Abstract: This paper presents a mathematical formulation of the optimal VAR source control problem in power system. This method employs a linearized objective function and constraints, and its approach is based on adjusting control variables. Transmission losses are considered as a function of voltage increments are related by modified jacobian matrix. Revised linear programming is used to calculate the voltage increments which minimize the transmission losses, and adjustments of control variables are obtained by a modified jacobian matrix. This method does not require any matrix inversion, it will save computation time and memory space. The proposed algorithm is applied to Ward-Hale 6 bus test system and numerical results verify the proposed method over the existing ones.

Keywords: VAR source, Reactive power, Voltage profile, Revised linear programming

1. Introduction

Now-a-days power system has grown into a very vast and complex system particularly with the development of integrated systems and integrated grids. Further the power that is in short supply compared to the ever increasing demand. It is generally accepted that the transmission loss in India is very much above the desirable limit. The important issue in power transmission is the basic requirement of delivering power to consumers without violating its permissible limits.

The quality and reliability of the power supply has to be maintained in the power system by maintaining the bus voltages in the permissible limits. Any changes to the system configuration by way of changes in power demands can result in higher or lower voltages of system. In order to enhance the voltage profile of the system, power systems are equipped with a lot of voltage controlling devices such as generators, tap changing transformers, shunt capacitors, synchronous condensers and static VAR compensators etc.[1-3]. This means that either by the variations of load or by the changes of network configurations, a real time control employing those controlling devices is required to reduce the problems caused by the perturbations.

Thus, in modern complex interconnected power systems, a coordinated procedure is needed to control voltage and reactive power flow in such a way as to minimize the transmission losses. Reactive power control or management can be defined as the control of generator voltages, variable transformer tap settings, switchable shunt capacitors and reactor bank in a manner that best achieves a reduction in system losses and voltage control.

Optimization is the process of maximizing the total effectiveness with a set of certain operating constraints of equalities and inequalities. The most common means of analyzing the voltage and reactive power flow problem for system planning is the standard load flows. Based on this, the system planner must analyze the violation of constraints on the system.

The revised Linear programming method employs a linearized objective function and constraints, and its approach is based on adjusting control variables. Initially base case
load flow calculation is done to determine the system state and violation of limit. Transmission losses are considered as a function of voltage increments which minimize the transmission losses, and adjustments of control variables are obtained by a modified jacobian matrix.

This method does not require any matrix inversion, it will save computation time and memory space, and hence can be implemented on very large-scale power systems. This approach would greatly simplify the application of decomposition methods in power systems planning and operations.

The objective function and the constraints are formulated from power flow equations. So they will be linear. Hence linearization of the objective function and constraints is done about the current operating point. The operating point is estimated using Newton Raphson load flow. From the LF studies, the violations are checked and necessary compensation requirements are implemented by revised Linear programming approach.

In the past, [4-9] a number of papers were published on the control of reactive power and voltage and necessary compensation techniques were developed to minimize the system real power loss and to improve the voltage profile.

2. System Modeling for Reactive Power Flow Analysis

The overall power flow can be divided into the following sub-problems.

1. Formulation of suitable mathematical network model. The model must describe adequately the relationships between voltages and powers in the interconnected system.

2. Specification of power and voltage constraints that must apply to the various buses of the network.

3. Numerical computation of power flow equations subjected to the above constraints. These computations give us, with sufficient accuracy, the values of all bus voltages.

4. When all the bus voltages have been thus determined, we must finally compute the actual power flows in all transmission lines.

5. Each bus of a power system is characterized by four variables, $P_i$, $Q_i$, $|V_i|$ and $\delta_i$. Depending on these known numbers of variables, buses can be classified.

2.1 The Load Modal

The effect of voltage variation on the power consumed by system loads depends to a great extent on the on the type of load that is being supplied by the power system. Reactive loads can be represented as a function of bus voltage magnitudes as follows

$$Q_d = Q_{ab} \left( \frac{V}{V_s} \right)^q$$

Initial or base case reactive power and voltage are denoted by $Q_{ab}$ and $V_s$ respectively. The characteristic of the load can specified by the value of $q$. The typical values for $q$ are:

- $q = 0$: constant power load
- $q = 1$: constant current load
- $q = 2$: constant impedance load

So, the variation of the load with respect to changes of bus voltage magnitude can be represented by

$$\Delta Q_d = \frac{\partial Q_d}{\partial V} \Delta V = \frac{qQ_{ab}}{V_s} \left( \frac{V}{V_s} \right)^{q-1} \Delta V$$

2.2 The Tap Changing Transformer Model

The modeling of voltage transformer taps must be incorporated in any discussions related to reactive power dispatch because they represent a vital part of the automatic control process. Transformer tap changing is more difficult to model since two buses are directly involved in the tap changing process.

Let us consider a transformer connecting buses $i$ and $l$ with tap $T_{il}$, as shown in Fig.1. This branch can be represented by an equivalent $\pi$ circuit as shown in Fig. 2.

![Fig.1 Modal of tap changing transformer](image-url)
The above equation should be added to other self admittance elements to form the overall admittance matrix (for 1 bus there is no effect).

From the fig. 2, the complex power injection to bus i is

\[ S_i = P_i + jQ_i = V_i (V_i^* (T_{il}^2 - T_{il}) y_{il}^*) \]

\[ = V_i^2 (T_{il}^2 - T_{il}) y_{il}^* \]  

\[ = V_i^2 (T_{il}^2 - T_{il}) g_{il}^* - j V_i^2 (T_{il}^2 - T_{il}) b_{il}^* \]

\[ Q_i = -b_{il} V_i^2 (T_{il}^2 - T_{il}) \]

Here, we will consider the effect of voltage changes on the calculation of \( \Delta t_i \) and \( \Delta t_l \). The derivatives of \( Q_i \) with respect to \( V_i \) and \( T_{il} \) are

\[ \frac{\partial Q_i}{\partial V_i} = -b_{il} V_i^2 (2T_{il} - 1) \]

\[ \frac{\partial Q_i}{\partial T_{il}} = -2b_{il} V_i (T_{il}^2 - T_{il}) \]

Since \( \Delta Q_i = -\Delta Q_i \),

\[ \Delta Q_i = \frac{\partial Q_i}{\partial V_i} \Delta V_i - \frac{\partial Q_i}{\partial T_{il}} \Delta T_{il} \]

So,

\[ \Delta Q_i = 2b_{il} V_i (T_{il}^2 - T_{il}) \Delta V_i - b_{il} V_i^2 (2T_{il} - 1) \Delta T_{il} \]


3.1 Objective Function

The objective is to minimize real power losses during the operation and control of a network. The real power loss \( P_L \) is represented by

\[ P_L = \sum_{k=1}^{n} G_k \left[ V_i^2 + V_j^2 - 2 V_i V_j \cos(\delta_i - \delta_j) \right] \]  \( (1) \)

\( G_k \) is the conductance of line k which is connected between buses i and l. In equation (1) the losses are represented by a nonlinear function of the bus voltages and phase angles which is indirectly a function of the controllable VAR sources.

In order to use the LP, the objective function is linearized as follows:

\[ \frac{\partial P_L}{\partial V_i} = G_k \left[ 2V_i - 2V_j \cos(\delta_i - \delta_j) \right] \]

\[ \frac{\partial P_L}{\partial V_j} = G_k \left[ 2V_i - 2V_j \cos(\delta_i - \delta_j) \right] \]  \( (2) \)

For every transmission line, the partial derivatives of \( P_L \) with respect to the voltages at buses i and l are calculated. Partial derivatives pertaining to a certain bus are summed to form the power loss sensitivities with respect to all bus voltages in the system. In our approach the objective function is linearized with respect to all bus voltages of the system. Equality constraints are presented for all non-generating buses. The power loss increment \( \Delta P_L \) is related to changes in bus voltages as follows:
$$\Delta P_L = \begin{bmatrix} \frac{\partial P_L}{\partial V_1} & \frac{\partial P_L}{\partial V_2} & \cdots & \frac{\partial P_L}{\partial V_{nb}} \\ \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_{nb} \end{bmatrix}$$

(3)

(4)

3.2. Network Performance Constraints

The following are the inequality and equality constraints imposed on different buses of power network:

$$Q^\text{min}_i \leq Q_i \leq Q^\text{max}_i$$

$$Q_i = Q_i^s$$

$$V^\text{min}_i \leq V_i \leq V^\text{max}_i$$

Where, \(i=1, 2, 3 \ldots nb\)

In equation (5), the first set of inequality constraints are for reactive power sources and tap changing transformer terminals. The equality constraints are for load and junction buses not connected to transformer terminals. These constraints can be rewritten in the form of increments:

$$\Delta Q_i^\text{min} \leq \Delta Q_i \leq \Delta Q_i^\text{max}$$

$$\Delta Q_i = \Delta Q_i^s$$

$$\Delta V_i^\text{min} \leq \Delta V_i \leq \Delta V_i^\text{max}$$

Where, \(i=1, 2, 3 \ldots nb\)

The formulation of the constraints follows two steps:

1. Formation of the modified jacobian matrix \(J''\)
2. Determination of limits.

4. Mathematical Statement of the Problem

Based on formulation of the objective function, constraints equations, and the modified jacobian matrix, the optimization problem is represented as follows:

Minimize \(\Delta P_L = M \Delta V\)  

Subject to

$$\Delta Q^\text{min}_i \leq \Delta Q_i \leq \Delta Q_i^\text{max}$$

$$J'' \Delta V = 0$$

$$\max(\Delta V^\text{min}_i - \Delta V_{\text{step}}, -\Delta V_{\text{step}}) \leq \Delta V \leq \min(\Delta V^\text{max}_i, \Delta V_{\text{step}})$$

Where,

\(J'', J''\) are sub matrix of the \(J''\) matrix

5. Solution Algorithm

1. Read the line data, Bus data, limits on the control variable and step size of control variable \(\Delta Q_{\text{step}}, \Delta T_{\text{step}}, \Delta V_{\text{step}}\)
2. Perform the base case load flow calculation using NR method to determine the state of the system with optimal real power generation schedule.
3. Check the performance of the system. If optimal adjustments of control variable are necessary go to next step, otherwise goto step (12)
4. Formulate the coefficients of objective function using load flow solution.
5. \(Q, T, V\) permitted to vary within these limits around the values, which are determined by power flow calculation. If any of these step sizes is beyond the limits, the value of limits will be used in the calculation.
6. Formulate the primitive jacobian matrix \(J\) using load flow equation.
7. Add load & Tap changing transformer effects to form modified jacobian matrix \(J''\)
8. Set up the equality and inequality constraints.
9. Formulate the LP problem for the given objective function and the set of constraints.
10. Solve the LP problem and use the results as base values for the next iteration
11. Check whether the real power loss in the system is significantly different from the previous iteration. If so, goto step (5), else goto next step.
12. Print the load flow results and status of the control variable.

6. Test System and Results

A test was conducted on a Ward-Hale 6-Bus system. The schematic diagram of Ward-Hale 6-Bus system is given in fig .3.
From the base case load flow results the voltage at bus 6 is 0.855 p.u and is in violation of limits of 0.9 p.u to 1.1 p.u.

Transformer taps: 
\[ t_{35} : 1.025 \]  
\[ t_{46} : 1.100 \]

Real power loss: 11.614 MW

This result indicates the initial tap position of transformer and injected VAR power source. Initial system real power losses are 11.614 MW. The proposed technique has been applied to improve this situation.

By applying proposed method, all the control variables have been adjusted. There are no limit violations and the load bus voltages are nearly 1.00 p.u. This is a saving of 1.543 MW or 13.2827% reduction in the system losses.

Transformer taps: 
\[ t_{35} : 1.007 \]  
\[ t_{46} : 0.994 \]

Real power losses: 10.071 MW

Reduction of real power losses: 1.543 MW

Percentage of real power losses reduction: 13.2827

The above procedure can be implemented using revised linear programming approach in order to effectively choose the controlling devices and achieve better voltage profile as well as reduction in real power losses.

7. Conclusion

A new method is presented to find optimal reactive power control to improve voltage profile by adjusting VAR sources and transformer tap positions. The inverse of Jacobian matrix (sensitivity matrix) is not required in this method, because the effect of all variables has been introduced in the Jacobian matrix. So, this method is time efficient and needs less memory space.

This method provides faster convergence in the optimal power dispatch problem than other conventional methods. It can be used as a tool to assist the power system operators to improve system voltage profile and reduce losses. The method has been tested on the Ward-Hale 6-Bus system and can be easily implemented to a larger system. The fast and reliable characteristics of the computations mentioned above present the possibility for on-line applications for reactive power-voltage control.

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