AN ELEGANT GAMMA BASED ALGORITHM FOR FIXED HEAD HYDROTHERMAL SCHEDULING

J. Sasikala 1 *                           ... using cost and  
water discharge rate coefficients of 
thermal and hydel plants respectively for a 
known values of γ
1.0 Introduction

The short range hydrothermal scheduling (HTS) is one of the most important and challenging optimisation problems in economic operation and control of interconnected power systems. The problem, due to insignificant operational cost of hydroelectric plants, is to effectively use the available water to maximise generation from hydro plants and reduce the cost of thermal generation. The HTS problem is therefore to optimally distribute generations among the thermal and hydel plants in such a way to minimise the total thermal production cost for a single day or a week, usually on an hourly basis, satisfying hydraulic and electrical operational constraints such as available water, generation limits, generation-demand balance, etc. The HTS boils down to a typical non-linear optimisation problem involving a non-linear objective function and a combination of linear and non-linear constraints [1]. The power generation in a short range HTS problem, is solely dependent on the water discharge on account of a fixed water head and the net head variation being ignored for relatively large reservoirs [2-6].

The short range HTS problem has been the subject of intensive research work during the past few decades. Several researchers have suggested many methods such as dynamic programming [7], network flow programming [8], mixed integer programming [9] and Lagrangian relaxation [10] to solve this difficult optimisation problem. Dynamic programming among these approaches has been found to tackle the complex constraints directly but suffers from the curse of dimensionality. The other methods have necessitated simplifications in order to easily solve the original model, which may lead to sub-optimal solutions with a great loss of revenue.

In recent years, heuristic optimisation techniques have aroused intense interest due to their flexibility, versatility and robustness in seeking global optimal solution. These evolutionary approaches such as genetic algorithms [11-16], simulated annealing [6], evolutionary strategy [17-19], particle swarm optimisation [20-21] and peak shaving [22] involve a large number of problem variables, which not only depend on the number of generating plants but also the number of intervals considered in the planning horizon and thus are highly inefficient.

A simple and elegant algorithm for solving fixed-head short range HTS problem is developed in this paper. This method eliminates the λ-iterations through an analytical approach in the λ − γ iteration method to enhance the solution speed. The proposed algorithm is tested on three HTS problems and the results are presented.

2.0 Problem Formulation

The main objective of HTS problem is to determine the optimal schedule of both hydro and thermal plants of a power system in order to minimise the total system operating cost, represented by the fuel cost required for the system’s thermal generation. It is intended to meet the forecasted load demand over the scheduling period, while satisfying various system and unit constraints. The HTS problem without accounting the losses is formulated as

Minimise \[ \Phi = \sum_{k=1}^{k_{\text{max}}} \sum_{t=1}^{n_{t}} t_{k} \cdot F_{ik} \left( P_{T,ik} \right) \] (1)

subject to the power balance constraint

\[ \sum_{i=1}^{n_{t}} P_{T,ik} + \sum_{j=1}^{n_{h}} P_{H,jk} - P_{DK} = 0 \]

\[ k = 1,2,\cdots,K_{\text{max}} \] (2)

and to the water availability constraint

\[ \sum_{k=1}^{k_{\text{max}}} Y_{ik} \left( P_{H,ik} \right) = V_{i,\text{av}} \quad ; \quad i = 1,2,\cdots,n_{h} \] (3)

with

\[ P_{T,\text{min}} \leq P_{T,ik} \leq P_{T,\text{max}} \quad ; \quad i = 1,2,\cdots,n_{t} \]

\[ P_{H,\text{min}} \leq P_{H,ik} \leq P_{H,\text{max}} \quad ; \quad i = 1,2,\cdots,n_{h} \] (4)

where

\[ F_{ik} (P_{T,ik}) = a_{i} \cdot P_{T,ik}^{2} + b_{i} \cdot P_{T,ik} + c_{i} \quad \text{S/h} \] (5)

\[ Y_{ik} (P_{H,ik}) = d_{i} \cdot P_{H,ik}^{2} + e_{i} \cdot P_{H,ik} + f_{i} \quad \text{m}^{3}/\text{h} \] (6)
2.1 Classical $\lambda - \gamma$ iteration method [1]

The augmented lagrangian function for the HTS problem is written as

$$
\Phi_T = \sum_{k=1}^{K_{\text{max}}} \left[ \sum_{i=1}^{n_T} t_k F_{ik} (P_{T,ik}) - \lambda_k \left( \sum_{i=1}^{n_T} P_{T,ik} + \sum_{j=1}^{n_H} P_{H,jk} - P_{Dk} \right) \right] + \sum_{i=1}^{n_H} \gamma_i \left[ \sum_{k=1}^{K_{\text{max}}} t_k Y_{ik} (P_{H,jk}) - V_i^\text{ref} \right]
$$

(7)

The co-ordination equation from the above function can be obtained as

$$
t_k \frac{\partial F_{ik}(P_{T,ik})}{\partial P_{T,ik}} = \lambda_k \tag{8}
$$

$$
\gamma_i t_k \frac{\partial Y_{ik}(P_{H,jk})}{\partial P_{H,jk}} = \lambda_k \tag{9}
$$

The above co-ordination equations along with constraint Eqs. 2, 3 and 4 is iteratively solved to obtain optimal HTS.

3.0 Proposed Methodology

The solution process of the $\lambda - \gamma$ iteration method involves time consuming three iterative loops, in which the $\lambda$-iterations itself accounts for two iterative loops in each $\gamma$-iteration. The solution speed can be enhanced, if $\lambda$-iterations is eliminated, thereby avoiding two iterative loops. An analytical non-iterative approach is developed instead of $\lambda$-iterations in the proposed approach.

The co-ordination equations for a system with two thermal and a hydro units are written from Eqs. 8 and 9 as:

$$
t_k \frac{dF_{ik}(P_{T,ik})}{dP_{T,ik}} = t_k (2 a_i P_{T,ik} + b_i) = \lambda_k \tag{10}
$$

Eqs. 15, 13 and 12 for a general system having $n_T$-thermal and $n_H$-hydro plants can be written as

$$
P_{T,jk} = \frac{P_{Dk} - (\beta_{T2} + \beta_{T3} + \ldots + \beta_{Tn_T} + \beta_{H1} + \beta_{H2} + \ldots + \beta_{Hn_H})}{(1 + \alpha_{T2} + \alpha_{T3} + \ldots + \alpha_{Tn_T} + \alpha_{H1} + \alpha_{H2} + \ldots + \alpha_{Hn_H})} = \frac{P_{Dk} - \beta}{(1 + \alpha)} \tag{16}
$$

$$
P_{T,jk} = \alpha_{T,j} P_{T,ik} + \beta_{T,j} ; \quad j = 2,3,\ldots,n_T \tag{17}
$$

$$
P_{H,jk} = \alpha_{H,j} P_{T,ik} + \beta_{H,j} ; \quad j = 1,2,\ldots,n_H \tag{18}
$$
where
\[
\alpha_T = \frac{a_1}{a_j}, \quad \alpha_H = \frac{a_1}{\gamma_j d_j},
\]
\[
\beta_T = \frac{b_1 - b_j}{2 a_j}, \quad \beta_H = \frac{b_1 - \gamma_j e_j}{2 \gamma_j d_j},
\]
\[j = 1, 2, \cdots, nh\]  

Eq. (16) can be solved for \( P_{T_{ik}} \) for a given power demand with a suitable initial value of \( \gamma \) for the hydel plants. Eqs. 17 and 18 are to be solved for the remaining thermal and hydro power generations respectively taking into account the generator power limits. If there is any limit violation of generation, the respective limit value is assigned as the generation for the violated plant; and the limit violated plant is omitted by treating the respective \( \alpha \) and \( \beta \) values zero and reducing the net power demand by the corresponding limit value in the process of computation for the remaining units. These computations are non-iterative in nature and are performed in the \( \gamma \)-iterative loop till the available volume of water is fully utilised in the planning horizon. This procedure avoids the \( \lambda \)-iterative loop in finding the optimal schedule and helps in reducing the overall computation time. A flow chart for solving the hydrothermal scheduling problem by this method is presented in Fig. 1.

### 4.0 Simulation Results

The proposed method (PM) is tested on three HTS examples, which are taken from Ref. [4]. The results of the PM are compared with that of classical \( \lambda-\gamma \) iteration method and EGA method in order to bring out the supremacy of the developed algorithm.

**Problem-1**

This problem under study consists of one thermal and one hydel plant. The system characteristics are

\[
P_{T_{ik}}(P_{T_{ik}}) = 0.001991 P_{T_{ik}}^2 + 9.606 P_{T_{ik}} + 373.7 \quad \text{\$/h}
\]

\[
P_{H_{ik}}(P_{H_{ik}}) = 0.0007749 P_{H_{ik}}^2 - 0.009079 P_{H_{ik}} + 61.53 \quad \text{M cubic ft/hour}
\]

The allowable volume of water for the dispatch period of one day is

\[V_1^{\text{avil}} = 2559.6 \quad \text{M cubic ft.}\]

The hourly power demand over the scheduling for a day is given in Table 1.

![Diagram](https://example.com/generic_diagram.png)

**Fig. 1 Flow chart for proposed \( \gamma \) iterative method**

<table>
<thead>
<tr>
<th>k</th>
<th>( P_{Dk} )</th>
<th>k</th>
<th>( P_{Dk} )</th>
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* Duration of each interval \( t_i = 1 \) hour

Table 1

Power demand at different intervals of Problem-1
The optimal thermal and hydel generations of each plant at each interval over the scheduling horizon for this problem, obtained by the PM, is shown in Fig. 2.

![Fig. 2 Optimal thermal and hydel generations over the scheduling period for Problem-1](image)

**Problem-2**

The second system under study is made up of one thermal and two hydel plants. The cost characteristics of thermal plants, the discharge characteristics of hydel plants and water available for each hydel plant of this problem are given below. The power demand at each time interval over the scheduling period is given in Table 2 for this problem.

\[
F_{1k}(P_{T1k}) = 0.01P_{T1k}^2 + 3.0P_{T1k} + 15.0 \text{ $/ h$}
\]

\[
Y_{1k}(P_{H1k}) = 0.000105P_{H1k}^2 + 0.03P_{H1k} + 0.2 \text{ $M\text{ cubic ft/ hour}$}
\]

\[
Y_{2k}(P_{H2k}) = 0.0001P_{H2k}^2 + 0.06P_{H2k} + 0.4 \text{ $M\text{ cubic ft/ hour}$}
\]

\[
V_1^{\text{av}} = 25.0 \text{ M cubic ft}
\]

\[
V_2^{\text{av}} = 35.0 \text{ M cubic ft}
\]

**Table 2**

<table>
<thead>
<tr>
<th>k</th>
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</table>

* Duration of each interval \( t_k = 1 \text{ hour} \)

The optimal thermal and hydel generations of each plant at each interval over the scheduling horizon for this problem, obtained by the PM, is shown in Fig. 3.

![Fig. 3 Optimal thermal and hydel generations over the scheduling period for Problem-2](image)

**Problem-3**

The third problem has two thermal and two hydel plants. The system characteristics and available water for each hydel plant are as follows:

\[
F_{1k}(P_{T1k}) = 0.0025P_{T1k}^2 + 3.2P_{T1k} + 25.0 \text{ $/ h$}
\]

\[
F_{2k}(P_{T2k}) = 0.0008P_{T2k}^2 + 3.4P_{T2k} + 30.0 \text{ $/ h$}
\]

\[
Y_{1k}(P_{H1k}) = 0.000216P_{H1k}^2 + 0.306P_{H1k} + 1.98 \text{ $M\text{ cubic ft/ hour}$}
\]

\[
Y_{2k}(P_{H2k}) = 0.000136P_{H2k}^2 + 0.612P_{H2k} + 0.936 \text{ $M\text{ cubic ft/ hour}$}
\]

\[
V_1^{\text{av}} = 2500.0 \text{ M cubic ft}
\]

\[
V_2^{\text{av}} = 2100 \text{ M cubic ft}
\]

**Table 3**

<table>
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<tr>
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* Duration of each interval \( t_k = 1 \text{ hour} \)
The hourly power demand over the scheduling period is given in Table-3. The optimal scheduling obtained by the PM is graphically represented in Fig. 4.

![Fig. 4 Optimal thermal and hydel generations over the scheduling period for Problem-3](image_url)

Table 4 Comparison of results with existing methods

<table>
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<th>Problem</th>
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<th>Fuel Cost ($/day)</th>
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The accuracy of the solution is evident from the fact that the fuel cost obtained by the PM is same as that of the λ−γ iteration method and nearer to that obtained by EGA.

5.0 Conclusion
An elegant gamma algorithm for solving HTS problem by reducing the number of iterative loops in the classical λ−γ iteration has been developed. The non-iterative procedure to replace λ -iterative loop is very simple and has served to reduce the computational burden without affecting the accuracy of the solution. The ability of the developed algorithm to converge at an optimal solution with smaller execution time will find its role as a powerful on-line tool in energy management systems.

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Reference


