_{1j}} S_{12j} \\
\sum_{L=1}^{p} \frac{L}{L_{1j}} S_{13j}
\end{bmatrix}
\]

(20)

For converter 2:

\[
\begin{bmatrix}
V_{A_{21n}} \\
V_{B_{21n}} \\
V_{C_{21n}}
\end{bmatrix} = \frac{E}{3} \begin{bmatrix} 2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix} \begin{bmatrix} \sum_{L=1}^{p} \frac{L}{L_{1j}} S_{21j} \\
\sum_{L=1}^{p} \frac{L}{L_{1j}} S_{22j} \\
\sum_{L=1}^{p} \frac{L}{L_{1j}} S_{23j}
\end{bmatrix}
\]

(21)

The three voltages of phase machine are described by the following matrix:

\[
\begin{bmatrix}
V_{A_{11A_{21}}} \\
V_{B_{11B_{21}}} \\
V_{C_{11C_{21}}}
\end{bmatrix} = \frac{E}{3} \begin{bmatrix} 2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix} \begin{bmatrix}
\sum_{L=1}^{p} \frac{L}{L_{1j}} S_{11j} - \sum_{L=1}^{p} \frac{L}{L_{1j}} S_{21j} \\
\sum_{L=1}^{p} \frac{L}{L_{1j}} S_{12j} - \sum_{L=1}^{p} \frac{L}{L_{1j}} S_{22j} \\
\sum_{L=1}^{p} \frac{L}{L_{1j}} S_{13j} - \sum_{L=1}^{p} \frac{L}{L_{1j}} S_{23j}
\end{bmatrix}
\]

(22)

Figure 21 shows the validation of the mathematical model correspond to the voltage \((V_{S_{11}} - V_{S_{21}}), (V_{S_{21}} - V_{S_{22}})\) and phase-to-phase machine voltage \(U_{1}\), which is 13 levels to supply with six 2-level cascaded inverters.

We used the mathematical model of three cascaded inverters to supply the induction motor. The simulation results are shown by figure 22.

The characteristics of the machine used:
- Nominal power \(P = 45\) kW.
- Speed \(n = 1450\) rpm.
- Resistance of stator \(R_s = 0.15\) Ω.
- Resistance of rotor \(R_r = 0.046\) Ω.
- Inductance of stator \(L_s = 17.9\) mH.
- Inductance of rotor \(L_r = 18.6\) mH.
- Mutual inductance \(M_{sr} = 17.2\) mH.

6. Advantage of the association open-end winding induction machine – 2-level cascaded inverters

If we consider that the induction machine (IM) and the open-end winding induction machine (OEWIM) have a power \(P = 45\) kW (55 kVA) for our application), the table II summarizes the power of each inverter, the current and the voltage of the switches \(V_{1}\) for the three phase inverter.

![Fig. 21. The phase-to-phase inverter voltage and phase-to-phase machine Voltage from the mathematical model.](image)

![Fig. 22. The phase to phase voltage of the machine, stator currents, speed, and torque, for a load torque \(Tr = kn^2\).](image)

<table>
<thead>
<tr>
<th>Machine Structure</th>
<th>Inverter Power</th>
<th>Current (A)</th>
<th>switches Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IM</td>
<td>P</td>
<td>80</td>
<td>540</td>
</tr>
<tr>
<td>OEWIM</td>
<td>P/2</td>
<td>80</td>
<td>270</td>
</tr>
</tbody>
</table>

| Table II. Dimensioning of the inverter. |
The table II shows the open-end winding induction machine is a good solution for the great power; indeed its two inverters are dimensioned to a half power of the machine. And more, this structure allows the operation in degraded mode but with only one degree of liberty, since a single failure in one of the two inverters is tolerated. The table III summarizes the power of each inverter, the current and the voltage of the switches for the two three phase cascaded inverters.

<table>
<thead>
<tr>
<th>Inverters</th>
<th>Inverter Power</th>
<th>Current I (A)</th>
<th>Switches Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inver 11 and Inver 21</td>
<td>P/2</td>
<td>80</td>
<td>V_{T11} = 270</td>
</tr>
<tr>
<td>Inver 12 and Inver 22</td>
<td>P/4</td>
<td>80</td>
<td>V_{T12} = 135</td>
</tr>
</tbody>
</table>

Table III. Dimensioning of the two three phase cascaded inverters

The table III shows the use of the two cascaded inverters supply for the open-end winding induction is not a constraint for the power segmentation since the cascaded inverters (Inv 12) and lower power (P / 4) that principal inverter (Inv 11), and more this structure improves the THD voltage and THD current, reduces torque undulations and especially increases the degrees of the liberty in degraded mode [5]. The table IV summarizes the power of each inverter, the current and the switches voltage for the six three phase cascaded inverters.

<table>
<thead>
<tr>
<th>Inverters</th>
<th>Inverter Power</th>
<th>Current I (A)</th>
<th>Switches Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inver 11 and Inver 21</td>
<td>P/2</td>
<td>80</td>
<td>V_{T11} = 270</td>
</tr>
<tr>
<td>Inver 12 and Inver 22</td>
<td>P/4</td>
<td>80</td>
<td>V_{T12} = 135</td>
</tr>
<tr>
<td>Inver 13 and Inver 23</td>
<td>P/3</td>
<td>80</td>
<td>V_{T13} = 180</td>
</tr>
<tr>
<td>Inver 14 and Inver 24</td>
<td>P/4</td>
<td>80</td>
<td>V_{T14} = 135</td>
</tr>
<tr>
<td>Inver 11 and Inver 21</td>
<td>P/6</td>
<td>80</td>
<td>V_{T15} = 90</td>
</tr>
<tr>
<td>Inver 12 and Inver 22</td>
<td>P/12</td>
<td>80</td>
<td>V_{T16} = 45</td>
</tr>
</tbody>
</table>

Table IV. Dimensioning of the six three phase cascaded inverters

Table IV shows that when increasing the number of cascaded inverters, the latter are of the power increasingly reduced, which does not render the very high cost of this structure type, on the other hand we increase of the degrees of system liberty of the machine supply in a degraded mode.

Notes:
We do not treat in this paper the cost of the inverters although it is evident that cascaded inverters especially for the number superior to 2 associated with the OEWIM become costliest, this is the price to pay to increase the degrees of liberty and consequently improve the continuity of service of the variable speed drive which is our principal objective.

7. Conclusion
We implemented the simulation model of the open-end winding asynchronous machine supplied by two voltage sources cascaded inverters in the Matlab Simulink environment.
The simulation results show that the open-end stator winding asynchronous machine has the advantage of increasing the level of phase-to-phase machine voltage. Indeed if we supply the machine by two « n » levels inverters, the level of phase to phase machine voltage is equal to N = 2n - 1.
The supply of the open-end winding induction machine by 2-level cascaded inverters clearly improves the THD voltage and current of stator.
If we want to increase the level voltage just add lower power inverters in cascade.
We developed a mathematical model generalized for the supply with 2-level cascaded inverters of this open-end stator winding machine. This model was validated in the « Matlab Simulink » environment, it reduced the simulation time and simplified the schemes realized with Power System Blocks.
The association open-end winding induction machine 2-level cascaded inverters ensure the power segmentation and modular inverters. It is also very interesting to improve the reliability of a variable speed drive system and without forgetting the redundant degrees it can offer.

References


