A NEW SIMULATED ANNEALING FOR SOLVING GEOMETRICAL SHAPE OPTIMIZATION OF A LINEAR ACTUATOR

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Abstract: In this paper, a new adaptive simulated annealing algorithm for geometrical shape optimization of electromagnetic devices is proposed. The adaptive simulated annealing is very good at finding the correct area of the solution under some hypotheses such as non-convexity and non-differentiability and its generation function is excellent at refining a solution repeatedly to the nearest maximum or minimum solution. The ASA algorithm has been applied on the geometrical shape optimization of a linear electromagnetic actuator. The non-linear finite element method and the adaptive simulated annealing algorithm have been used to maximize the magnetic force versus displacement. To have the quality of this new algorithm, the performances of ASA are compared with other algorithm such as genetic algorithm (GA) in term of accuracy of the solution and computation time. The reached results suggest that the proposed algorithm ASA has excellent effectiveness in finding best solution.

Key words: Optimization, Genetic Algorithm, Adaptive Simulated Annealing, Linear Actuator, Finite Element Method.

1. Introduction

Shape optimization is part of the field of optimal control theory [1]. The typical problem is to find the shape which is optimal in that it minimizes a certain cost functional while satisfying given constraints. The functional to be optimized is called the goal function, or objective function, and is usually provided by the user as a black-box procedure that evaluates the function on a given state. Furthermore, this function is dependent on the design variables, which are the unknown system parameters.

Stochastic algorithms may find the global minimum of the objective function with a few hypotheses such as non-convexity, non-differentiability, etc. These methods are very simple to implement on design problems to converge to the solution with a high probability. Among the principal advantages of these procedures are their aptitudes to locate global solution without the make necessary derivatives. The stochastic methods are based on a set of points and to modify them with the probabilistic process to assure the best solution.

The simulated annealing is one of the most common stochastic methods for solving inverse problems through the search of solution space. There is a class of general stochastic strategies, which often use randomized search for example. They can be applied to a wide range of problems, but good performance is never guaranteed [2]. In addition, if the objective functions are under some hypotheses such as non-convexity and non-differentiability, it cannot afford the adequate fidelity of the inverse problem because the convergence to an optimal solution cannot theoretically be guaranteed after a great number of function evaluations. Also, it gives us bad results and run very slowly or demand more computation time for finding the solution in some problems.

For achieve good solutions in a reasonable amount of time, a new simulated annealing method has been developed by us to apply it in our geometrical shape optimization problem. This very fast simulated annealing method gives us an optimal solution of any optimization problems with a significantly lower number of function evaluations than those required by stochastic methods such as the Tabu search, genetic algorithms and simulated annealing [3]. The adaptive simulated annealing is very good at finding the adaptive solution area of the optimal point, tolerant of local maxima and minima, and the new generation function is excellent at refining a solution systematically to the nearest maximum or minimum (best solution). The algorithm is planned so that facilitates a global search and escapes the local minima. This improved algorithm can be worked adequately when the cost function is multimodal and under some hypotheses. The differences with standard SA are that the ASA uses a much faster annealing schedule and employs a reannealing scheme to adapt itself.

The objective of this paper is to propose the new fast optimization method for solving inverse electromagnetic problem in electrical engineering such as the geometrical shape optimization problem. In this case, the ASA method has been implemented on a linear electromagnetic actuator for its optimum design to maximize force magnetic.

The test configuration chosen for the evaluation of our algorithm is shown in Fig. 1. It was originally introduced in [4], and it consists of a core and a plunger, made of iron, and a copper coil. The application studied is a linear actuator with a
cylindrical core mobile. The actuator takes the form of a solenoid valve (or electromagnet). When energized, a magnetic force appears and has the tendency to displace the plunger about a position that minimizes the energy of the system. In the contrary case (de-energized), the plunger is returned to its original position by the spring action and the valve returns to its resting state. This device has been the subject of an internal study in electrical engineering laboratory in Grenoble, conducted by Saldanha in the context of industrial collaboration. Considering the symmetry, the model is only designed for the half (see Fig. 1).

![Coup axisymmetric linear actuator](image)

**Fig. 1. Coup axisymmetric linear actuator**

2. Adaptive Simulated annealing

2.1. Simulated annealing

Before giving a detailed description of ASA, first we shall explain the fundamental terminology of SA. Simulation optimization by simulated annealing was first described by Kirkpatrick et al [3], and is based on work by Metropolis et al [2] in the area of statistical mechanics. SA is inspired from the heating process of a crystalline structure. The metal is slowly lowered until it achieves its regular crystal pattern. At each temperature level, the simulation process must proceed long enough for the system to reach a steady state or equilibrium. This process makes a sequence of state for the final temperature with regular crystal pattern.

A simulated annealing optimization starts with a metropolis Monte Carlo simulation for state-space variables at a high temperature. This means that a relatively large percentage of the random steps that result an increase in energy will be accepted. After a sufficient number of Monte Carlo steps, or attempts, the temperature is decreased. The acceptance of the novel result is according to the Metropolis’s condition based on the Boltzmann’s probability [3]. SA algorithm contains two steps: the first, perform search while the temperature is decreasing. The second determine the acceptance. The acceptance probability of solution point \( i \) is defined by:

\[
P = \exp \left( \frac{E_j - E_i}{kT} \right)
\]

Where \( K \) is the Boltzmann’s constant and \( T \) is the temperature of the heat bath, \( E_j \) is the current energy state for the system and \( E_i \) is a subsequent energy state. If \( E_j - E_i \leq 0 \), \( j \) is accepted as a starting point for the next iteration; otherwise, solution \( j \) is accepted with Boltzmann’s probability (1). The above procedure is repeated \( nt \) time until temperature \( T \) is reduced. The aim of the Metropolis’s succession is to authorize the system to attain thermal equilibrium. It should be noted that classical optimization algorithm only accept improved design and never accept a worse design. In simulated annealing, the condition \( E_j - E_i \geq 0 \) gives the algorithm a chance of get out of a local minimum.

2.2. Adaptive Simulated Annealing

In practice, a geometric cooling schedule is generally utilized to have SA settle down at some solution in a finite amount of time. It has been proved by some authors that by carefully controlling the rate of cooling of the temperature, SA can find the global optimum. However, this requires infinite time. Fast annealing and very fast simulated reannealing (VFSR) or adaptive simulated annealing (ASA) are each in turn exponentially faster and overcome this problem. The first simulated annealing employed Gaussian distribution as a generator and was proposed by Kirkpatrick. In 1987, Szu and Dartly [3] proposed a fast simulated annealing by using Cauchy/Lorentzian distribution. Another modification of the SA, the so-called adaptive simulated annealing was proposed by Ingber [5] and was designed for optimization problem in a constrained search space. For \( x^k \) a parameter in dimension \( i \) at annealing time \( k \) with rang \( x^k \in \left[ x^i_{\text{min}}, x^i_{\text{max}} \right] \) the new value is generated by:

\[
x^i_{k+1} = x^i_k + \lambda(x^i_{\text{max}} - x^i_{\text{min}})
\]

Where \( x^i_{\text{max}} \) and \( x^i_{\text{min}} \) are the maximum and minimum of the \( i^{th} \) domain. This is repeated until a legal \( x_i \) between \( x^i_{\text{max}} \) and \( x^i_{\text{min}} \) is generated. \( \lambda_i \) \( (\in [-1,1] ) \) the random variable generated by the following generating function:

\[
g(\lambda_i) = \frac{1}{2(1 + T_i)} \cdot \frac{1}{\ln \left( 1 + \frac{1}{T_i} \right)}
\]

The \( i \) values and \( T_i \) are identifies the parameter index and temperature. To find \( \lambda_i \) one most find the normalized cumulative probability distribution of \( g(\lambda_i) \). The cumulative probability distribution can be defined as:

\[
g(\lambda_i) = \frac{1}{2} + \frac{\text{sign}(\lambda_i)}{2} \cdot \ln \left( 1 + \frac{1}{T_i} \right)
\]

To simplify this generating function \( \lambda_i \) for a uniform distribution is preferred. A normal uniform distribution is defined as follow:

\[
u(\lambda_i) = \frac{1}{2} + \frac{\text{sign}(\lambda_i)}{2}
\]

Where \( u_i \) \( (\in [0,1] ) \) is the uniform distribution function and is the cumulative of a uniform distribution. Each parameter is generated using a
cumulative function. In this case, by the idea of Ingber it can be seen to choose $g(\lambda_i) = u_i$. Then, to calculate $\lambda_i$ according to the preceding distribution, we can apply this formulation:

$$\lambda_i = \text{sign}(u_i - 0.5) T (1 + \frac{I}{T} |2^{u_i - 1} - 1|)$$  \hspace{1cm} (5)

The new generation distribution function in ASA has much fatter tails than Gaussian and Cauchy generation function. Temperature $T$ is a key element in the cooling system in the ASA algorithm. After every generated points, annealing takes place with a new annealing schedule. A global optimum can be obtained statistically if the annealing schedule is:

$$T_k = T_0 \exp(-c_k k ^{1/n})$$  \hspace{1cm} (6)

Where $c_i$ is a user-defined parameter whose value should be selected according to the guidelines in reference [6], but $n$ is the dimension of the space under exploration. The same type of annealing schedule should be used for both the generating function and the acceptance function $I(l + P)$.

Reannealing in ASA algorithm periodically rescales the generating temperature in terms of the sensitivities $s_i$ calculated at the most current minimum values of the cost function. After every acceptance points, reannealing takes place by the first calculating the sensitivities:

$$s_i = \frac{\partial E}{\partial x_i}$$  \hspace{1cm} (7)

The annealing time is adjusted according to $s_i$, based on the heuristic concept that the generating distribution used in the relatively insensitive dimension should be wider than that of the distribution produced in a dimension more sensitive to change.

3. Formulation of finite element method and force calculation

3.1. Field Equation

All electromagnetic phenomena are governed by Maxwell’s equations. The differential form of Maxwell’s equations can be expressed as:

$$\nabla \times H = J$$  \hspace{1cm} (8)

$$\nabla \cdot B = 0$$  \hspace{1cm} (9)

$$\nabla \times E = -\frac{\partial B}{\partial t}$$  \hspace{1cm} (10)

where $\nabla$ is the Laplace operator, $H$ is the magnetic field intensity, $B$ the magnetic induction intensity, $J$ the electrical current density and $E$ is the electrical field intensity.

For isotropic medial material, the constitutive equations to Maxwell’s equations are:

$$B = \mu [H] H$$  \hspace{1cm} (11)

$$J = \sigma \cdot E$$  \hspace{1cm} (12)

where $\mu$ and $\sigma$ are the magnetic permeability and electric conductivity of the medium electromagnetic field respectively.

For two dimensional problems, the magnetic vector potential $A$ is the obvious choice in most instances.

The divergence condition on $B$ implies the existence of a vector potential defined by:

$$B = \nabla \times A$$  \hspace{1cm} (14)

The magnetic field of electromagnetic actuator can be considered as a magnetostatic problem. Substituting (14) to (8) we obtain:

$$\nabla \times (\frac{1}{\mu} \nabla \times A) = J$$  \hspace{1cm} (15)

where $\mu$ has only a component in the direction $\varphi$.

This also the direction of $J$. With these conditions, Eq. (15) becomes:

$$\nabla \cdot (\frac{\partial A}{\partial z} \frac{\partial A}{\partial z} + \frac{\partial A}{\partial r} \frac{\partial A}{\partial r}) = -J$$  \hspace{1cm} (16)

The term $\nabla \cdot (\frac{\partial A}{\partial z} \frac{\partial A}{\partial r})$ creates an asymmetry in the elemental matrix, when Galerkin’s method is applied, because this term depends only on coordinate $r$. To eliminate this inconvenience we introduce a new variable $V$ related to $A$ which $V = r A$. Eq. (16) becomes:

$$\frac{\partial V}{\partial z} \frac{\partial V}{\partial z} + \frac{\partial V}{\partial r} \frac{\partial V}{\partial r} = J$$  \hspace{1cm} (17)

Where $V$ is the modified magnetic vector potential.

3.2. Finite Element Method

The finite element method is one of the most numerical methods used to solve differential equations. The FEM is widely used by scientists and engineers. The general principle of the finite element method consists in the division of the solution domain into small sub-domains or segments, known as “finite elements”. In this method, the equation is discretized in space by the Galerkin’s method. After discretization of the domain, the vector potential has been approximated using first-order triangular elements. In each element, the vector potential varies according to $i_{\lambda} (l + P)$:

$$V = \sum_{i = 0}^{\infty} V_{n} N_{n}$$  \hspace{1cm} (18)

Where $V_{n}$ are the node values of $V$ and $N_{n}$ are first order polynomials.

Applying the Galerkin’s method to Eq. (17), we have:

$$\int_{s_{i}} N_{n} \left[ \frac{\partial V}{\partial z} \frac{\partial V}{\partial z} + \frac{\partial V}{\partial r} \frac{\partial V}{\partial r} \right] dr dz + \int_{s_{j}} J_{n} dr dz = 0$$  \hspace{1cm} (19)

After assembling all the elementary equations, a differential system of equations is obtained which may be written as:

$$[M][V] = [F]$$

where $[M]$ is the global coefficient matrix, $[V]$ is the matrix of nodal magnetic vector potentials and $[F]$ is nodal currents (forcing functions) which are given by:

$$M_{ij} = \int_{s_{i}} V_{n} N_{n} \int_{r_{j}} dr dz$$  \hspace{1cm} (20)

$$F_{i} = \int_{s_{i}} J_{n} dr dz$$  \hspace{1cm} (21)
The Gaussian elimination algorithm is then used to solve the above banded matrix equation. The field solution is used to calculate the magnetic induction $B$. More details about the finite element theory can be found in [7].

3. Magnetic Force Calculation

The most important parameter of electromagnetic actuator, magnetic force, can be calculated by means of the nonlinear virtual work method. For the vector potential formulation, the local magnetic force is calculated on the nodes. Only the elements surrounding a node have changed their energy by moving virtual node. The energy of the system compared to a virtual displacement is given by:

$$ W = \int_\Omega \left( \int_0^B H \ dB \right) \ d\Omega \quad (22) $$

The force in a direction is given by the derivation of the magnetostatic energy system compared to a virtual displacement:

$$ F = -\frac{\partial W}{\partial q} $$

The magnetic force on a node $i$ is [8]:

$$ F = -\sum_i \left( \frac{\Omega}{2} \left( \frac{\partial B^2}{\partial q} \right) + \frac{\Omega}{\partial q} \left( \frac{\partial B}{\partial q} \right) \right) \quad (23) $$

Where $\Omega$ is the surface area of a triangle, $\partial v_e/\partial B^2$ is computed from the equation that represents the characteristic magnetization of ferromagnetic materials, $v_e$ the reluctivity of element and $q$ is the virtual displacement.

4. Shape Optimization Problem

Fig. 2 shows the flow chart of the optimization procedure. The initial dimensions of the linear electromagnetic actuator are used as starting point in the optimization. In the first step, the Finite element and nonlinear virtual work is utilized to obtain the force magnetic global of the device. To calculate the objective function of design parameters, the non linear finite element method package must be able to accept parameters generated by ASA, to perform the finite element method computation automatically, and to return the value of the objective function to the ASA algorithm. During each iteration, the dimensions of actuator are determined by ASA method as shown in Fig.2 indication. In the second step, if the results do not meet the termination criteria, the dimensions of the actuator are modified for the next iteration.

5. Results of Simulation and Discussion

The geometry of the actuator is illustrated by six design parameters $x_i$ ($i = 1...6$) selected to change the shape of the actuator (see Fig. 3). The dimensions $x_i$ have to be optimized in order to guarantee a maximum magnetic force versus displacement.

![Fig. 3 Design variable of the actuator](image)

The finite element Simulation of the global magnetic force of the electromagnetic actuator currently used has been carried out with saturation taken into account. The core and the plunger iron core are constructed from steel M19, whose magnetization characteristic is plotted in Fig. 4. This material is characterized by a curve $B(H)$ nonlinear, giving the magnetic field as a function of magnetic induction (see Fig. 4). The reluctivity is approximated by the following expression:

$$ \nu(B^2) = \tau + (\delta - \tau) \frac{B^{2\eta} + \zeta}{B^{2\eta} + \delta} \quad (24) $$

Where $\tau$ (1.25e-4) is the reluctivity at low values of $B$, $\delta$ (0.425) is the reluctivity of highly saturated materials and $\eta$ (5.22) and $\xi$ (21300) are the parameters determining the transitions between these two values. The finite element method (FEM) considering the saturation effect of the magnetic material is used, and is computed by using the Newton–Raphson method. The errors in the solution obtained are analyzed, the mesh is refined, and the problem is solved again. The procedure is repeated until the solution error is smaller than a predefined value.

Our axsiymmetrical model is based on the 2D-element finite method (2D-FEM) which permits to calculate the global magnetic force of electromagnetic actuator. The mesh is automatically generated by dividing the geometry into discrete elements. Standard triangular elements are applied here. The open boundary was set at a radius of $R_c$ (exterior radius) using the Dirichlet condition. The generated mesh had
approximately 3196 nodes or 6268 first order triangular elements. It is important to select an adequate mesh to represent correctly the electromagnetic phenomena and then, to reduce the numerical errors that can influence the convergence of the optimization process.

By using our finite element method program, Fig. 6 shows the equipotential lines of magnetic vector potential \( \mathbf{A} \). The problem was solved on a PC with P4 2.4G® CPU under Matlab 7 workspace using the Partial Differential Equation Toolbox for the finite element meshes generation.

Fig. 6 shows the experimental results of the magnetic global force acting on the moving parts using the finite element method and nonlinear virtual work method [8]. The results shows that when the air-gap between the magnetic circuit and the moving parts is near zero, the force acting on the plunger (moving parts) reaches the maximum value of 45.8 N. As the air-gap distance is increased, the force action on the plunger decreases.

The objective of optimization is to maximize the maximum magnetic force (45.8 N), with a global constant volume. This optimization consist of minimizing an objective function, which is the error between the target magnetic force (100 N) and a magnetic force \( F_z \) calculated using ASA and FEM-code and compare its optimal solution with genetic algorithm.

For this optimization problem, we define the cost function as the difference between the target magnetic force and the magnetic force calculated by the finite element method and adaptive simulated annealing method. Generally, the optimization is considered as a nonlinear problem to locate a solution \( x \) that minimizes the following cost function:

\[
 f(x) = \sqrt{\frac{1}{n_p} \sum_{i=1}^{n_p} \left( \frac{F_i(L_i(x), p)}{F_0} - 1 \right)^2} 
\]

Where \( F_0 \) is the desired magnetic force (here \( F_z = 100 \) N), is the magnetic force exerted on the plunger core by considering the gap \( L_z(i) \) (0.1 mm to 0.35 mm with a step of movement 0.05 mm) and \( n_p \) is equal to the number of design variable.

The values of \( p = (x_1, x_2, x_3, x_4, x_5, x_6) \) are optimized through minimization of this objective function. Eq.(25) is minimized by using the new hybrid FEM-ASA. The actuator design also needs to satisfy the following constraints:

- The excitation coil current density is 5.71 A.mm\(^{-2}\).
- Maximum flux density in the magnetic circuit and plunger core (\( B_s \leq 1.93 \ T \)).
- The equality constraints:
  \[
  g_f(x) : x_4 + x_5 - 0.433 \ L_e(x) = 0 \quad (26)
  \]
  \[
  g_2(x) : \pi \ R_e(x)^2 \ L_e(x) - 7.363 e - 6 = 0 \quad (27)
  \]
  \[
  g_f(x) : 2 \pi (r_1 + x_1 + r_2) x_4 - \pi (r_1 + x_2 + r_2)^2 - r_1^2 = 0 \quad (28)
  \]
The design optimization problem is to respect these constraints and to minimize the objective function \( f \). In engineering practice, a narrower range is always preferred for accuracy in inverse solution and for computational efficiency. The lower and upper bounds of the parameter \( x \) of the problem are:

\[
\begin{align*}
  x_{1}^{\text{min}} & = 1.37 \text{ mm} & x_{1}^{\text{max}} & = 3.13 \text{ mm} \\
  x_{2}^{\text{min}} & = 3.00 \text{ mm} & x_{2}^{\text{max}} & = 15.0 \text{ mm} \\
  x_{3}^{\text{min}} & = 0.03 \text{ mm} & x_{3}^{\text{max}} & = 1.20 \text{ mm} \\
  x_{4}^{\text{min}} & = 1.00 \text{ mm} & x_{4}^{\text{max}} & = 3.50 \text{ mm} \\
  x_{5}^{\text{min}} & = 1.00 \text{ mm} & x_{5}^{\text{max}} & = 8.00 \text{ mm} \\
  x_{6}^{\text{min}} & = 1.00 \text{ mm} & x_{6}^{\text{max}} & = 8.00 \text{ mm}
\end{align*}
\]

Using the exterior penalty function method, the constrained is converted in to an unconstrained problem to minimize the objective function shown in Eq. (27). By this idea, the objective function of the design problem is replaced by the following function [9]:

\[
\phi(x, m, k) = f(x) + m \sum_{j}^{m} \max(0, h_{j}(x))
\]

Where \( f \) is the objective function and \( h \) is the inequality constraints. But, \( m \) is the penalty coefficient.

<table>
<thead>
<tr>
<th>Test N°</th>
<th>P \text{ (mm)}</th>
<th>Initial dimensions</th>
<th>Optimized dimensions</th>
<th>Magnetic force</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>x1</td>
<td>2.130</td>
<td>3.004</td>
<td>89.94 N</td>
</tr>
<tr>
<td></td>
<td>x2</td>
<td>7.730</td>
<td>7.892</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x3</td>
<td>0.630</td>
<td>0.943</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x4</td>
<td>1.840</td>
<td>2.510</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x5</td>
<td>4.630</td>
<td>2.831</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x6</td>
<td>4.160</td>
<td>1.863</td>
<td></td>
</tr>
<tr>
<td>Iteration</td>
<td></td>
<td></td>
<td>101</td>
<td></td>
</tr>
<tr>
<td>CPU time</td>
<td></td>
<td></td>
<td>30e3 s</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>x1</td>
<td>1.800</td>
<td>3.102</td>
<td>91.45 N</td>
</tr>
<tr>
<td></td>
<td>x2</td>
<td>10.10</td>
<td>7.891</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x3</td>
<td>1.100</td>
<td>0.949</td>
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<td></td>
<td>x5</td>
<td>1.900</td>
<td>2.783</td>
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<tr>
<td></td>
<td>x6</td>
<td>1.500</td>
<td>1.962</td>
<td></td>
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<tr>
<td>Iteration</td>
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<td></td>
<td>57</td>
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</tr>
<tr>
<td>CPU time</td>
<td></td>
<td></td>
<td>15e3 s</td>
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</tr>
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</table>

As shown in Tab. 1, the global force magnetic of the optimized electromagnetic actuator has improved, and the volumes of the magnetic circuit and the plunger (moving parts) of the device have increased. The Fig. 7 shows the change of the static force magnetic by the adaptive simulated annealing approach. The force magnetic has augmented by 96.37 % for test I and 99.67 % for test II (see Tab.1 and Fig.8), while the volume of coil has decreased about 30 %. Consequently, the flux density on the plunger has increased and the reluctance of the actuator has reduced. All of these are useful in reducing the manufacture cost of the actuator. Moreover, after the shape optimization, the volume of the coil has diminished. These make the new actuator become more robust. For example, the configuration of the optimized actuator is shown in Fig. 8.

The force magnetic during the iterations process is shown in Fig. 9 obtained by the FEM-ASA algorithm for the test I. As shown in this figure, the error of the desired force magnetic and the magnetic force exerted on the plunger core is small. Thus, the new hybrid method (ASA with FEM) is confirmed our goal.

Now we compare this new method with a global search method as the genetic algorithm that determines the design parameters in any problem. The genetic algorithm (GA) is an optimization and search technique based on the principles of genetics and natural selection. The method was developed by John Holland over the course of the 1960s and 1970s and finally popularized by one of his students, David Goldberg, who was able to solve a difficult problem involving the
control of gas-pipeline transmission for his dissertation [10]-[11].

For this optimization method, the code has been programmed in Matlab with the Genetic algorithm toolbox [12] the parameters used are selected as follow: The tests start with number of population equals to 60 with 20 generations. Each generation stores the best fitness string, and at the end gives us the best candidate. A binary encoding is used. The crossover probabilities are equal 0.61 and 0.72 for the test I and II respectively. In both tests the mutation probabilities were 0.001 and 0.01. Also, the method of tournament selection is used. The convergence criteria used in the present work is when the percentage difference between the average values of all the designs and the best value in the population reaches a very small specified value.

The Tab.2 shows the results for maximizing the magnetic force using the genetic algorithm and CPU time. The computation time varies with the precision of calculation, and especially with the initial population. For these solutions, is not sure that we have the finest solution if the procedure is finished by the limit number of generations.

<table>
<thead>
<tr>
<th>Test</th>
<th>P (mm)</th>
<th>Optimized dimensions</th>
<th>Magnetic force</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>x1</td>
<td>2.131</td>
<td>71.32 N</td>
</tr>
<tr>
<td></td>
<td>x2</td>
<td>7.911</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x3</td>
<td>0.891</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x4</td>
<td>2.323</td>
<td></td>
</tr>
<tr>
<td></td>
<td>x5</td>
<td>2.594</td>
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</tr>
<tr>
<td></td>
<td>x6</td>
<td>1.720</td>
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</tr>
<tr>
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<td>2.798</td>
<td>72.10 N</td>
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<td></td>
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<td>1.988</td>
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</tr>
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<td></td>
<td>x5</td>
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</tr>
<tr>
<td></td>
<td>x6</td>
<td>2.002</td>
<td></td>
</tr>
</tbody>
</table>

It is expected to combine adaptive simulated annealing and a numerical method (FEM) so as to provide an ideal performance for the optimization procedure, which is often vital in nonlinear problems. As such, not only can the global optima be ensured but results can also be obtained at a reasonably fast speed (see Tab.1 and Tab.2). The other advantages of ASA are the capability to escape from the local optima.

With ASA optimization, the convergence to an optimal solution can theoretically be guaranteed after a number of iterations. Interestingly, when a combination of adaptive simulated annealing and the finite element method was applied, an even better result was achieved. This can be explained with the fact that the ASA method has different strength. The adaptive simulated annealing is very good at finding the correct area of the solution, tolerant of local maxima and minima, and the new generation function (see Eq. (4)) is excellent at refining a solution systematically to the nearest maximum or minimum (best solution). The new algorithm is better equipped for global optimization because it is more aggressive in the exploration of the search space. This algorithm can be worked adequately when the cost function is multimodal and not derived for the design parameters.

In our results, the value of the force magnetic versus the moving parts was improved by about 98% (see Tab.1), which means that the cost function decreased. The results presented here show that the performance of the electromagnetic devices can be substantially improved if combined ASA with FEM. When
compared the FEM-ASA with the FEM-GA, the numerical results show that the adaptive simulated annealing gives us an excellent convergence in a minimal CPU time (see Tab.1 and Tab. 2). It is evident from the above results that adaptive simulated annealing is superior to on this problem, both in terms of optima found and speed convergence. Whilst genetic algorithm is thorough, it does not appear to be able to adequately search the full space, and slowly converges to final solution. In contrast, the new generation function and annealing schedule of adaptive simulated annealing consistently gives us better results, especially when using new random-search technique.

6. Conclusion

In this paper we have presented a new optimization algorithm for solving inverse electromagnetic problem (IEP). The new algorithm is an extension of the traditional simulated annealing algorithm. It is based on a simulated annealing algorithm extended by a search technique to improve the parameters of the function that may keep high diversity and reduce the likelihood premature convergence.

Stochastic algorithms are extensively used for geometrical shape optimization problems, but they need several function evaluations and its convergence rate is short. To attain fast convergence, adaptive simulated annealing is a good approach because it can, under the non-convexity and non-differentiability, repeatedly adjust the adaptive solution space and rapidly converges to global solution.

When used to solve the optimization problem in the geometrical design of a linear actuator, which its objective function is under some hypotheses such as non-convexity and non-differentiability, adaptive simulated annealing can not only obtain the global optimal solution but also the convergence history showed that the ASA converged to the optima faster than the genetic algorithm.

Finally, the new ASA algorithm can be extensively used in any other situation to solve different optimization problems of electromagnetic devices.

References
12. Genetic Toolbox user’s guide, for use with MATLAB, the Math Works Inc.