MATHEMATICAL MODELING OF COMPOSITE FILTER FOR POWER QUALITY IMPROVEMENT OF ELECTRIC ARC FURNACE DISTRIBUTION NETWORK

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Abstract: This paper presents mathematical modeling of composite filter (CF) for power quality improvement of electric arc furnace (EAF) distribution network. The composite filter is consisting of a shunt passive filter (SHPF) connected with a lower rated voltage source pulse width modulation (PWM) converter based series active power filter (SAPF). A state-space averaging model of a CF is constructed to analyze its system stability by traditional control strategy taking into account the effect of the time delay. The control strategy adopted for composite filter operation is based on simultaneous detection of source current and load voltage harmonic based on the vectorial theory dual formulation of instantaneous reactive power. Simulation for a typical EAF distribution network with a composite power filter has been carried out to validate the performance. Simulation results are shown in an attempt to verify the mathematical model of the filter. The simulations have been carried out in MATLAB environment using SIMULINK and power system block set toolboxes.

Index Terms— Active filter, harmonics, harmonic distortion, arc, mathematical modeling, stability analysis

1. Introduction

The increasing popularity of EAF in metallurgical industries to melt scrap causes significant impacts on power system and on electrical power quality. EAF is one of responsible source for deteriorating the power quality in the connected network. An EAF is chosen as an industrial non-linear load to demonstrate power quality problems.

Harmonic distortion in power distribution systems can be suppressed using two approaches namely, passive and active powering. The passive filtering is the simplest conventional solution to mitigate the harmonic distortion [1]. Although simple, the use passive elements do not always respond correctly to the dynamics of the power distribution systems. Although simple and least expensive, the passive filter inherits several shortcomings [2]. Combined operation of SAPF with traditional SHPF i.e. composite filter (CF) is found one of the solutions to overcome the disadvantages of the existing passive filters. It controlled to act as a harmonic isolator between the source and nonlinear load by injection of a controlled harmonic voltage source [3-5]. A control algorithm for a three-phase series APF is proposed as shown in Fig. 1.

The control strategy is based on the vectorial theory dual formulation of instantaneous reactive
power [5], so that the voltage waveform injected by the active filter is able to compensate the reactive power and the load current harmonics and to balance asymmetrical loads. Here an attempt is made to apply the same theory for unbalanced and non-sinusoidal voltage conditions for randomly varying load as an electric arc furnace (EAF).

System stability during CF operation is also one of the important factors from control aspects. Choice of a suitable gain to ensure a good compensation is a tedious and difficult task while keeping the system stable for different load and source condition [6-8]. The analysis and experimental results have pointed out that a time delay can cause a stability problem [9-11, 13]. Hence, a state-space averaging model of composite filter is constructed to analyze the stability problem by traditional control strategy taking into account the delay time. Bode plot of open-loop transfer function concludes that larger time delay could results into phase angle loss seriously leading to system unstable and poor filter performance. A new leading compensating control strategy discussed in [12-14] is applied for enhancing the system stability, for improving the filter performance and for reducing the power rating of the active part. The simulations have been carried out in MATLAB environment using SIMULINK and power system block set toolboxes to validate the same. The paper is organized as follows: Second section deals with the mathematical modeling of EAF as non-linear load. Third section discusses the stability analysis. Fourth section shows various calculations involved in stability analysis. Fifth section deals with the analysis of simulation results. Sixth section concludes the paper.

2. EAF Modeling as Non-linear Load

A. Main circuit configuration

The system configuration of a CF consists of a SAPF with a SHPF is shown in Fig. 2.

Main circuit configuration can be divided into four parts: (a) an ac main (b) SAPF (c) SHPF (d) EAF as non-linear load. SAPF can also be further subdivided into three parts: (a) a three phase bridge Inverter (generating harmonic voltage for compensation) (b) a Second order Carrier-wave filter (applied to filter the switching frequency component) (c) a Coupling transformer (connected between the electric network and the load with transformation ratio n).

B. System Modeling

The output of voltage of the inverter \( v_i \) changes between \( \pm V_{dc} \) when the IGBTs are switching. The inverter output voltage is represented by:

\[
v_i = V_{dc} \cdot (2 \cdot V - 1)
\]

Where, \( V = [0,1] \), represents the input discrete values of IGBTs. The controlled output voltage \( v_c \), so generated will constrain the harmonics produced by the load. The state-Space averaged model of \( v_i \) in the stationary frame of reference is described as follows:

Applying KCL at node we get:

\[
f_c \cdot \frac{dv_c}{dt} = \frac{1}{n} \cdot i_s + i_f
\]

Applying KVL at node we get:

\[
V_{dc} \cdot (2 \cdot V - 1) - i_f \cdot R_f - v_c - L_f \cdot \frac{di_f}{dt} = 0
\]

\[
\therefore L_f \cdot \frac{di_f}{dt} = [v_c + i_f \cdot R_f - V_{dc} \cdot (2 \cdot V - 1)]
\]

\[
v_s = \frac{1}{n} \cdot v_c - v_o - L_s \cdot \frac{di_c}{dt} = 0
\]

Fig. 2 Main system configuration
Let the state variables are,

\[ x_1 = v_c \Rightarrow x_1 = \frac{dv_c}{dt} \]  
\[ x_2 = i_r \Rightarrow x_2 = \frac{di_r}{dt} \]  
\[ x_3 = i_s \Rightarrow x_3 = \frac{di_s}{dt} \]

Re-writing the equations (2-4) using equations (7-9):

\[ \dot{x}_1 = 0 \cdot x_1 + \frac{1}{C_f} \cdot x_2 + \frac{1}{n \cdot C_f} \cdot x_3 \]  
\[ \dot{x}_2 = \left( -\frac{1}{L_f} \right) \cdot x_1 + \left( -\frac{R_f}{L_f} \right) \cdot x_2 + 0 \cdot x_3 + \frac{V_{dc}}{L_f} \cdot (2 \cdot V - 1) \]  
\[ \dot{x}_3 = \frac{1}{L_s} \cdot x_1 + 0 \cdot x_2 + \left( -\frac{Z_F}{L_s} \right) \cdot x_3 + \left( \frac{v_s}{L_s} + \frac{i_s \cdot Z_F}{L_s} \right) \]

The equivalent state-space model can be presented as:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} = \begin{bmatrix}
0 & \frac{1}{C_f} & \frac{1}{n \cdot C_f} \\
\frac{1}{L_f} & -\frac{R_f}{L_f} & 0 \\
\frac{1}{n \cdot L_s} & 0 & -\frac{Z_F}{L_s}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 \\
0 & \frac{1}{L_f} & 0 \\
0 & \frac{1}{L_s} & \frac{Z_F}{L_s}
\end{bmatrix}
\begin{bmatrix}
V_{dc} \\
v_s \\
i_L
\end{bmatrix}
\]

\[ y = x_4 = v_c = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{dc} \\ v_s \\ i_L \end{bmatrix} \] (14)

Equation (13) is a state equation and equation (14) is an output equation. Comparing them with the standard form,

\[ \dot{x} = A \cdot x + B \cdot u \]  
\[ y = C \cdot \tilde{x} + D \cdot \tilde{u} \]

These yields:

\[ A = \begin{bmatrix} 0 & \frac{1}{C_f} & \frac{1}{n \cdot C_f} \\
\frac{1}{L_f} & -\frac{R_f}{L_f} & 0 \\
\frac{1}{n \cdot L_s} & 0 & -\frac{Z_F}{L_s}
\end{bmatrix} \]  
\[ B = \begin{bmatrix} 0 \\
0 \\
0
\end{bmatrix} \]  
\[ C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \]  
\[ D = \begin{bmatrix} 0 & 0 \end{bmatrix} \]  

Now Transfer Function of the system is given by:

\[ G(s) = C[sI - A]^{-1}B + D \]

3. Stability Analysis

Fig. 3 shows an equivalent single-phase circuit of the system considering current and voltage harmonics produced by EAF.

![Fig. 3 Single phase equivalent circuit of main system](image)

![Fig. 4. Single phase equivalent harmonic circuit of main system](image)
Fig. 4 shows single-phase equivalence harmonic circuit of system under consideration. The EAF is represented by harmonic current and harmonic voltage source. From the equivalent circuit shown in Fig. 4, it is obtained:

\[ v_o = (i_{sh} - i_{lh}) \cdot Z_F \]

\[ v_{AF} = K \cdot i_{sh} \] (22)

Applying KVL:

\[ v_{sh} - i_{sh} \cdot Z_s - v_{AF} - v_o - v_{lh} = 0 \]

\[ v_{sh} - i_{sh} \cdot Z_s - (K - i_{sh}) \cdot Z_F = v_{lh} = 0 \]

\[ v_{sh} - i_{sh} \cdot Z_s - K \cdot i_{sh} - i_{lh} \cdot Z_F + i_{lh} \cdot Z_F - v_{lh} = 0 \]

\[ (v_{sh} - v_{lh}) - i_{sh}(Z_s + K + Z_F) + i_{lh} \cdot Z_F = 0 \]

\[ Z_{lh} = \frac{Z_s}{Z_s + K + Z_F} + \frac{Z_F}{Z_s + K + Z_F} \] (24)

With respect to the performance of the compensation, the system behaves like a closed-loop control system. Equation (24) can be represented by traditional closed-loop model of composite filter as shown in Fig. 5. Therefore, the analysis in the s-domain could be developed with the help of block diagram shown in Fig. 6.

Fig. 5 Block diagram of closed loop model of CF

\[ [v_{sh}(s) \cdot v_{lh}(s)] \]

\[ i_{lh}(s) \rightarrow Z_{v}(s) \rightarrow 1 \rightarrow Z_{i}(s) \rightarrow Z_{v}(s) \rightarrow i_{l}(s) \]

Fig. 6 Reduced block diagram of closed loop model of CF

The open-loop transfer function \( k(s) \) from input source current to output voltage can be obtained as:

\[ k(s) = \frac{v_c(s)}{i_{s}(s)} = k_c(s) \cdot A(s) \cdot \frac{K_{PWM}}{n} \cdot k_v(s) \cdot k_f(s) \] (25)

where, \( k_c(s) = \frac{k_c}{T_c \cdot s + 1} \), \( A(s) = \frac{s^2 + \omega_0^2}{s^2 + m \cdot \omega + \omega_0^2} \), \( k_v(s) = \frac{k_v}{T_v \cdot s + 1} \). (Generally, \( T_v = \frac{1}{5} \) of carrier wave period (neglecting the sidebands of the switching frequency and the dead time), \( k_f(s) = \frac{R_f \cdot C_f \cdot s}{L_f \cdot C_f \cdot s^2 + R_f \cdot C_f \cdot s^2 + 1} \), it can be alternatively expressed as:

\[ \frac{1}{T^2 \cdot s^2 + 2 \cdot \xi \cdot T \cdot s + 1} \], where \( T = \sqrt{L_f \cdot C_f} \) (that controls the shape of the transfer function in the vicinity of the \( f_0 = \frac{1}{2 \cdot \pi} \) respectively.

\[ L_f = \frac{n \cdot k}{2 \cdot \pi \cdot f_v} \] (26)

where, \( k = \left( \frac{3 \cdot V_i^2}{S} \right) \),

\[ f_c = \sqrt{\frac{1}{L_f \cdot \pi \cdot C_f}} \Rightarrow C_f = \frac{1}{(2 \cdot \pi)^2 \cdot L_f \cdot (f_c)^2} \] (27)

\[ \xi = \frac{R_f \cdot \sqrt{C_f}}{L_f} \Rightarrow R_f = 2 \cdot \xi \cdot \frac{L_f}{\sqrt{C_f}} \] (28)

Where, \( \xi \) = damping factor.

\[ 1.6 \leq \frac{L_f}{\sqrt{C_f}} \leq 0.8 \] (29)

4. Calculations

A three-phase 550/50 Hz, 100 MVA power rating an electric arc furnace is considered as a non-linear load. The detailed specifications are as follows:

Table 1 EAF Distribution Network Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>550 V</td>
<td>S</td>
<td>1 MVA</td>
</tr>
<tr>
<td>f</td>
<td>50 Hz</td>
<td>C_{dc}</td>
<td>200 µF</td>
</tr>
<tr>
<td>S</td>
<td>10 MVA</td>
<td>V_{dc}</td>
<td>400 V</td>
</tr>
<tr>
<td>R_s</td>
<td>0.1 Ω</td>
<td>f_c</td>
<td>25 kHz</td>
</tr>
<tr>
<td>L_s</td>
<td>110 µs</td>
<td>n</td>
<td>25</td>
</tr>
<tr>
<td>R_c</td>
<td>100 µs</td>
<td>\xi</td>
<td>0.6</td>
</tr>
<tr>
<td>L_c</td>
<td>0.23 mH</td>
<td>k_c</td>
<td>0.2</td>
</tr>
<tr>
<td>\tau_c</td>
<td>100 µs</td>
<td>m</td>
<td>100</td>
</tr>
<tr>
<td>k_v</td>
<td>40</td>
<td>K_{PWM}</td>
<td>200/16</td>
</tr>
<tr>
<td>\tau_v</td>
<td>8 µs</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Neglecting Kelvin effect, eddy current effect and attenuation of filter’s gain, the maximum gain will be calculated using (26) to (29) [10]:

\[
k = \left( \frac{3 \cdot V^2}{S} \right) \cdot \left( \frac{550/\sqrt{3}}{10^3} \right) = 0.0303
\]

\[
k_c(s) = k_c \cdot \tau_c \cdot s + 1 = \frac{0.2}{100 \cdot 10^{-6} \cdot s + 1} = \frac{0.2}{1 \cdot 10^{-4} \cdot s + 1}
\]

\[
A(s) = \frac{s^2 + \omega_0^2}{s^2 + m \cdot s + \omega^2} = \frac{s^2 + 100 \cdot s + (2 \cdot \pi \cdot 50)^2}{s^2 + 10 \cdot s + (2 \cdot \pi \cdot 70)^2}
\]

\[
A(s) = \frac{s^2 + 314^2}{s^2 + 10 \cdot s + 314^2}
\]

\[
k_v(s) = \frac{k_v}{T_v \cdot s + 1} = \frac{40}{8 \cdot 10^{-6} \cdot s + 1}
\]

\[
L_f = \frac{n \cdot k}{2 \cdot \pi \cdot f_v} = \frac{25 \cdot 0.03025}{2 \cdot \pi \cdot 25 \cdot 10^3} = 4.8144 \times 10^{-6} \text{ H}
\]

\[
f_F = \left( \frac{2 \cdot \pi}{s} \right) \cdot L_f \cdot \left( 1 - \frac{1}{f_v F} \right) = \left( \frac{2 \cdot \pi}{4.816 \times 10^{-6}} \right) \cdot \left( \frac{25 \cdot 10^{-3}}{10} \right)
\]

\[
c_f = \left( 2 \cdot \pi \right)^2 \cdot L_f \cdot \left( 1 - \frac{1}{f_v F} \right) = \left( 2 \cdot \pi \right)^2 \cdot \left( 4.816 \times 10^{-6} \right) \cdot \left( \frac{25 \cdot 10^{-3}}{10} \right)
\]

\[
R_f = 2 \cdot \xi \cdot \sqrt{c_f} = 2 \cdot \left( 0.6 \right) \cdot \sqrt{8.424 \times 10^{-4}} = 0.0908 \Omega
\]

\[
k_f(s) = \frac{R_f \cdot c_f \cdot s}{L_f \cdot c_f \cdot s^2 + R_f \cdot c_f \cdot s + 1} = \frac{8.424 \times 10^{-4}}{7.639 \times 10^{-5}} \cdot s^2 + \frac{0.09073}{7.639 \times 10^{-5}} \cdot s + 1
\]

\[
= \frac{8.424 \times 10^{-4} \cdot s^2 + 0.09073 \cdot s + 1}{7.639 \times 10^{-5} \cdot s^2 + 7.639 \times 10^{-5}}
\]

\[
K_p(s) = 0.530 \times 10^{-4} \cdot s + 1
\]

Now the transfer function of feedback path with phase-lead compensator can be given by:

\[
K'(s) = (0.530 \times 10^{-4} \cdot s + 1) \cdot K_p(s)
\]

5. Simulation Results

Simulation results of Bode plot for functions \( \kappa'(s) \) and \( \kappa(s) \) are obtained by (30) and (32) respectively. Obtained simulation results are shown in Fig. 7 and Fig. 8 respectively.

![Fig. 7. Magnitude and Phase Bode Plot for function k(s)](image1)

![Fig. 8. Magnitude and Phase Bode Plot for function k(s)](image2)

Comparison of simulated results shown in Fig. 8 with Fig. 7 shows that the gain margin and the phase margin of the open loop transfer function can be improved a lot. Also, the cut-off frequency boosted when, the lead compensation is adopted. Quantified results of gain margin and phase margin are tabulated in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Gain margin</th>
<th>Phase margin</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa'(s) )</td>
<td>0.506 dB</td>
<td>1.45 deg</td>
</tr>
<tr>
<td>( \kappa(s) )</td>
<td>50.1 dB</td>
<td>40.9 deg</td>
</tr>
</tbody>
</table>

It can be seen that the gain margin and the phase margin has increased considerably by lead compensation strategy.
Compared with the traditional control strategy, the proposed control strategy improves the system stability compared with the traditional control strategy. Simulation verifies the validity of the developed theory. This work does not include experimental analysis. The real-time implementation of the proposed work can be given a purview as future work. One can analyze the stability considering advanced stability analysis methods to get rid of shortcoming of traditional control strategy presented in this paper as future work.

7. Nomenclature

- $n =$ transformation ratio
- $\gamma_c =$ controlled inverter output voltage
- $V_{dc} =$ dc bus voltage
- $i_s =$ source current
- $i_L =$ load current
- $L_f =$ carrier filter inductance
- $R_f =$ carrier filter resistance
- $C_f =$ carrier filter capacitance
- $i_L =$ inductor current through $L_f$
- $v_s =$ source voltage
- $v_m =$ voltage in the secondary of the coupling transformer
- $v_i(t) =$ input reference voltage
- $v_s =$ output voltage seen by the load
- $i_{sh} =$ harmonics components of source voltage
- $v_f =$ fundamental components of source voltage
- $i_{gy} =$ harmonics components of load voltage
- $i_f =$ fundamental components of current injected by load
- $i_{sh} =$ harmonics components of current injected by load
- $Z_s =$ source impedance
- $Z_f =$ passive filter impedance
- $k_c(s) =$ transfer function of the sensor modulating circuit
- $k_c =$ gain of the sensor modulating circuit
- $T_c =$ time constant of the sensor modulating circuit

Comparison of Fig. 9 and Fig. 10 shows improved voltage wave at PCC as well as total harmonic distortion (THD) after lead compensation application. Quantified results are tabulated in Table 3. THD has improved by 77.35% after lead compensation application. Maximum gain is calculated by neglecting Kelvin effect, eddy current effect and attenuation of filter’s gain, which could be a limitation of this analysis.

6. Conclusion

In this paper, a state-space averaging model has been constructed to analyze the stability of CF used for PQ improvement of EAF distribution network. It is difficult to obtain a better filter performance confined to the stability by the traditional control strategy, taking into account the time delay in the control circuit. The analysis has indicated that the time delay can cause a stability problem. Therefore, a phase-lead compensation strategy has been proposed to eliminate the effect of time delay. It suggests method of lead compensation for $k_c(s)$ to increase the value of the cut-off frequency. The increase of gain margin and phase margin can clearly clarify that the proposed control strategy improves the system stability compared with the traditional control strategy.
\( A(s) = \text{Transfer function of the harmonics calculating circuit} \)

\( m = \text{internal gain of the harmonics calculating circuit} \)

\( K_{PWM} = \text{Inverter gain of the harmonics calculating circuit} \)

\( k_v(s) = \text{transfer function of the inverter} \)

\( k_s = \text{gain determined by the speed of the processor/related software} \)

\( T_v = \text{time constant determined by the speed of the processor/related software} \)

\( k_f(s) = \text{Transfer function of the output carrier filter circuit} \)

\( \xi = \text{damping factor (chosen between 0.4 to 0.6)} \)

\( s = \text{apparent output of the load} \)

\( v = \text{phase voltage} \)

\( f_{cf} = \text{cross-over frequency of the filter} \)

\( f_c = \text{carrier frequency} \)

**References**


