Direct Torque Control of Matrix Converter Fed Five Phase Interior Permanent Magnet Synchronous Motor Based on Space Vector Modulation

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**Abstract:** Matrix Converters (MCs) have many advantages over conventional inverters such as sinusoidal input and output waveforms, operation with close to unity input power factor and more compact size. Direct Torque Control (DTC) is known as one of the popular methods in controlling AC drives. This paper presents Indirect Space Vector Modulation (ISVM) model for a three to five -phase MC. Then, DTC is proposed for a five-phase Interior Permanent Magnet Synchronous Motor (IPMSM), fed by a three to five -phase MC. The simulation results of the proposed drive system show very low flux and torque ripples, sinusoidal input and output currents, constant switching frequency and operation with close to unity input power factor.

**Keywords:** DTC, matrix converter, ISVM, five phase IPMSM

**1. Introduction**

Application of Matrix Converter (MC) in controlling AC drives has received considerable attention nowadays. Due to some advantages such as generating more voltage vectors, sinusoidal input and output waveforms, operation in close to unity input power factor and absence of DC-link capacitor, it can be competitive with traditional Voltage Source Inverters (VSIs).

One of the most popular methods in controlling variable speed drives is Direct Torque Control (DTC) [1]. DTC scheme for matrix converter drive was first presented in [2].

Space Vector Modulation (SVM) is one of the preferred modulation methods for three -phase MCs [3-6]. SVM strategy enables the control of input power factor independent from output power factor, achieves maximum voltage transfer ratio without the need of third harmonic injection and reduce harmonics. Multiphase drives are used in high reliability areas such as aerospace applications, ships electric propulsion, electric vehicles and defence. Some of the important advantages of multiphase motors over traditional three phase ones are categorized as: improvement of torque per RMS ampere, increasing of torque frequency pulsations, reducing stator current per phase without the need of increasing voltage per phase, lowering the DC link current harmonics and good fault tolerances [7-8]. This paper presents Indirect Space Vector Modulation (ISVM) model for a three to five -phase MC. Then, DTC is proposed for a five-phase Interior Permanent Magnet Synchronous Motor (IPMSM), fed by a three to five -phase MC. Sinusoidal input and output currents, which is obtained by utilization of a harmonic elimination strategy for the three to five -phase MC, close to unity input power factor, significant torque ripple reduction, good flux and speed responses and constant switching frequency are the results of the proposed drive system.

**2. Three to five - phase MC**

A three to five -phase MC is a single stage converter that connects 3 input phases to 5 output phases as illustrated in Fig.1. Each power switch is equipped with two anti-parallel connected IGBTs and fast diodes and has a switching function which is defined as follows:
\[ S_{jk}(t) = \begin{cases} 0 & \text{switch, } S_{jk} \text{ is open} \\ 1 & \text{switch, } S_{jk} \text{ is closed} \end{cases} \quad (1) \]

\[ j = \{a, b, c\}, \quad k = \{A, B, C, D, E\} \]

The input phase should never be short-circuited; therefore, only one switch can be on-state in each leg at any instant:

\[ S_{ak} + S_{bk} + S_{ck} = 1 \quad (2) \]

And for the inductive load, output of the MC should not be open-circuited at any time. Output line-to-line voltages can be deduced as:

\[ U_{out.L} = T_L \cdot u_{in.ph} \quad (3) \]

\[
\begin{bmatrix}
U_{AB} \\
U_{BC} \\
U_{CD} \\
U_{DE}
\end{bmatrix} = 
\begin{bmatrix}
S_{aA}(t) - S_{bB}(t) & S_{bA}(t) - S_{bB}(t) & S_{cA}(t) - S_{cB}(t) \\
S_{aA}(t) - S_{cC}(t) & S_{bB}(t) - S_{cC}(t) & S_{dD}(t) - S_{cC}(t) \\
S_{cC}(t) - S_{aA}(t) & S_{bB}(t) - S_{dD}(t) & S_{eE}(t) - S_{dD}(t) \\
S_{dD}(t) - S_{aA}(t) & S_{aA}(t) - S_{bB}(t) & S_{eE}(t) - S_{cC}(t)
\end{bmatrix}
\begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix}
\]

A three to five-phase MC produce \(3^5 = 243\) output voltage space vectors with respect to the mentioned constraints. Among these vectors, 93 vectors are the so-called stationary vectors that have fixed directions. In the case of three to five-phase MC, as well as five-phase VSIs, the output vectors form three concentric decagons. So that, 30 output voltage space vectors are large vectors, 30 vectors are medium and the last 30 vectors are small vectors in stationary reference frame \(D = Q - Z_1 - Z_2\), as shown in Fig. 2.

3. Indirect Space Vector Modulation

The ISVM model considers matrix converter as a two-stage converter scheme. First stage is called the “rectification” stage that supplies the constant imaginary DC-link and the second stage is the “inversion” stage, which is analogous to a conventional VSI and is used to generate output voltages with variable magnitude and frequency for the load [6]. In order to find the transfer matrix of a three to five-phase MC, the relation between the input and output voltages should be observed. Input phase voltages are expressed as:

\[
\begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix} = u_{im} \begin{bmatrix}
\cos(\alpha_1 t) \\
\cos(\alpha_1 t - 120^\circ) \\
\cos(\alpha_1 t + 120^\circ)
\end{bmatrix}
\]

And the output line-to-line voltages are defined as (6), Where \(u_{im}\) is considered as the maximum input phase voltage, \(U_{om}\) is the maximum output phase voltage and \(\phi_i\) is an arbitrary angle.
If we identify $T_{ph,L}$ as the transfer matrix of the three to five-phase MC, it will be derived as (7), where $m$ is the modulation index and $0 \leq m \leq 1$.

$$U_{out,L} = T_{ph,L} \cdot U_{in,ph}$$

$$T_{ph,L} = m \begin{bmatrix}
\cos(\omega_o t - \phi_o + 54^o) \\
\cos(\omega_o t - \phi_o + 54^o - 72^o) \\
\cos(\omega_o t - \phi_o + 54^o - 144^o) \\
\cos(\omega_o t - \phi_o + 54^o + 144^o) \\
\cos(\omega_o t - \phi_o + 54^o + 72^o)
\end{bmatrix}$$

Fig. 4 shows the proposed topology of the ISVM model for a three to five-phase MC.

**3.1 Inversion stage**

The inversion stage is defined as a five phase VSI, supplied by an imaginary DC link. The output VSI voltage space vectors in stationary reference frame ($D-Q-Z_1-Z_2$) are calculated using (8):

$$V^D-Q_o = \frac{2}{5}(V_{AB} + V_{BC}e^{\frac{j2\pi}{3}} + V_{CD}e^{\frac{j4\pi}{3}} + V_{DE}e^{\frac{j4\pi}{5}} + V_{EA}e^{\frac{j2\pi}{3}})$$

$$V^{Z-Z_2}_o = \frac{2}{5}(V_{AB} + V_{BC}e^{-\frac{j2\pi}{3}} + V_{CD}e^{\frac{j2\pi}{3}} + V_{DE}e^{-\frac{j4\pi}{5}} + V_{EA}e^{\frac{j4\pi}{5}})$$

Fig. 4. ISVM model for a three to five-phase MC
The five-phase voltage space vectors in both $D-Q$ and $Z_1-Z_2$ planes are shown in Fig. 5. The following properties can be summarized from this figures:

- The ratio of the amplitudes of voltage vectors is $1:1.618:1.618^2$ from the small vectors to large vectors.
- The inverter states with connection configurations of $\{5-0, 0-5\}$ produce zero voltage vectors. In other words, if all the upper (lower) switches are connected simultaneously to a same phase, the output voltages will be zero.
- The inverter states with connection configurations of $\{4-1, 1-4\}$ produce one medium vector in $D-Q$ plane and one medium vector in $Z_1-Z_2$ plane.
- The inverter states with connection configurations of $\{3-2, 2-3\}$ produce one large voltage vector in $D-Q$ plane and one small voltage vector in $Z_1-Z_2$ plane, or produce one small voltage vector in $Z_1-Z_2$ plane, and one large voltage vector in $Z_1-Z_2$ plane.
- The outer decagon of the $D-Q$ plane is mapped into the inner decagon of the $Z_1-Z_2$ plane and vice-versa. The medium decagon of $D-Q$ is mapped into the medium decagon of $Z_1-Z_2$ plane.

The major drawback of a five phase VSI is the generation of low order harmonics in $Z_1-Z_2$ plane. The low order harmonic voltages in $Z_1-Z_2$ plane will produce huge harmonic currents due to the small winding resistance and inductance [8]. As we know, large and medium vectors in a same branch in $D-Q$ plane will stand in reverse direction in $Z_1-Z_2$ plane. So, if a relation is found between the dwell time of the selected large and medium voltages in $D-Q$ plane to cancel each other at $Z_1-Z_2$ plane in each sampling period, harmonic voltages and currents will not be produced in $Z_1-Z_2$ plane.

Suppose that output voltage vector lies in the $K_n$ sector. 4 adjacent voltages, 2 medium and 2 large vectors, and a zero voltage vector, are chosen to reconstruct the desired output voltage (Fig. 5.a). For each branch, $U_{LK}$ and $U_{MK}$ are the amplitudes of large and medium voltages of the same branch of the $K_n$ sector and $T_{LK}$ and $T_{MK}$ are the dwell time of them in $D-Q$ plane. In order to make zero output voltage in $Z_1-Z_2$ plane:

$$T_{LK} = 1.618 T_{MK}$$ (9)

According to Fig. 5.a, reference output voltage in $K_n$ sector is reconstructed by combination of adjacent vectors in $D-Q$ plane. The dwell times of the selected voltages in each sector are:

$$d_M = \frac{T_M}{T_S} = m_v \sin(\frac{\pi}{5} - \theta_{out})$$
$$d_{M+1} = \frac{T_{M+1}}{T_S} = m_v \sin(\theta_{out})$$ (10)
$$d_{L+1} = 1.618 d_{M+1}$$
$$d_{oV} = \frac{T_{oV}}{T_S} = 1 - (d_L + d_M + d_{L+1} + d_{M+1})$$

In (10), $\theta_{out}$ is the angle of the output voltage vector within the respective sector, $T_S$ is the sampling period and $m_v$ is the voltage modulation index.
3.2 rectification stage

Rectification stage of a three to five-phase MC is analogous to a three-phase one. The rectification stage hexagon is illustrated in Fig.6. To reconstruct the input current space vector in a specified sector, two adjacent vectors and a zero vector are synthesized. The input current vector will be defined as:

$$I_{in} = \frac{2}{3}(i_a + i_b e^{j \frac{2\pi}{3}} + i_c e^{j \frac{4\pi}{3}})$$  

(11)

If the input current is in the Kth sector (Fig. 6), the duty cycles of each adjacent vector are derived as follows:

$$d_\mu = \frac{T_\mu}{T_s} = m_c \sin\left(\frac{\pi}{3} - \theta_{in}\right)$$

$$d_v = \frac{T_v}{T_s} = m_c \sin(\theta_{in})$$

$$d_{oc} = \frac{T_{oc}}{T_s} = 1 - (d_\mu + d_v)$$

(12)

Where $\theta_{in}$ is the angle of the input current space vector within the respective sector and $m_c$ is the current modulation index.

Suppose that both output voltage and input current vectors stay in sector.1 of the $D-Q$ plane. The averaged output line-to-line voltages are defined as follows:

$$\begin{bmatrix}
U_{AB} \\
U_{BC} \\
U_{CD} \\
U_{DE} \\
U_{EA}
\end{bmatrix} = T_{VSI(o_1)} \cdot \begin{bmatrix}
\frac{d_L + d_{L+1}}{2} \\
\frac{d_{M+1}}{2} \\
-\frac{d_L - d_{M+1}}{2} \\
-\frac{d_M - d_{L+1}}{2} \\
\frac{d_M}{2}
\end{bmatrix} V_{DC}$$

(13)

Where $T_{VSI(o_1)}$ is defined as the inversion stage transfer matrix and $\theta_{out}$ in sector.1 is found as:

$$\theta_{out} = (\omega_{o_1} t - \phi_a + 54^\circ) + 18^\circ$$

(14)

Fig.6. Rectification stage hexagon

The averaged input currents in the sampling period are calculated as bellow:

$$\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} = \begin{bmatrix}
d_\mu + d_v \\
-d_\mu \\
-d_v
\end{bmatrix} I_{DC} = T_{VSR(o_1)}^T I_{DC}$$

(15)

Where $T_{VSR(o_1)}$ is defined as the rectification stage transfer matrix and the angle of input current space vector in sector.1 is:

$$\theta_{in} = \omega_{i_1} t - \phi_i + 30^\circ$$

(16)

In ISVM model, the transfer function of a matrix converter is derived from the multiplication of inversion stage and transpose of the rectification stage matrices, which means:

$$\begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix} = \begin{bmatrix}
U_{AB} \\
U_{BC} \\
U_{CD} \\
U_{DE} \\
U_{EA}
\end{bmatrix} = T_{VSI(o_1)} \cdot T_{VSR(o_1)}^T \begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix}$$

(17)
For the first sector of rectification and inversion stages, (18) will be defined as:

\[
\begin{bmatrix}
U_{AB} \\
U_{BC} \\
U_{CD} \\
U_{DE} \\
U_{EA}
\end{bmatrix} = \begin{bmatrix}
d_L + d_{L+1} \\
d_M - d_{M+1} \\
d_L - d_{M+1} \\
d_M - d_{L+1} \\
d_M
\end{bmatrix} \begin{bmatrix}
\cos(\omega_t - \phi_e + 54^\circ) \\
\cos(\omega_t - \phi_e + 54^\circ - 72^\circ) \\
\cos(\omega_t - \phi_e + 54^\circ - 144^\circ) \\
\cos(\omega_t - \phi_e + 54^\circ + 144^\circ) \\
\cos(\omega_t - \phi_e + 54^\circ + 72^\circ)
\end{bmatrix} \begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix}
\]  

Substitution (14) and (16) in (18):

\[
\begin{bmatrix}
U_{AB} \\
U_{BC} \\
U_{CD} \\
U_{DE} \\
U_{EA}
\end{bmatrix} = m \begin{bmatrix}
\cos(\omega_t - \phi_e) \\
\cos(\omega_t - \phi_e - 120^\circ) \\
\cos(\omega_t - \phi_e + 120^\circ)
\end{bmatrix} \begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix}
\]  

Eq.(19) is similar to (7), which means that the ISVM model can be applied for a three to five-phase MC, and:

\[m = m_e m_r\]  

Multiplication of inversion and rectification stages will produce new duty cycles, such as:

\[d_M = d_M d_M\]  

Input current space vectors are used to show which phases supply positive (P) and negative (N) terminals of the imaginary DC link (Fig.4). For example, if \(I_{ab}\) is chosen, input phase (a) connects to terminal (P) and input phase (b) connects to terminal (N).

In inversion stage, upper switches connect to terminal (P) and lower ones connect to terminal (N). Suppose voltage vector 11001 is selected in inversion stage and \(I_{ab}\) is chosen for Imaginary DC link, the MC voltage vector which is made is “aabba”.

Fig. 7 shows the placement of the MC vectors in the half sampling period, when both input current and output voltage space vectors are in sector 1:

\[
\begin{bmatrix}
T_2 \\
T_{2a} \\
T_{2b} \\
T_{2c} \\
T_{2d} \\
T_{2e}
\end{bmatrix} = \frac{T}{2}
\]

Fig.7 Double sided modulation strategy during half sampling period

4. Proposed DTC scheme

Fig. 8 illustrates the block diagram of the proposed drive system. Torque and flux estimation unit is defined from the following equation:

\[
V_{Ds} = r_x i_{Ds} + \frac{d\lambda_{Ds}}{dt}
\]

\[
V_{Qs} = r_y i_{Qs} + \frac{d\lambda_{Qs}}{dt}
\]

\[
T_e = \frac{5p}{2} [\lambda_{Ds} i_{Qs} - \lambda_{Qs} i_{Ds}]\]

Where \(r_x\) is the stator winding resistance, \(i_{Ds}, i_{Qs}\), \(\lambda_{Ds}\) and \(\lambda_{Qs}\) are the stator currents and stator fluxes in D-Q reference frame.

In order to find the viability of the proposed drive system, simulink model has been arranged using matlab/simulink. The sampling period is 150\mu s for the proposed scheme.

The parameters of the IPMSM are given in table I.

<table>
<thead>
<tr>
<th>P</th>
<th>Ld</th>
<th>Lq</th>
<th>B</th>
<th>J</th>
<th>Rs</th>
<th>(\psi_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>18 mh</td>
<td>42 mh</td>
<td>0.005</td>
<td>0.025</td>
<td>0.7</td>
<td>0.5 (wb)</td>
</tr>
</tbody>
</table>

Fig.9 shows torque and flux magnitudes at no load operation and 600 rpm.

As can be seen in Fig.9, torque and flux ripples are small and track their reference very well. Fig.10 shows estimated torque, flux, speed and stator currents at rated load and 600 rpm. The results emphasize the good performance of the drive system. In Fig. 10d, unfiltered input current is in phase with the input voltage with close to unity input power factor, as one of the outstanding features of MC. Five phase stator currents are sinusoidal in Fig.10e, due to the zero output.
voltage in $Z_1 - Z_2$ plane. Fig.10f shows the harmonic spectra of the stator phase current. Dominant harmonics are around 6.6 KHZ, which is determined by the SVM sampling time.

Fig. 8. Block diagram of the proposed DTC scheme

Fig. 9. Simulation results for the proposed scheme at no load and 600 rpm (a) Estimated torque (b) Estimated flux

Fig. 10. Simulation results for the proposed scheme at rated load and 600 rpm (a) Estimated torque, (b) Estimated flux, (c) Speed response, (d) Five phase stator currents, (f) stator current harmonic spectrum

The dynamic behaviour of the torque has been tested with the reference step torque command from 15 Nm to -15 Nm and from -15 Nm to 10 Nm at 600 rpm.

In Fig. 11a, from 0 s to 0.5 s, drive is in motoring operation and the filtered input current is in phase with input voltage. When the torque is reversed from 15 Nm to -15 Nm at 0.5 s, the filtered input current becomes out of phase with the voltage by 180° (Fig. 11b). This is true because MC passes electric power from motor to the grid during
Regenerative braking operation. When the drive is switched back to motoring operation at 0.8s, the current becomes in phase with input voltage again.

Finally, a triangular command speed is applied to the motor at no load operation. In Fig. 12, the speed accelerates from 0 rpm to 600 rpm in 0.3 s and then decelerates from 600 rpm to -600 rpm during t= 0.3 s to t=0.7 s. As can be seen, the speed tracks its reference during reversal very well.

5. Conclusion
This paper investigates the use of DTC with a three to five-phase MC for a five phase IPMSM. The ISVM model is applied for the modulation of the three to five-phase MC. Simulation results show that the proposed drive system inherits well-known features of DTC such as fast response, almost ripple free operation and robustness and also, achieves outstanding features of MC such as sinusoidal input and output waveforms, operation with close to unity input power factor and regenerative braking capability. Harmonic elimination strategy in inversion stage reduces the voltage harmonics to zero in $Z_1 - Z_2$ plane and also, constant switching frequency is derived from the proposed drive system.

REFERENCES


