Leakage fluxes and mechanical forces calculation on the single phase shell-type transformer winding under over currents by 2-D and 3-D finite element methods

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Abstract: This paper is focused on the investigation of leakage flux and resulted mechanical forces distribution and magnitude on windings of transformer in the over current conditions. Three dimensional (3-D) and 2-D computer modeling for a real 10MVA 66/11kV single-phase shell-type transformer is employed. The transformer current is supposed to change from normal condition to 25 per unit continuously. 2-D and 3-D finite element methods are used to calculate the magnetic leakage fluxes and electromagnetic forces. Relation between current, magnetic leakage flux and forces are realized and three dimensional descriptions of mentioned parameter is shown. Results indicate 2-D finite element method is not efficient accuracy for calculation of radial component of leakage flux and axial component of forces. Radial component of force related to any conductor is very depended on the radial position of that and axial component of force is very depended on the axial position of conductor on the winding. Results also indicate a linear relation between leakage magnetic flux and current and a nonlinear relation between current and forces.

Key words: single phase shell-type transformer, electromagnetic forces, finite element methods, leakage magnetic flux, over current

1. Introduction
Failure of transformer due to over current (such as inrush and short circuit currents) is a major concern of transformer users. One of the major reasons for fault in power transformers is deterioration of winding and insulation of conductor due to the oscillation resulted from the electrodynamics forces. There are continuous efforts by manufactures and users to improve the short circuit and inrush withstand performance of transformer. The short circuit strength of a transformer enables it to survive through fault currents due to external short circuits in a power system network; an inadequate strength may lead to a mechanical collapse of winding, deformation damage to clamping structures and may eventually lead to an electrical fault in the transformer itself [1]. In [2] electromechanical forces for short-circuit cases and also for inrush current through the winding, using the finite-element methods is calculated. In this reference is shown that the forces exerted on the winding due to inrush current in many regions are larger than those due to short-circuit currents. In [3], the forces have been calculated using the finite-element-method [FEM] for both inrush current and short circuit currents cases and the results are compared. In many literatures to avoid complexity of the simulation model, the actual three dimensional geometry of three-phase power transformer was simplified into a one phase, two-dimensional equivalent axial symmetry geometry. In [4] based on the expressions of transient three-phase short-circuit current, the formulas of radial and axial electromagnetic forces acting on the transformer winding have been derived. A 3-D analysis of the magnetic field and electromagnetic forces due to the short circuit current has been done using methods [4]. Different techniques based on theory of images and the FEM enabling to calculate the static electromagnetic forces on the winding of power transformer have been compared [5]. In the [6] the electromagnetic forces acting in a power transformer under short-circuit in the winding have been determined. In [7] FEM has been used to evaluate the forces exerted on the coils of a single-phase shell-type transformer. It has been shown that the amplitudes of the axial forces in the yoke have a considerable difference with that of the outer point. It has been confirmed that a 3-D analysis is necessary to compute the end forces and examine asymmetries outside the window that are not possible with 2-D analysis. The goal of this paper in first is calculation of radial and axial components of leakage magnetic flux then calculation of electromagnetic force components for over current and yielding to relationship between the current, magnetic flux leakage and force components in the winding. A simple analytical model is offered and computer modeling for 2-D and 3-D finite element methods on a real single phase shell-type transformer is done.
2. Analytical modeling

In the power transformers, current density has an azimuthally direction and produces radial and axial leakage flux on the conductors. Current density in the winding is affected by the leakage magnetic flux and produces forces. These forces don’t have azimuthally component. When the current increases the leakage magnetic flux in the winding and magnetizing flux in the core will be increased. Therefore forces in the winding will be increased. In the over current such as inrush and short circuit currents, the current density increases severally and pay attention to nonlinearity of magnetizing curve of core, the core saturates. Saturation of core leads to increase the core reluctance while winding and gap reluctance remain constant. Therefore, in this situation the magnetic flux is more interested in passing from winding and gap and lower interested in passing from core. Really, leakage flux in the inrush current (in the primary winding) and in the short circuit current (in primary and secondary) severally will be increased. Consequently big current density and big leakage flux in the winding of transformer will produce very big force in the winding. In power transformers, radial and axial components of current density are ignorable. Leakage magnetic flux has radial and axial components. Figure1 shows a simple two dimensional description of leakage magnetic flux distribution in the transformer window. As seen in this figure the region between primary and secondary windings is very similar to solenoid inside. Dashed region in the figure 1 is a solenoid and its center has a constant magnetic field.

Consider equation (1) for m parallel solenoid with different diameter and equal height. Therefore the resultant magnetic field intensity in the region between the two winding is equal to:

\[ H_{z_{\text{max}}} = NI / \frac{h}{m} \]  

(2)

Considering completely circular cross section for conductors m can be calculated from equation (3).

\[ m = \sqrt{\frac{N \pi d}{4h}} \]  

(3)

Where d is the primary or secondary winding diameter.

\[ H_{\text{z}_{\text{max}}} \]  

is the maximum magnitude of the magnetic field intensity in the window. Internal solenoids approximately don’t have any effect on the outer regions. Therefore magnetic field intensity on the inner conductors of internal winding and on the outer conductors of external winding has minimum magnitude. Minimum magnitude of the magnetic field intensity in the window can be calculated from equation (4)

\[ H_{\text{z}_{\text{min}}} = NI / \frac{m h}{m} \]  

(4)

Pay attention to monotonous distribution of conductors on the winding diameter, the parallel solenoids will produce linear axial magnetic field intensity in term of radial distance. Axial magnetic intensity on the outer winding is given by equation (5).

\[ H_z = \left(\frac{NI}{h}\right)[1 + \left(\frac{r}{d}\right)^2 \left(1 - \frac{m}{m}\right)] \]  

(5)

The current density has only azimuthally component and we can summarize the equation (6) in the equation (7).

\[ \nabla \times \vec{H} = \vec{J} \]  

(6)

\[ \frac{\partial H_z}{\partial z} - \frac{\partial H_r}{\partial r} = J_\theta \]  

(7)

\[ H_z \]  

is specified in the equation (5). The radial component of magnetic field intensity in the center of winding height is negligible therefore we will have a boundary condition corresponding to equation (8).

\[ H_z \bigg|_{z = h/2} = 0 \]  

(8)

Finally radial component of magnetic field intensity will be defined in the equation (9)

\[ H_r = \left(\frac{NI}{hd m}\right)(z - h/2) \]  

(9)

\[ J_\theta \]  

is the azimuthally component of current density and is calculated from equation (10).

\[ J_\theta = NI / dh \]  

(10)
The forces will be vertical to the magnetic field intensity and have only radial and axial components in any point of windings. Equations (12) and (13) show the radial and axial components of forces.

\[
\vec{F} = \vec{J} \times \vec{B}
\]

\[
F_r = J_\theta \times B_z
\] (12)

\[
F_z = -J_\theta \times B_r
\] (13)

\(\vec{B}\) is the magnetic flux density vector and calculable from equation (14).

\[
\vec{B} = \mu_0 \vec{H}
\] (14)

Finally the equations (15) and (16) show the axial and radial components of forces.

\[
F_r = \mu_0 d J_\theta \left[1 + \left(\frac{L}{d}\right)^2 \frac{1-m}{m}\right]
\] (15)

\[
F_z = -\mu_0 L J_\theta \left(\frac{z-h/2}{m}\right)
\] (16)

As seen in these equations the forces have a nonlinear relationship with current density. Radial component of force in the conductors with lower radial distance on the outer winding also in the conductors with higher radial distance on the internal winding have higher amounts. In the winding window the conductors that close to the core receive smaller radial forces than other part of winding. Axial forces on the central conductors of winding height are negligible and in the two ends are equal and maximum with opposite directions.

3. Finite element modeling

In this paper two dimensional and three dimensional modeling of a single phase shell-type transformer is done by finite element methods. Specification and dimensions of proposed single phase shell-type transformer are presented in the table 1.

| Specification and dimensions of proposed single phase shell-type transformer |
|-------------------|-------------------|
| Primary rated voltage(kV) | 66 |
| Secondary rated voltage(kV) | 11 |
| Number of turns of primary winding | 1200 |
| Number of turns of secondary winding | 200 |
| Rated power (MVA) | 10 |
| Window area(m×m) | 1×1 |
| LV winding cross-section(m×m) | 0.167×1 |
| HV winding cross-section(m×m) | 0.167×1 |
| Yoke dimensions (m×m) | 0.5×0.25 |
| Core limb(m×m) | (0.5×0.5) |

Three dimensional modeling and meshing is showed in the figures 2 and 3 respectively. As seen in the figure 2, the winding in the azimuthally directions is divided to 36 volume. Any volume consists of 160 equal elements. Meshing of winding is done with 11520 elements. Figures 2 and 3 don’t show the surrounding and insulators.

In the finite element modeling, the magnetization characteristic of core corresponding to Fig.4 is used. In order to achieve the relationship between current, leakage magnetic flux and forces, simulations are done in the current range from 1 to 25 per unit. Situations similar to the short circuit current are considered and supposed that primary and secondary current change similarly. At first the leakage magnetic flux distributions then the forces are discussed.

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Figure 2. Three dimensional scheme of the modeled transformer by finite element methods (only core and windings)

Figure 3. Meshed single phase shell-type transformer (windings and core)

Figure 4. Magnetizing curve of core in the finite element modeling
3.1. Leakage flux in the windings
In normal operating conditions, the magnetic flux density in the core is bigger than magnetic flux density in the window. When current increases, core begins to saturation and magnetic flux density in the window will be increased. For example figure 5 shows a three dimensional distribution of magnetic flux density vector in the core and winding for 10 per unit current. As seen in this figure the leakage flux density is very increased and in some regions of winding equal to the magnetizing flux density in the core. This figure indicates the maximum leakage flux density in the central part of winding height and in the air gap between primary and secondary winding.

Figures 6 and 7 show the axial and radial components of leakage magnetic flux density on the half of outer winding conductors for 10 per unit current. As seen in the figure 6 maximum axial component of leakage magnetic flux density take place in the lozenge area on the inner conductor of outer winding. The center of this lozenge is located on the conductors between two neighbor core limbs. In the corners the axial magnetic leakage flux density has minimum magnitude. As seen in the figure 6, axial leakage flux density in the inner conductors of outer winding has maximum magnitude and outer conductors have minimum magnitude. In the inner winding, the radial distribution of axial leakage flux density is opposite with that on the outer winding. In the inner winding, the mentioned lozenge takes place on the outer conductors of winding. Figure 7 shows the radial component of magnetic leakage flux density in the center of windings’ height is zero. Two ends of windings’ height have maximum and equal magnitude of radial component (with opposite directions). Comparing figure 6 and 7 indicates the radial component of leakage magnitude flux density in the winding except of two ends is bigger than axial component of leakage magnetic flux density.

3.2. Forces in $\theta$ direction
Many researches and literature are used from a 2-D computer modeling to calculate the leakage magnetic flux and forces. These models consider the impact of radial and axial position of any conductor on the forces. Three dimensional FEM modeling in this paper indicates that azimuthally positions effect on the forces. Three dimensional modeling shows the forces distribution in the r, z and $\theta$ directions. Figure 8 shows the force vector in the inner and outer winding in the 10 per unit current. As seen in this figure forces in the $\theta$ direction aren’t constant. In the conductors that situated in the space between tow neighbor limbs, forces are bigger than the conductors that situated outer the space between two neighbor limbs. This is result of big magnetic
leakage flux that exists in the space between two neighbor limbs. This space is near to the core. The core sets up paths with lower magnetic reluctance (even at saturation situation) related to the outer free space of transformer. Therefore the forces on the conductors that exist in the space between two neighbor limbs are maximum magnitudes. The corners have minimum magnitudes.

2-D computer modeling doesn’t consider the impact of 0 position of conductors on the force calculations. The figure 8 shows the force vectors on the inner and outer winding in 10 per unit current. As seen in this figure the radial components are much higher than axial component because force vector is in the radial direction. The forces on the inner winding are in negative radial direction and on the outer winding in positive radial direction.

Figure 8. Force vector on the inner and outer windings in 10 per unit current ($N/m^2$)

Figure 9. Radial and axial components of magnetic flux density in the middle radial distance and upper end ($\varepsilon = 1$ and $r = \delta/2$) conductors of outer winding for 10 per unit current. As seen in this figure magnetic flux density in $\theta$ direction isn’t constant. Magnetic flux density in $\theta$ direction has a sinusoidal waveform (especially axial component). The changes of axial component of magnetic flux density are very higher than corresponding radial component. Therefore on the similar radial and axial positions the axial force is almost constant but the radial forces change. This is very clear in the figure 10.

Figure 10. Radial and axial components of force in the middle radial distance and upper end conductors of outer winding for 10 per unit current ($\varepsilon = 1$ and $r = \delta/2$)

3.3. Forces in $r$ and $z$ directions
The axial and radial components of forces are shown in figures 11 and 12 respectively. As seen in these figures radial force distribution is similar to axial leakage magnetic flux distribution and axial force distribution is similar to radial leakage magnetic flux. This result is from constant current density in the windings. Figure 12 indicates a positive (outward) radial force on the outer winding. As seen in the figure 11 the axial component of force is negative in the upper end and positive in the lower end of outer winding. Really the radial force pushes the outer winding outward and axial force at two ends compress the winding to the height center of winding. As seen in the figure 12 radial component of force in the height center of winding and in the conductors that located between two neighbor limbs are maximum magnitudes. The corners have minimum magnitudes.
The axial component of magnetic leakage flux and radial component of force in the center of height \( z = \frac{H}{2} \) and \( \theta = 0 \) in term of radial distance for 10 per unit current are shown in the figures 11 and 12 respectively. In these figures the result of analytical method, 2-D and 3-D finite element methods is shown. As seen in these figures the three mentioned methods confirm another. The analytical method is completely linear and follows the ideal conditions.

The radial component of magnetic leakage flux and axial component of force in \( r = \frac{d}{2} \), \( \theta = 0 \) in term of winding height is shown in figures 13 and 14 respectively. As seen in the figure 13 two ends of height winding have maximum magnitude of radial magnetic flux density and the center of height has minimum magnitude of radial flux density. Therefore the axial force in the two ends has maximum magnitude with opposite direction together. As seen in the figure 14 the 2-D FEM method calculates smaller axial force than 3-D FEM method because of don’t considering \( \theta \) position effect on the radial magnetic leakage flux density.
3.4. Relationship between the over current, leakage flux and forces

As mentioned before increasing the current in the transformer winding increases the leakage flux and force in the windings. The relationship between the maximum magnitude of leakage flux components and current also relationship between the maximum magnitude of force components and current in the outer winding of mentioned single phase shell-type transformer are shown in the figures 17 to 20. These figures indicate the results for three mentioned methods.

As seen in the figures 17 and 18 the radial and axial components of magnetic leakage flux density have linear manner. The radial and axial components of force have nonlinear manner and proportional with squared current. The calculated curves in the figures 17 to 20 can be used in the calculation of instantaneously changes of force in short circuit conditions. These results can be used in the dynamic modeling of force in the short circuit current. This work can be used on the inrush current dynamic modeling but the secondary winding current must be ignored in the computer modeling.
Figure 19. Relationship between the maximum magnitude of radial force and current

Figure 20. Relationship between the maximum magnitude of axial force and current

4. Conclusions

In this work a simple analytical model, 2-D and 3-D finite elements methods are used. Magnetic leakage flux and forces on the winding calculate on the three directions $r, \theta, z$. The results indicate the force in the radial direction is bigger than forces in axial direction. The conductors on the maximum radial distance of inner winding and on the minimum radial distance on the outer winding receive the maximum radial forces. The axial forces compress the winding to the center of height of winding. The radial forces compress the inner winding to the core and outer winding to the outward. The forces also depended on the $\theta$ position of conductors. The conductors on the space between two neighbor limbs have maximum magnitude forces. The 2-D model cannot calculate the $\theta$ direction effect and calculates the axial force with big error. The relationship between the current, leakage flux density and force in the 1 to 25 per unit current is calculate and realized the leakage flux have linear and force nonlinear relationship with current.

References: