STUDY OF TRANSIENT AND STABILITY OF SYNCHRONOUS RELUCTANCE MOTOR USING COMBINED FEM-CIRCUIT MODEL CONSIDERING CROSS MAGNETIC SATURATION

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Abstract: In this paper, a combined FEM-circuit model of the Synchronous Reluctance Motor (SynRM) is presented, in which the lumped circuit parameters are calculated numerically using FEM. The inductance cross-magnetic saturation is considered and used as lookup table in the circuit model. This model combines the advantages of both methods; the accuracy of FEM and ability to include geometrical and physical effects relevant to non-linearity, in addition to the speed of circuit models and ability of simulating controller/drive applications or simulating the effect of unbalanced/non-ideal supply. The dynamic operation and stability limits at supply voltage sag are also investigated to demonstrate the importance of including non-linear cross-magnetic effects in circuit models using the proposed approach.

Key words: FEM-Circuit model, SynRM, Cross-magnetic saturation, dynamic operation

1. Introduction
Synchronous Reluctance Motor (SynRM) has attracted the researchers and manufacturers in the last few decades, as it can become a viable alternative of the induction motor in industry, especially in variable speed drive applications [1]. The main advantages of the SynRM are the relatively low cost, robust design, the lack of need of rare-earth magnets and the better field weakening capability compared to permanent magnet machines. It has theoretically no rotor losses due to the absence of rotor cage, which leads to lower rotor temperatures compared to induction machines. Its operation depends basically on the saliency of the rotor [2-4]. The basic design principle for a SynRM rotor is to create a magnetic structure that provides minimum magnetic reluctance in one direction (d-axis) and maximum reluctance in the other direction (q-axis). A reluctance torque is produced to align the rotor at the minimum reluctance position with respect to the rotating magnetic field. The saliency ratio \(L_d/L_q\) is the main factor which decides the performance of the reluctance machine, where \(L_d\) and \(L_q\) are the d-axis and q-axis inductances, respectively. The higher the saliency ratio; the better is the performance of the machine, in terms of torque, efficiency and power factor. The main disadvantage of the SynRM is its low power factor compared to induction or permanent magnet machines, which can be improved by adding permanent magnets in the flux barriers, which is called the PM-Assisted SynRM [3]. Many researchers have concerned finding an optimal design for the rotor to increase this ratio [2, 4-7], through studying the effect of various design parameters, such as rotor shape and material, number of barriers, barriers geometry and dimensions, etc., on the performance of the motor.

Beside the design methodology, there is a strong need of modeling the SynRM as fully and as accurately as possible. This accurate model improves the estimation and study of the
Machine behavior and dynamics, in addition to the determination of the machine’s operating figures such as electromechanical torque, power-losses and efficiency. Various modeling techniques are used with varieties in accuracy level and model calculation time. Modeling of the electrical machines can be performed by analytical methods, lumped circuit models, Finite Element Method (FEM) and coupled multi-physics modeling [8]. Numerical modeling techniques, such as FEM, are valuable in exploring the overall behavior of the machine considering all possible physical effects simultaneously, such as nonlinear magnetic behavior especially at overload operation, slotting effects on slot harmonics and torque ripples, etc. On the other hand, FEM is time and computationally consuming. In addition, it is difficult to include the effect of solid state drives and controllers in speed control applications in FEM. Therefore, coupled or combined methods can be used to include various effects and to increase the speed of model computation. One of the effective coupling techniques is combining the analytical circuit model of the machine with a numerical technique in order to reduce the computational efforts, for example for dynamic simulations of motor or observer models for the machine control. The coupling of methods in this case is beneficial to get both an accurate and simple overall model of the machine [9-11].

The operation and control of the SynRM is highly affected by the magnetic saturation caused by the nonlinear magnetic characteristics of the ferromagnetic material constructing the main parts of the machine. Inductances \( L_d \) and \( L_q \) are not only dependent on their self-currents, \( i_d \) and \( i_q \) (the \( d \)- and \( q \)-axis currents respectively), but also on the mutual of these current components as they both share the same flux path. This effect is called cross-saturation, \( L_d(i_q,i_d) \) and \( L_q(i_q,i_d) \). Therefore, accurate representation of the magnetic saturation is required to obtain a powerful model of the machine. Many papers have focused on determining the cross-saturation effect on the machine performance, and its control [12-17]. The non-linearity of \( L_d \) and \( L_q \) including self and mutual inductance can affect the observed performance of the SynRM particularly when it comes to the output torque, power factor, voltage sag and stability limits.

In this paper, a combined FEM-Circuit method is developed to study the performance of SynRM. Generally, the approach used to develop such model is applicable to any type of machine. First, FEM is used to study the motor performance and calculate flux distribution and developed torque. Second, FEM is used to calculate the lumped parameters of the circuit model of the machine, considering nonlinear magnetic properties of the core material. Cross-saturation effect including non-linearity due to ferromagnetic saturation is calculated from FEM and used as lookup table in circuit model. The circuit model is then used to investigate the motor performance and stability limit in simple and fast way under unbalanced/non-ideal supply. The developed model can include non-linear core effects as well as effect of defects and faults. The model can also identify the effect of the change of any design parameter on all observed motor performance features as well as losses.

The paper aims at demonstrating that through coupled modeling, the effect of non-linearity of core material can be included in a robust computational efficient model, as well as other effects discussed above. The main outcome is a circuit model which can accurately predict SynRM performance (as well as other types of electrical machines). For instance, the implementation of such model shows that non-linearity can affect performance crucial parameters of SynRM such as torque capability, stability limit and power factor, which further shows the importance of coupled modeling approach which achieves appropriate balance between computation speed and accuracy. These effects for instance are essential in applications related to assessment of the motor performance in design stage in applications such as in control drive operation or electric vehicle.
2. Modeling and Methodology

2.1. Overall approach

The used approach in this paper is to calculate the inductances of the lumped circuit model using static FEM, which is less time and resource consuming than the dynamic or transient FE calculation. The sequence of calculating the cross-saturated inductances is presented in the flowchart in Figure (1). First, the considered values of \(dq\)-currents are transferred to the equivalent \(abc\)-currents. Then, the FEM is used to calculate the \(abc\)-fluxes in the machine, which in turn converted to the \(dq\)-fluxes. Finally, the inductances are obtained by dividing the \(dq\)-fluxes by the \(dq\)-currents.

In the next subsections the machine model and operating figures are presented. First, a transient FEM calculation is used to determine the motor torque and overall performance. Then, the circuit model is introduced in both \(abc\) and \(dq\) equations. Finally, the static FEM analysis is used to calculate the lumped parameters for the circuit model, including the cross magnetic saturation, and the combined FEM-Circuit model is presented.

2.2. Finite Element Computations and Model

Finite Element Method (FEM) is a widely used numerical technique in electrical machine modeling, design, analysis and optimization. FEM is used to solve the magnetic field non-linear Partial Differential Equation (PDE) derived from the Maxwell’s equations combined to the materials properties, and their coupling with other phenomena in electromagnetic structures, such as electric circuits, and mechanical motional equations.

In this paper a 4-pole SynRM is considered. A standard motor model is extracted from Infolytica MotorSolve 4.1 software package, and its dimensions are exported to the FEM software to evaluate the motor torque. The rated values, basic machine design data and geometries are illustrated in Table 1. Figure (2) presents one-half of the motor under study, showing the phase-distribution and the used solution mesh analysis.

The used core material is Ma-19 29 Ga Silicon Steel, with nonlinear magnetic properties as presented in the \(B-H\) curve shown in Figure (3). The magnetic flux density and flux lines at rated current operation are presented at both \(d\)- and \(q\)-positions, as shown in Figure (4).
Table (1): Basic specifications of the design values of the machine

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>2500 W</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>380 V</td>
</tr>
<tr>
<td>Rated current</td>
<td>10 A</td>
</tr>
<tr>
<td>Rated speed</td>
<td>1500 rpm</td>
</tr>
<tr>
<td>Number of poles</td>
<td>4</td>
</tr>
<tr>
<td>Stator outer diameter</td>
<td>150 mm</td>
</tr>
<tr>
<td>Stator inner diameter</td>
<td>83.3 mm</td>
</tr>
<tr>
<td>Number of stator slots</td>
<td>24</td>
</tr>
<tr>
<td>Rotor outer diameter</td>
<td>81.7 mm</td>
</tr>
<tr>
<td>Rotor inner diameter</td>
<td>13.3 mm</td>
</tr>
<tr>
<td>Number of flux barriers</td>
<td>3</td>
</tr>
</tbody>
</table>

The motor torque versus torque angle is calculated by transient FEM analysis at different stator current and is presented in Figure (5). The instantaneous developed torque at different stator current versus time is presented in Figure (6). In addition to the average torque and ripples (defined as the difference between max and min torques) are illustrated in Figure (7). The percentage torque ripples range from 17% to 21%.

2.3. Lumped Circuit Model of SynRM

A) ABC Model

For a three-phase cageless-rotor SynRM, the motor equations in the phase coordinates can be represented as [18]
Where  

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c \\
\end{bmatrix}
= 
\begin{bmatrix}
R_s & 0 & 0 \\
0 & R_s & 0 \\
0 & 0 & R_s \\
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c \\
\end{bmatrix}
+ 
p
\begin{bmatrix}
\lambda_a \\
\lambda_b \\
\lambda_c \\
\end{bmatrix}
\]  
(1)

The phase inductances vary sinusoidally with the rotor position, such that

\[
L_{aa} = L_t + L_m + L_q \cos(2\theta_r)  
\]  
(3)

\[
L_{bb} = L_t + L_m + L_q \cos(2\theta_r + 2\pi/3)  
\]  
(4)

\[
L_{cc} = L_t + L_m + L_q \cos(2\theta_r - 2\pi/3)  
\]  
(5)

\[
L_{ab} = -\frac{L_m}{2} + L_q \cos(2\theta_r - 2\pi/3)  
\]  
(6)

\[
L_{bc} = -\frac{L_m}{2} + L_q \cos(2\theta_r)  
\]  
(7)

\[
L_{ac} = -\frac{L_m}{2} + L_q \cos(2\theta_r + 2\pi/3)  
\]  
(8)

$L_{ab}$, $L_{bb}$ and $L_{cc}$ are the self-inductances of abc-phases respectively, while $L_{ab}$, $L_{bc}$ and $L_{ac}$ are the mutual inductances between phases. $\theta_r$ is defined as the angle between $d$-axis of the rotor and the axis of phase-$a$.

The electromechanical developed torque is obtained as

\[
T_e = \frac{p}{2} [i_a \quad i_b \quad i_c] \begin{bmatrix}
0 & L_{abc}(\theta_r) \\
L_{abc}(\theta_r) & 0 \\
\end{bmatrix} \begin{bmatrix}
i_a \\
i_b \\
i_c \\
\end{bmatrix}
\]  
(9)

### B) d-q Model

When applying Park’s transformation, the dq-model equations can be obtained as

\[
V_d = R_s i_d + \frac{dL_d}{dt} - \omega_r \lambda_q  
\]  
(10)

\[
V_q = R_s i_q + \frac{dL_q}{dt} + \omega_r \lambda_d  
\]  
(11)

Where,

\[
\lambda_d = L_d i_d  
\]  
(12)

\[
\lambda_q = L_q i_q  
\]  
(13)

\[
L_d = L_t + L_{ad}  
\]  
(14)

\[
L_q = L_t + L_{aq}  
\]  
(15)

And the developed torque with the mechanical equation are represented as

\[
T_e = \frac{3}{2} P (L_d - L_q) i_q i_d  
\]  
(16)

\[
T_e - T_L = \frac{L_d}{p} \frac{d\omega_r}{dt} + \frac{B}{p} \omega_r  
\]  
(17)

The dq-model inductances are related to the phase inductances as

\[
L_{ld} = \frac{3}{2} (L_m + L_q)  
\]  
(18)

\[
L_{lq} = \frac{3}{2} (L_m - L_q)  
\]  
(19)

These equations, either in abc-frame (equations 1-9) or dq-frame (equations 10-17), can be used to estimate the motor performance during transient and steady-state operation, especially at the application of inverter-fed variable speed motor, or unbalanced/non-ideal voltage supply.

### 3. Combined FEM-Circuit Model of SynRM

Solving the circuit model equations to get the motor performance requires the determination of the lumped circuit parameters. The lumped parameters of the circuit model can be determined by experimental tests, classical analytical expressions or by the FEM based on the motor geometries and known materials properties. Determining the parameters using FEM is an accurate method, compared to analytical method, as it can consider the non-linear material properties, motor geometry and physics which are usually neglected by the analytical models for simplifying the calculations. These non-linear effects such as saturation, slot harmonics and coupled physics are crucial for predicting the machine behavior. All such effects can be included in the approach used in the present paper.
3.1. Parameters Identification Using 2D FEM

The most effective parameter in determining the SynRM performance is the inductance. In this paper, inductances are calculated from static FEM including magnetic saturation and they are included in the circuit model as lookup table as function of current.

If the abc-model (equations 1-9) is used in determining the motor performance, in such case, phase inductances should be calculated, and the saturation effect will be represented in the saturated values of leakage and mutual inductances, $L_b$, $L_m$, and $L_0$. Phase inductances are estimated using static FEM, by injecting current in one phase and calculating the self and mutual fluxes. Inductances can be then calculated by dividing the fluxes by the current. These steps are repeated at different stator current to evaluate the effect of saturation. For coil of phase-A is carrying the rated current, the phase inductances waveforms are shown in figure (8). The saturated mutual and leakage inductances are shown in Figure (9) versus the stator current.

For $dq$-model (equations 10-17), the cross magnetic saturation is evaluated as illustrated above in the procedure in figure (1). The results of the static FEM procedure of calculating the inductances $L_d(i_d,i_q)$ and $L_q(i_q,i_d)$ are presented in Figures (10) and (11), showing the cross saturation effect in both $d$- and $q$-axes fluxes and inductances. It is clear that the saturation is more noticeable in the $d$-axis inductance due to the longer magnetic path of the flux compared to that of the $q$-axis.

Another important circuit parameter is the stator resistance ($R_s$). It can be calculated in two methods. The first one in using the classical analytical formula of resistance as in equation (20)

$$R_s = \frac{N_s l_s}{\sigma a_s}$$  \hspace{1cm} (20)

Where $N$ is number of turns per phase, $l_s$ is the mean length of the stator turn, $\sigma$ is the electrical conductivity of the used copper and $a_s$ is the cross-section area of the stator conductor. The other method is to calculate the resistance from the calculated copper losses from the FEM analysis. The obtained results from both calculation methods are similar (after compensating the end winding resistance and losses that is not calculated in 2D FEM). The stator resistance was calculated as 1.7 $\Omega$/phase (% difference between the two methods above = 0.6%).

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Fig. 8. FEM analysis of $L_{aa}$, $L_{ab}$ and $L_{ac}$ at rated current

Fig. 9. Saturated mutual and leakage inductances versus current
3.2. Motor Transient Operation and Stability Analysis

After obtaining the lumped circuit parameters from FEM, they are used in MATLAB Simulink model to estimate the motor performance in a simple and fast way. The values of the cross-saturated inductances are generated using a lookup table in the model, based on the instantaneous values of dq-currents as determined in the FEM model.

Figures (12) and (13) illustrate the torque and speed responses, respectively, when the motor is loaded by 10 N.m. from no-load. It compares the transient response of the motor model using constant values of the inductances $L_d$ and $L_q$, against the response when using the cross-saturated values, $L_d(i_d, i_q)$ and $L_q(i_q, i_d)$, as estimated from FEM analysis. It can be shown that the cross-saturation improves the dynamic response of the motor and settles the torque and speed faster. It was also found that the cross-saturation not only affects the transient response of the motor, but also increases its stability limits and loading capability. For the constant-inductance model, it was found that the maximum limit of the applied load can be 12 N.m., then the motor loses its synchronism and its speed reduces to zero. On the other hand for the cross-saturated model, the torque capability of the motor increases up to 20 N.m. (i.e. about 170% of the constant-inductance model, matching the results presented in [14]). This is illustrated in Figures (14) and (15), showing the torque capability of both models and the speed response of both models. The impact of considering non-linear cross magnetic effects on predicted performance of the machine is shown to be crucial.
In addition, the response and transient stability of both models of SynRM are studied at the case of voltage sag in the supply. A three-phase voltage sag of 80% for duration of 0.5 second is applied while the motor is loaded by the rated torque of 10 N.m, as shown in Figure (16) for the waveform of phase-A voltage \( V_a \). It can be shown from figures (17) and (18) that the cross-saturated model can withstand this voltage sag level and the motor continues its normal operation after the recovery of the nominal voltage. On the other hand, due to the voltage sag, the fixed-inductance model showed less stability to this disturbance and the motor stops. It was found that the minimum voltage sag level that the fixed-inductance model can withstand is only 88%.
From the presented results, it can be shown that cross magnetic saturation plays a crucial role in determining the SynRM performance and stability limits, which in turns affect the evaluation and simulation process of the motor in different applications. For example, SynRM was presented as a powerful alternative in electric vehicle (EV) application due to its low losses, compact design and fast torque response, which are very important requirements for such an application. The exact study and estimation of the motor characteristics, performance and stability limits can affect its assessment in EV application when considering the torque capability for high acceleration requirements and stability against battery voltage drop during operation [19, 20].

4. Conclusion

In this article, a combined FEM-Circuit model for the SynRM is introduced. The model includes non-linearity of core material and cross saturation. The parameters of the lumped circuit model are calculated using static FEM. Cross magnetic saturation is modeled and the values of the cross-saturated inductances are stored as a lookup table in the model. The model is then used to study the transient performance of the motor and its stability limits. It was shown that the torque capability is increased and oscillations are decreased when the cross-saturation is considered. In addition, the capability of the motor to withstand voltage sag is increased. Including non-linearity and cross saturation increases the torque capability by 170% and stability against voltage sag by 150%, which agrees with near results from other articles in literature.

Such coupled model which can include non-linear effects in fast efficient circuit model can allow designers to use the non-linearity of the core positively to improve dynamic response of the machine. The model can also reflect the effect of each design parameter on dynamic behavior of the machine which can be easily included in a full system simulation such as with drivers, controllers and electric vehicle. The modeling approach is flexible enough to include design parameters and dimension variations, leading to better prediction of motor behavior in circuit model which can include motor drives and control systems.

References


8. _: The art of modelling Electrical Machines, Technical article, Institute of Electrical


