Abstract—This paper presents the performance evaluation of a new digital protection for symmetrical faults occurred during the occurrence of power swing. The proposed digital protection is based on extracting the first level low frequency and high frequency sub-band contents present in the $d-q$-axis three phase instantaneous current components. This characterization helps to provide enough information to efficiently detect and discriminate power swing and symmetrical faults during power swing in two machine system. The utilized Wavelet Packet Transform (WPT) is realized by a half-band digital low and high pass filter, whose coefficients are determined by the Daubechies $db4$ wavelet basis functions. The WPT coefficients contain information that offer accurate, fast, and reliable detection in discriminating power swing and symmetrical fault. This detection method is immune to the power swing slip frequency, fault inception time and fault location. To test the proposed method, several power swings and faults are numerically simulated in MATLAB/SIMULINK.

Key words: Digital protective relays, $d-q$-axis components, power swing, symmetrical fault, wavelet packet transform and transient disturbances.

NOMENCLATURE

- $x_d[n]$  Discrete signal
- $WT$  Wavelet Transform
- $WPT$  Wavelet Packet Transform
- $d-q$  Direct- and- quadrature -axis rotating reference frame
- $j$  Level of decomposition in WT and WPT
- $A_{1d}[n]$  Low frequency content present in $x_d[n]$  
- $D_{1d}[n]$  High frequency content present in $x_d[n]$  
- $A_{1dq}[n]$  First level low frequency subband of $dq$ – WPT
- $D_{1dq}[n]$  First level high frequency subband of $dq$ – WPT
- $\Psi$  Non-zero value of $A_{1dq}[n]$  
- $D_{a3}[n]$  Third-level subband frequencies (high-low-high) of phase A current
- $D_{d3}[n]$  Third-level subband frequencies (high-high-low) of phase A current
- $DFT$  Discrete Fourier Transform
- $FFT$  Fast Fourier Transform
- $I_d[n]$  Direct axis current
- $I_q[n]$  Quadrature axis current
- $\Omega_1, \Omega_2, \Omega_3$  Continuous frequency components in rad/sec
- $g[n]$  Low pass filter coefficients
- $h[n]$  High pass filter coefficients
- $B_L$  Low frequency half band
- $B_H$  High frequency half band
- $\omega_1, \omega_2, \omega_3, \omega_4, \omega_5$  Discrete frequencies
- $\lambda_1$  Threshold
- $\lambda$  Non zero value of $A_{1dq}[n]$  
- $M$  Length of filter
- $db4$  Daubechies 4 wavelet
- $f_1, f_2, f_3$  Continuous frequencies in Hz
- $V_m$  Maximum value of voltage
- $\theta_\lambda$  Phase angle of input voltage
- $R$  Resistance
- $L$  Inductance
- $T$  Transformation matrix
Several fault detection methods based on the wavelet transform (WT) are proposed in [2, 3, 11]. In [2], Brahma introduced the use of WT to detect the symmetrical fault, but the sampling rate of 40.96 kHz is needed to satisfy all of the studied cases. In [3], traveling wave based protection along with WT is used. High sampling rate of 10 kHz and 5th level decomposition is used to detect symmetrical fault with fault detection time of 3 ms. In [11], 20 kHz sampling rate and 8th level wavelet decomposition is used to detect fault and power swing with the response time for fault detection is 1.5 cycles which is slow.

Performances of WPT-based digital and microprocessor based protective relays have demonstrated significant capabilities for identifying and responding to faults in [12,13].

The $d-q$ WPT-based digital relays have been tested for power transformers, motor drives and microgrids [14-17]. This $d-q$ WPT-based digital protection method has shown simplified realization without compromising the accuracy and speed of response. In this paper, the WPT in the $d-q$ axis reference frame has been used to detect symmetrical fault during power swing. Power swing is low frequency oscillation which can be detected by using first level approximation of $d-q$ WPT whereas symmetrical fault is high frequency oscillation which can be detected by using first level details $d-q$ WPT.

2. The $d-q$ WPT-Based Disturbance Detection and Classification Method

2A. Wavelet packet transform

The WPT is one of the most effective and accurate methods for processing signals associated with transient disturbances. It provides extended signal decomposition and it is used to extract frequency contents of the processed signal over narrow frequency sub-bands. Decomposition of the discrete signal $x_d[n]$ using WPT is shown in Fig.1.

**P** Park transformation

$\alpha_a, \alpha_b, \alpha_c$ Time functions of $3\Phi$ quantities

$\alpha_d, \alpha_q, \alpha_0$ Time functions of $d-q0$ components

$T_s$ Sampling time

1. Introduction

Distance relays used for transmission line protection operate on the basis of measured apparent impedance. Power swing cause change in electrical load impedance under steady state condition. During power swing, the apparent impedance measured by a distance relay may move in to the tripping zone of the distance relay characteristics, causing unnecessary trip [1]. Power swing creates oscillations in active and reactive powers following severe disturbances such as line faults, loss of generation and switching heavy loads.

Asymmetrical faults are unbalanced and it produces negative and zero sequence components, makes it possible to detect fault during the power swing. During symmetrical fault and power swing, these components are not present. Symmetrical fault is the most severe fault involving largest current and it occurs rarely compared to unsymmetrical fault. It is difficult for distance relay to distinguish a power swing from a symmetrical fault. To detect symmetrical faults during power swing, several methods have been proposed [2]. It is not possible to detect symmetrical fault during a power swing based on negative and zero sequence components because both power swing and symmetrical fault are both balanced phenomena in [3]. The basic theory of relay to distinguish the power swing from a fault is the speed of impedance moving during the power swing is slower than during the fault condition.

In [4], Fast Fourier Transform (FFT) is used to detect symmetrical fault using frequency components of instantaneous three phase active power, the minimum time required is half cycle and 1 kHz sampling frequency is required. Su et al. [5] used swing center voltage (SCV) to detect symmetrical faults occurred during power swing. Lin et al. [6] presented a method for detecting symmetrical faults by measuring three-phase active and reactive powers. Lotfifard et al. [7] presented a method based on the dc component of three-phase currents, extracted by the Prony method.

Wavelet-based signal-processing techniques are effective tools for power system transient analysis [8-10].
These frequencies will be relocated by

\[ \alpha = h - \frac{\alpha}{2} \]

The relocation of frequencies can be illustrated if one considers two frequencies \( f_1 \) and \( f_2 \), such that \( f_1 < f_s \) and \( f_2 > f_s \). In [12], these frequencies will be relocated by the abc-to-dq0 transformation as shown in Fig.2.

\[ T = \sqrt{\frac{2}{\pi}} \begin{bmatrix} \cos \delta \cos (\frac{2\pi}{3}) & \cos (\frac{\delta + 2\pi}{3}) \\ \sin \delta \sin (\frac{2\pi}{3}) & \sin (\frac{\delta + 2\pi}{3}) \end{bmatrix} \]

where angle \( \delta = 2\pi f_s t = \Omega t \) with \( f_s \) being the fundamental frequency in \( \alpha_d(t), \alpha_q(t), \) and \( \alpha_c(t) \).

The equations (4) and (5) are taken from [17]. The relocation of frequencies can be illustrated if one considers two frequencies \( f_1 \) and \( f_2 \), such that \( f_1 < f_s \) and \( f_2 > f_s \). In [12], these frequencies will be relocated by the abc-to-dq0 transformation as shown in Fig.2.

\[ \begin{align*}
|ABC\text{ Quantities}| & \quad |dq\text{ Quantities}|
\end{align*} \]

Fig.2. Mapping and relocating frequencies using abc-to-dq0 transformation

Relocating and converting continuous frequency components present in 3 phase quantities to discrete frequencies of the \( d-q \) axis components is shown in Fig.3. Mapped and relocated discrete frequencies \( \omega_1 \) and \( \omega_2 \) are related to \( f_1 \), whereas \( \omega_3 \) and \( \omega_4 \) are related to \( f_2 \).
3. Proposed Symmetrical Fault Detection Method

A flowchart of the proposed method in a logical pattern is shown in Fig. 4. 0dq transformation is performed on instantaneous three phase currents and WPT is performed on d–q components.

Fig. 3. Relocating continuous frequency components to discrete frequencies of the d-q axis components

The transformation P in [13] relocates all frequency components present in $a_d(t)$, $a_q(t)$, and $a(t)$, and so that

$$f_c \rightarrow \left\{ \begin{array}{ll} 0 & \{f_c < \frac{f_1 - f_2}{2} \} \end{array} \right.$$  \hspace{1cm} (6)

$$f_1 \rightarrow f_{11} = |f_1 - f_2| \hspace{1cm} (7)$$

$$f_2 \rightarrow f_{12} = |f_2 - f_1| \hspace{1cm} (8)$$

Low frequency half-band $B_L: 0 \leq \omega < \frac{\pi}{2}$

High frequency half-band $B_H: \frac{\pi}{2} \leq \omega < \pi$

$$A_{dq}[n] = \left\{ \begin{array}{ll} \psi sinoidal, \ power \ swing \hspace{1cm} \psi \leq \lambda 1, Normal \hspace{1cm} \psi > \lambda 1, \ Fault \end{array} \right.$$  \hspace{1cm} (9)

$$D_{dq}[n] = \left\{ \begin{array}{ll} \psi sinoidal, \ power \ swing \hspace{1cm} \psi \leq \lambda 1, Normal \hspace{1cm} \psi > \lambda 1, \ Fault \end{array} \right.$$  \hspace{1cm} (10)

where $\lambda 1$ is threshold.

$A_{dq}[n]$ using circular convolution operation is given by

$$A_{dq}[n] = \sum_{k=0}^{M-1} g[k]X_{dq}[n-k] \hspace{1cm} (11)$$

$D_{dq}[n]$ using circular convolution operation is given by

$$D_{dq}[n] = \sum_{k=0}^{M-1} h[k]X_{dq}[n-k] \hspace{1cm} (12)$$

where M is the length of $h[k]$ and $g[k]$. $M=8$

$X_{dq}[n]$ is given as:

$$X_{dq}[n] = \left( I_d[n] \right)^2 + \left( I_q[n] \right)^2 \hspace{1cm} (13)$$

where $I_d[n]$ and $I_q[n]$ are the $d-q$-axis components of the 3Ø components. These components are calculated using the following equations.

$$I_d[n] = \sum_{k=0}^{\frac{N-1}{2}} \left( i_d[n] \cos(\delta) + i_q[n] \cos \left( \delta - \frac{\pi}{3} \right) + i_0[n] \cos \left( \delta + \frac{\pi}{3} \right) \right) \hspace{1cm} (14)$$

$$I_q[n] = \sum_{k=0}^{\frac{N-1}{2}} \left( i_d[n] \sin(\delta) + i_q[n] \sin \left( \delta - \frac{2\pi}{3} \right) + i_0[n] \sin \left( \delta + \frac{2\pi}{3} \right) \right) \hspace{1cm} (15)$$

The Daubechies db4 is employed as wavelet basis functions as in [12]. The eight coefficients of db4 of the LPF and the HPF are as follows:

$$g[k] = \{-0.0106 \ 0.0329 \ 0.0308 \ -0.1870 \ -0.0280 \ 0.6309 \ 0.7148 \ 0.2308\} \hspace{1cm} (16)$$

$$h[k] = \{-0.2304 \ 0.715 \ -0.631 \ -0.028 \ 0.187 \ 0.031 \ -0.0329 \ 0.0106\} \hspace{1cm} (17)$$

Fig. 4. Flowchart for the proposed method
If the $D_{1dq}[n]$ is greater than threshold, then fault is detected. If $A_{1dq}[n]$ is not equal to zero, power swing is detected. This method is insensitive for fault distance, and power swing slip frequency.

3A. Two machine system

In a double frequency system, during power swing, the three-phase current oscillates at the slip frequency. After the fault inception, it oscillates at the nominal frequency (50 or 60Hz). To demonstrate the proposed symmetrical fault detection method, a series of tests by using MATLAB/SIMULINK to a two machine 400-kV transmission system [4] shown in Fig.5. was conducted. The length of the transmission line is 150 km. Symmetrical faults are applied on various distances on transmission line. The system details are given in the Appendix. Table 1 indicates the different cases fault simulation with different values of power swing taken from [4]. The sampling frequency chosen is 2 kHz. If sampling frequency is further increased, the speed of fault detection will be increased. The WPT is carried out every 0.5 ms. The fault can then be detected in quarter power cycle.

\[ R_L + L \frac{d}{dt} \] shown [\sin \beta] [50 km].

Fig.6, shows the model for phase A when symmetrical fault occurs [4]. $V_m \cos(\omega t + \theta_A)$ is the input voltage for the lumped model. is current flowing through the circuit. The phase A current is governed by equation

\[ RL + L \frac{d}{dt} = \frac{V_m \cos(\omega t + \theta_A)}{\sqrt{R^2 + \omega^2 L^2}} \] (18)

\[ i_A(t) = K_A e^{-(\frac{R}{2L})t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \beta_A) \] (19)

where $\beta_A$ can be found by

\[ \beta_A = \tan^{-1}\left(\frac{R \cos \theta_A + \omega L \sin \theta_A}{\omega L \cos \theta_A - R \sin \theta_A}\right) \] (20)

Where $K_A$ is a constant obtained from the initial condition by

\[ K_A = i_A(0) - \frac{V_m}{(R^2 + \omega^2 L^2)} \sin \beta_A \] (21)

Phase B and C currents can be written accordingly as

\[ i_B(t) = K_B e^{-(\frac{R}{2L})t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \beta_B) \] (22)

\[ i_C(t) = K_C e^{-(\frac{R}{2L})t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \beta_C) \] (23)

\[ \beta_B = \beta_A + \frac{2\pi}{3} \] (24)

\[ \beta_C = \beta_A + \frac{2\pi}{3} \] (25)

\[ K_B = i_B(0) - \frac{V_m}{(R^2 + \omega^2 L^2)} \sin \beta_B \] (26)

\[ K_C = i_C(0) - \frac{V_m}{(R^2 + \omega^2 L^2)} \sin \beta_C \] (27)

Table 1

<table>
<thead>
<tr>
<th>Case</th>
<th>Power swing slip frequency (Hz)</th>
<th>Fault distance to relay R1(km)</th>
<th>Fault inception time (s)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
<td>1.50</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>75</td>
<td>1.75</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>145</td>
<td>2.00</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>5</td>
<td>0.75</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>75</td>
<td>0.87</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
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<td>7</td>
<td>3</td>
<td>5</td>
<td>0.83</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>75</td>
<td>0.91</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>145</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>5</td>
<td>0.62</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>75</td>
<td>0.68</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>145</td>
<td>0.75</td>
</tr>
<tr>
<td>13</td>
<td>5</td>
<td>5</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Fig.5. Simulated two machine system.

3B. Three phase currents after symmetrical fault inception

To extract the phase currents during symmetrical fault, lumped model for faulty system is used.

Fig.6. Lumped model for Phase A after symmetrical fault inception.
4. Simulation results of two machine system

The simulation results of the two machine system for the different cases in Table 1 are shown in the following figures.

In Fig. 7, the A1dq[n] is having non-zero sinusoidal oscillations and it indicates the presence of power swing during time period 1-5s. For normal conditions, A1dq[n] is zero and it indicates there is no power swing during the time duration 5-10s. The results show that the first stage approximation A1dq[n] itself detects the power swing. The details D1dq[n] shows the impulse signal at the starting and end of the swing. Hence, A1dq[n] is used to detect power swing and D1dq[n] is used to detect the fault. Due to the impulse signal at the starting and end of the swing, a threshold is fixed to operate for fault condition.

In WPT method, the Dad3[n] indicates the sinusoidal signal during power swing, the Dda3 does not detect the power swing. Hence, Dad3[n] is used to detect power swing and Dda3[n] is used to detect fault condition. In DFT method, the DFT signal indicates the sinusoidal signal during power swing and for the remaining time duration, the DFT signal has constant magnitude.

Fig. 8. Three phase currents, d-q-axis components Id and Iq, High-frequency sub-band D1dq[n], Dda3[n] of WPT and DFT of phase A current measured in case 1 in table 1

Fig. 9. Three phase currents, d-q-axis components Id and Iq, High-frequency sub-band D1dq[n], Dda3[n] of WPT and DFT of phase A current measured in case 2 in table 1
Fig. 10. Three phase currents, \(d-q\)-axis components \(I_d\) and \(I_q\), High-frequency sub-band \(D_{1dq}[n]\), \(D_{da3}[n]\) of WPT and DFT of phase A current measured in case 3 in table 1.

Fig. 11. Three phase currents, \(d-q\)-axis components \(I_d\) and \(I_q\), High-frequency sub-band \(D_{1dq}[n]\), \(D_{da3}[n]\) of WPT and DFT of phase A current measured in case 4 in table 1.

Fig. 12. Three phase currents, \(d-q\)-axis components \(I_d\) and \(I_q\), High-frequency sub-band \(D_{1dq}[n]\), \(D_{da3}[n]\) of WPT and DFT of phase A current measured in case 5 in table 1.

Fig. 13. Three phase currents, \(d-q\)-axis components \(I_d\) and \(I_q\), High-frequency sub-band \(D_{1dq}[n]\), \(D_{da3}[n]\) of WPT and DFT of phase A current measured in case 6 in table 1.
Fig. 14. Three phase currents, $d-q$-axis components $I_d$ and $I_q$, High-frequency sub-band $D1dq[n]$. $Dda3[n]$ of WPT and DFT of phase A current measured in case 7 in table 1.

Fig. 15. Three phase currents, $d-q$-axis components $I_d$ and $I_q$, High-frequency sub-band $D1dq[n]$. $Dda3[n]$ of WPT and DFT of phase A current measured in case 8 in table 1.

Fig. 16. Three phase currents, $d-q$-axis components $I_d$ and $I_q$, High-frequency sub-band $D1dq[n]$. $Dda3[n]$ of WPT and DFT of phase A current measured in case 9 in table 1.

Fig. 17. Three phase currents, $d-q$-axis components $I_d$ and $I_q$, High-frequency sub-band $D1dq[n]$. $Dda3[n]$ of WPT and DFT of phase A current measured in case 10 in table 1.
From Figs. 8-20. $D1dq[n]$ are non-zero values for fault duration and it is zero for normal time durations. The first stage of $D1dq[n]$ itself detects the symmetrical fault during power swing. In WPT method, $Dda3$ signal is non-zero for symmetrical fault and zero for normal time durations and during power swing. In DFT method, the DFT coefficients are non-zero for normal, power swing and fault conditions. But the values of coefficients during fault condition are high.

In Fig. 7. $D1dq[n]$ have considerable value during power swing conditions. Hence, threshold is necessary so that the relay will not operate for power swing. By comparing the $D1dq[n]$ with a predetermined threshold $\lambda$, this method can detect a symmetric fault occurring during a power swing within quarter power cycle. The threshold value was determined by a systematic numerical study of a series of cases for this power system. In two machine system, for all (1-13) cases, the maximum threshold value is 6000, i.e $\lambda_1 = 6000$ for two machine system

$|D1dq[n]| > \lambda_1$, fault is detected  
$|D1dq[n]| < \lambda_1$, fault is not detected

Comparisons of results are shown in table 2 and table 3.
Table 2
Comparison of results for various transforms for two machine system

<table>
<thead>
<tr>
<th>Transforms</th>
<th>Normal</th>
<th>Power swing</th>
<th>Symmetrical fault during power swing</th>
</tr>
</thead>
<tbody>
<tr>
<td>dq WPT</td>
<td>A1dq[n] is zero</td>
<td>A1dq[n] is non-zero sinusoidal oscillation, D1dq[n] is non-zero and greater than threshold</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D1dq[n] is zero</td>
<td>D1dq[n] is less than threshold</td>
<td></td>
</tr>
<tr>
<td>WPT</td>
<td>Dda3 is zero</td>
<td>Dad3 is non-zero sinusoidal oscillation, Dda3 is zero</td>
<td></td>
</tr>
<tr>
<td>DFT coefficients</td>
<td>non-zero and constant value</td>
<td>sinusoidal oscillation</td>
<td>non-zero and it is greater that of normal</td>
</tr>
</tbody>
</table>

Table 3
Comparison of the $d$–$q$ WPT, WPT and DFT Methods

<table>
<thead>
<tr>
<th>Criterion</th>
<th>$d$–$q$ WPT</th>
<th>WPT</th>
<th>DFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity to Sampling Frequency</td>
<td>None</td>
<td>Sensitive</td>
<td>Sensitive</td>
</tr>
<tr>
<td>Inputs</td>
<td>$I_d$ and $I_q$</td>
<td>$i_a$, $i_b$, $i_c$</td>
<td>$i_a$, $i_b$, $i_c$</td>
</tr>
<tr>
<td>Analog implementation</td>
<td>1 HPF</td>
<td>3 LPFs and HPFs</td>
<td>3 LPFs</td>
</tr>
<tr>
<td>Inception angle</td>
<td>Insensitive</td>
<td>Insensitive</td>
<td>Sensitive</td>
</tr>
<tr>
<td>Level of Fault current</td>
<td>Insensitive</td>
<td>Insensitive</td>
<td>Sensitive</td>
</tr>
<tr>
<td>Memory requirement</td>
<td>Less</td>
<td>More</td>
<td>More</td>
</tr>
</tbody>
</table>

5. Conclusion
A novel scheme for detecting symmetrical faults occurring during power swings was presented based on the value of details of $d$–$q$-WPT coefficients of the three-phase currents after the fault inception within quarter power cycle. The presented digital protection has been structured to extract the high-frequency sub-band present in the $d$–$q$-axis components of the 3Φ currents. The proposed method has several advantages that are superior to the existing methods. The new method is very fast. While using a sampling frequency of only 2 kHz, it can detect the fault within quarter power cycle. Employment of a single-stage WPT, which has simplified the implementation and removed any constraints on the sampling frequency of measured currents. The $d$–$q$ WPT-based digital protection has been simulated for performance evaluation on two machine system through a systematic numerical study of more than 10 disturbance conditions. In all tests, power swing frequency, fault locations have not affected responses of the $d$–$q$ WPT-based digital protection. Finally, the $d$–$q$ WPT-based digital protection has shown performance superiority over other digital protection methods in terms of response speed, simplicity, memory requirements. The distinct features and performance capabilities of the $d$–$q$ WPT-based digital protection support its applications for discriminating power swing and symmetrical fault in two machine system.

APPENDIX
The parameters of the two machine system used for simulation (Fig. 5) are given.

Source impedance
\[
Z_{1S} = 2.21 + j25.04\Omega \\
Z_{0S} = 4.90 + j31.51\Omega
\]

Transmission line: Distributed model with parameters
\[
Z_{1L} = 0.03 + j0.34\frac{\Omega}{km} \\
Z_{0L} = 0.28 + j1.04\frac{\Omega}{km}
\]
\[
C_{1L} = 12.74\frac{\mu F}{km},\quad C_{0L} = 7.75\frac{\mu F}{km}
\]

References