Modeling and Design of PWM based Sliding Mode Controller for Active Clamp Forward Converter

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Abstract: This paper presents the step by step design procedure to model and design of sliding mode controller for Forward Converter with Active Clamp circuit (ACFC). Brief review on working principle and mathematical modeling of Converter is first given. The importance of sliding mode controller for forward converter and design procedure of the considered controller is described in detail. And the validity of designed controller and achievement of desired compensation is confirmed by the obtained results. Finally the results are analyzed by applying disturbances at source end and at load side.

Key words: Active Clamp, Forward Converter, Sliding Mode Controller, state Space Modeling.

1. Introduction.
Among Switch Mode Power supplies used in various low voltage high current applications where there are large front end voltage fluctuations like Distributed Power System Servers, Automotive, Desktop Computers, Laptop Computers and Telecommunication Systems, the buck derived single switch forward converter is most promising topology up to few hundred watts due to its simplicity and robustness[1-4]. But the transformer in a forward converter topology does not inherently reset in each switching cycle, number of reset techniques have been evolved for forward converter. Between which Active clamp reset approach is most attracting one, by improving the efficiency of the converter. It solves some problems in conventional forward converter like voltage stress on the switch, core reset, voltage spikes caused by the transformer leakage inductance and low duty cycle [5-8]. The basic circuit diagram of ACFC with low side clamp circuit is shown in Fig.1.

And to improve the performance of the basic forward converter with active clamp circuit different types of controllers like voltage mode controller, peak current mode controller and average current mode controllers are designed [9-11]. But the complexity in the process of design of the controller also increases because of high switching frequency converters are having strong nonlinearity in the components of the converter, liner small signal models restricted the validity of the controller. The main difficulty encountered in modeling a DC-DC converter is that the characteristics of the converter are nonlinear, i.e. they are having the variable structured systems. So there is a need of controllers for satisfactorily operation of DC-DC converters. The conventional solutions for controller requirements are based on classical control theory or modern control theory. Controllers designed on the basis of classical control theory require precise linear mathematical models of the plants. These controllers have poor performance under parameter variation, non-linearity and load disturbance etc. Further, the models obtained by state-space averaging methods are only useful for small signals. On the other hand, modern control theory-based controllers such as state-feedback controllers, self-tuning controllers, hysteresis control, Pulse-width Modulation based sliding-mode controllers (SMC), Proportional + Integral + Derivative (PID) controllers and model reference adaptive controller’s etc are very successful in providing good control performance for complex systems. These controllers are also needed mathematical calculations but sensitive to parameter variations. The accuracy of these controllers is verified by comparing the simulation results with the responses obtained from ACFC using MATLAB/SIMULINK.

The main objective of this paper is to discuss the design issue of Sliding Mode Controller (SMC) for Active Clamp Forward Converter and to verify the validity of the designed controller by comparing with the results of conventional Controller. In section 2, outline of the working Principle and Mathematical Modeling of the ACFC is presented. Design procedure of the proposed controller is presented in section 3. In section 4 analysis and Simulation Results are presented and finally, the
conclusion about the designed controller is presented in Section 5.

2. Working Principle and Mathematical Modeling

Consider an ACFC as shown in Fig.1, where \( L \) is the leakage inductance, the lump capacitance; \( C \) is the sum of parasitic capacitance of MOSFET, \( Q_1 \) and the transformer. We assume to be at steady state, i.e. \( C \) is already charged to \( V_c \).

2.1. Working Principle of ACFC

Outline of the working principle of the ACFC explained in two stages.

Stage 1: The MOSFET, \( Q_1 \) is turned on at \( t_0 \), the current ramps up in the magnetizing inductor. The secondary diode \( D_1 \) conducts and \( D_0 \) is blocked as shown in Fig.2. The leakage capacitor \( C \) is discharged and \( V_{DS} \) of \( Q_1 \) is almost zero. The leakage inductor is crossed by the reflected output inductor current which peaks to \( I_n \) plus \( \frac{I_n}{N} \).

Stage 2: As the clamp diode is now conducting, the controller can activate the auxiliary switch, \( Q_2 \) in zero-voltage condition as shown in Fig.3. The controller must thus generate a gate signal slightly delayed after turning off the main switch. The magnetizing current crosses zero where it changes direction, thanks to the auxiliary switch allowing conduction in both directions. The clamp voltage decreases until the magnetizing current reaches its maximum negative value at \( t_1 \). At that time, the controller instructs the auxiliary switch to open at \( t_1 \).

2.2. State Space Averaged Model of ACFC

There are two operating modes of ACFC in a switching period, in which converter can be denoted using two linear state space equations (1,2), where \( x \) is the vector of state variables, \( u \) is the vector of independent sources; \( A_1 \), \( B_1 \), \( A_2 \) and \( B_2 \) are respective system matrices in each of the two operating modes.

\[
\dot{x} = A_1 x + B_1 u (n=1,2) \\
y = C_1 x + D_1 u 
\]

Where \( x = [V_C \; I_L \; V_{CC} \; I_{m}]^T \); \( y = V_{out} \); \( u = V_g \)

The state space averaged model of the converter is determined by taking weighted averaged of the equations (1) and (2) as

\[
\dot{x} = A x + B u 
\]

Where \( A = dA_1 + (1 - d)A_2 \); \( B = dB_1 + (1 - d)B_2 \)

We now perturb and linearize the converter wave form this quiescent operating point: Each variable is written as a sum of steady state or DC component and small signal or AC component as,

\[
x = X \pm \delta x \quad u = U \pm \delta u \quad D = d + \delta d 
\]

Where \( \delta x \ll X \); \( \delta u \ll U \); \( \delta d \ll D \)

Substituting equations (4) & (5) in equation (3) and neglecting the higher order terms, the equation which relates small changes in variables is,

\[
\delta x = AX + B\delta u + E\delta d 
\]

Where \( E = (A_1 - A_2)X + (B_1 - B_2)U \)

\[
A_1 = \begin{bmatrix} -\frac{1}{R_{L_0}} & 0 & 0 \\ 0 & \frac{1}{C_0} & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \quad B_1 = \begin{bmatrix} 0 \\ \frac{1}{N_{L_0}} \\ 0 \end{bmatrix} 
\]

\[
A_2 = \begin{bmatrix} -\frac{1}{R_{L_0}} & 0 & 0 \\ 0 & \frac{1}{C_0} & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \quad B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} 
\]
\[
\begin{bmatrix}
\frac{d}{dt} V_c \\
\frac{d}{dt} I_L \\
\frac{d}{dt} V_C \\
\frac{d}{dt} I_m
\end{bmatrix} =
\begin{bmatrix}
\frac{-1}{R_0 C_0} & 0 & 0 & 0 \\
\frac{-1}{R_0} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{C_0} & \frac{1}{L_m} \\
0 & 0 & \frac{-1}{C_0} & 0
\end{bmatrix}
\begin{bmatrix}
V_c \\
I_L \\
V_C \\
I_m
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{1}{L_m} & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
V_g \\
\delta V_o \\
V_{ref}
\end{bmatrix}
(9)
\]

With the help of designed mathematical model, Block diagram representation of ACFC is drawn and presented in Fig.5.

Fig.5. Block diagram representation of ACFC

A power stage is designed by assuming input and output parameters. The parameter values for the simulation of ACFC are shown in Table-1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Voltage</td>
<td>(36-72)V Nominal: 48V</td>
</tr>
<tr>
<td>Output Voltage</td>
<td>5V</td>
</tr>
<tr>
<td>Output power</td>
<td>100W</td>
</tr>
<tr>
<td>Switching Frequency</td>
<td>100KHz</td>
</tr>
<tr>
<td>High Frequency Transformer</td>
<td>130W:12:3 Lm=100µH</td>
</tr>
<tr>
<td>Output Filter</td>
<td>Loo=8µH Coo=590µF</td>
</tr>
</tbody>
</table>

Table 1. Design parameters of ACFC

With the help of designed parameters, ACFC is simulated in P-Sim Software and open loop response of ACFC is shown in Fig.6.

Fig.6. Open loop response of ACFC

3. Methodology to design Sliding Mode controller for DC-DC Converters

The complete discussion about the theory of SM control, equivalent control, and the relationship of SM control and duty ratio control is presented in [14-19]. Here we will discuss the dc-dc converter modeling and the detailed procedure for designing the SM controller for Active Clap Forward Converter, which are operated in continuous conduction mode (CCM).

3.1 Mathematical Model of an Ideal SM PID Voltage Controlled for a DC-DC Converter

The first step to the design of an SM controller is to develop a state-space approach to Active Clap Forward Converter which is model in terms of the desired control variables (i.e., voltage and/or current etc.) by applying the Kirchhoff’s voltage and current laws [15]. Here the output voltage is taken as the control variable for the controller. The SM controller presented here is a second-order proportional integral derivative (PID) SM voltage controller. Fig.7 shows the schematic diagram of the PID Sliding Mode Voltage Controller (SMVC) of ACFC converter in the conventional PWM configuration. Here C, L and Ro denote the capacitance, inductance, and instantaneous load resistance of ACFC respectively. Lc, Lm and Ro are the capacitor, induct, and load currents respectively; Vg, δVo and Vref are the input voltage, the sensed output voltage and reference voltages respectively; δ is the scaling factor which is defined as \( \delta = \frac{V_{ref}}{V_{od}} \) and u=0 or 1 is the switching state of the power switch Sw.

Fig.7. Schematic diagram of the PID Sliding Mode Voltage Controller of ACFC converter
The basic expression for ACFC PID SMVC converter, the control variables expressed in the general form as:

\[ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} (V_{ref} - \delta V_c) \\ \frac{dt}{dt} (V_{ref} - \delta V_c) \\ \int (V_{ref} - \delta V_c) \ dt \end{bmatrix} \]  \tag{10}

Here, the control variables \(x_1, x_2\) and \(x_3\) represent the voltage error, the rate of change of voltage error, and the integral of voltage error respectively. By substituting the ACFC state space operational model under continuous conduction mode (CCM) into eq. (10) gives the following control variable description:

\[ x_{acfc} = \begin{bmatrix} x_1 = \frac{\delta V}{RC} + \int \frac{V_{ref} - \delta V_c}{LC} \ dt \\ x_2 = \frac{\delta V}{RC} + \int \frac{V_{ref} - \delta V_c}{LC} \ dt \\ x_3 = \frac{\delta V}{RC} + \int \frac{V_{ref} - \delta V_c}{LC} \ dt \end{bmatrix} \]  \tag{11}

The state differentiation of eq. (11) gives the state-space descriptions required for ACFC.

\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{1}{R_C} \\ 0 & 0 & \frac{1}{R_C} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{R_C} \delta q \\ \frac{1}{R_C} \delta q \end{bmatrix} u + \begin{bmatrix} 0 \\ -\frac{1}{R_C} \delta q \\ \frac{1}{R_C} \delta q \end{bmatrix} \]  \tag{12}

The state-space representation of eq. (12) can be written in the standard form as: \( \dot{x} = Ax + Bu + D \).

### 3.2. Controller Design

The basic idea of SM control is to design a certain sliding surface in its control law that will follow the reference path state variables towards a desired equilibrium. The designed SM controller must satisfy the following three conditions [14-16].

#### 3.2.1 To Meet Hitting condition

The control law which is satisfying hitting condition that follows switching functions as:

\[ u = \frac{1}{2} (1 + \text{sgn}(S)) \]  \tag{13}

and \( u = 1 \) when \( S > 0 \), \( u = 0 \) when \( S < 0 \).

Where \( S \) is the instantaneous state variable Trajectory reference path, and is defined as:

\[ S = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 \]  \tag{14}

Here \( \lambda_1, \lambda_2, \text{and} \lambda_3 \) representing the control parameters which are also called as sliding coefficients.

#### 3.2.2 To Meet Existence condition

After determining the switching states, whether \( u = 1 \) or 0, the next step is to verify whether the sliding coefficients \( \lambda_1, \lambda_2, \text{and} \lambda_3 \) follow the existence condition [7]. Here Lyapunov’s direct method is used to determine the ranges of the employable sliding coefficients. This is possible by checking the approachability condition of the state trajectory path as:

\[ \lim_{S \to 0} \dot{S} < 0 \]  \tag{15}

By solving this equation for ACFC, it will give the existence condition as:

\[ 0 < -a_2 \delta q \left( \frac{\lambda_3}{\lambda_2} - \frac{1}{R_C} \right) i_C + \text{NL} \delta q \left( \frac{\lambda_3}{\lambda_2} - \frac{1}{R_C} \right) (V_{ref} - \delta V_c) \]  \tag{16}

#### 3.2.3 To Meet stability condition

In addition to the existence condition, the selected sliding coefficients must ensure the stability condition [7]. The selection of sliding coefficients is based on the desired dynamic response of the converter. The sliding surface equation (14) is relating the sliding coefficients to the dynamic response of the converter during SM operation is:

\[ \lambda_1 x_1 + \lambda_2 \frac{dx_2}{dt} + \lambda_3 \int x_1 \ dt = 0 \]  \tag{15}

The equation (15) can be rearranged into a standard second-order form as:

\[ \frac{dx_2}{dt} + \lambda_2 \frac{dx_2}{dt} x_2 + \lambda_3 \int x_1 \ dt = 0 \]  \tag{16}

By comparing the Equation (16) with standard second order form we get:

\[ \omega_n = \frac{\lambda_2}{\lambda_3} \frac{\sqrt{2}}{2} \sqrt{\lambda_2 \lambda_3} \]  \tag{17}

By observing equation (17), it is clear that the sliding coefficients are dependent on the bandwidth with the existence condition for the PWM based controllers. The design equations mentioned in (17) are applicable for all other types of second order SMVC converters.

### 3.3 Implementation of PWM Based SMVC for ACFC

The HM technique in SM control requires only the control equations (13) and (14). The linear PWM based SM controller requires the relationship of the two control techniques which are to be developed as shown in the fig. 4.3 of the pulse-width modulation technique [15-16]. The PWM-based PID SMVC ACFC converter controller structure is shown in Fig. 8. The design of the linear PWM based SM controller can be performed in two steps.

- The equivalent control signal \( u_{eq} \) [15] is used instead of \( u \), which is a function of discrete input function is derived from the invariance condition by setting the time differentiation of (13) as \( \dot{S} = 0 \).

- The equivalent control function \( u_{eq} \) is mapped on the duty cycle function of the pulse-width Modulator. For the PWM based SMVC ACFC converter, the derivations for the equivalent control and duty cycle control techniques are as follows [15]:

By equating \( S = 1^T A x + 1^T B u_{eq} + 1^T D = 0 \), it gives equivalent control signal. Here \( u_{eq} \) is in between 0 and 1. The obtained control law is:

\[ 0 < -a_2 \delta q \left( \frac{\lambda_3}{\lambda_2} - \frac{1}{R_C} \right) i_C + \text{NL} \delta q \left( \frac{\lambda_3}{\lambda_2} - \frac{1}{R_C} \right) (V_{ref} - \delta V_c) + \delta V_e < \delta V_{ref} \]  \tag{17}

With the designed sliding coefficients, ACFC is simulated in P-Sim software and schematic diagram of ACFC with SMVC is shown in Fig.8.

![Schematic diagram of the ACFC converter with Sliding Mode Voltage Controller](image-url)
4. Results and Discussion

Table 1 shows the specifications of the ACFC converter. The PWM based SM controller is designed to give a second order system response at settling time of $T_s = 250 \mu\text{Sec}$. From eq. (17), the sliding coefficients are determined as $\frac{\lambda_1}{\lambda_2} = 8000$ and $\frac{\lambda_3}{\lambda_2} = 15999961.18$. Finally control parameters can be calculated as $\gamma p_1 = -\delta L_{0}\left(\frac{\lambda_1}{\lambda_2} - \frac{1}{R_{p}C_{0}}\right) = -0.009776$ and $\gamma p_2 = N L_{0}C_{0}\frac{\lambda_3}{\lambda_2} = 0.302$. The simulation results of ACFC with sliding mode controller are as follows. For getting the closed-loop simulation results the equivalent voltage control signal which is generated from the sliding surface is compared with the ramp signal. The generated PWM Signals are shown in Fig. 9. And delay is provided between switching transitions of Mosfets Q1 and Q2, as shown in Fig. 10.

A Step change in Input Signal is applied by changing the input from the actual designed value at a Time of 2 msec, the closed loop SMVC ACFC converter results are shown in Fig. 11. Similarly, Load disturbance is providing by applying step change of 2A at a Time of 4msec. Finally the variation of output voltage with the change in load resistance from full load to Quarter load is observed and obtained results are shown in Fig. 12, in Fig. 13 and in Fig. 14. The Ripple quantity in output wave forms is shown in Fig. 15.
Fig. 12. Output Voltage and Current Wave form of ACFC with Sliding Mode Controller at Quarter load and with load disturbance

Fig. 6. Ripple Quantity in Output Voltage and Output Current of ACFC with Sliding Mode Controller

5. Conclusion.
A Step by Step design process to design Sliding Mode Voltage Controller is presented in this paper. State Space averaged Model is more useful to design Control law for Sliding Mode Controller. The operation of ACFC in Continues Conduction Mode with the SMVC is discussed. The SMVC for ACFC is implemented by employing a certain sliding surface. With the help of designed parameters and control constraints, ACFC is simulated in P-Sim Software. And to verify the validity of designed controller disturbance is applied in different aspects and results are analyzed. From design processes and obtained results, it is concluded that design processes is easy for the systems are having nonlinearity like ACFC and designed Sliding mode controller is more robust as compared to other conventional Controllers.

References
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