A SIMPLE STATE FEEDBACK LINEARIZATION CONTROL OF MULTILEVEL ASVC

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Abstract: This paper presents a modeling analysis of a nonlinear control strategy for a three-phase multilevel Advanced Static Compensator (ASVC). The nonlinear state-space model of the multilevel ASVC is obtained from the DQ0 reference frame. The input/output feedback linearization is then applied and the state feedback linearization control law is obtained. The model that was obtained was linearized and decoupled in two independent subsystems. The stabilizing controllers were designed based on the linear method. The simulation analysis was performed by MATLAB. The system performance was tested through a sudden change of the load from inductive to capacitive mode.

Key words: ASVC (Advanced Static Var Compensator), NPC (Neutral Point Clamping), Nonlinear Controller.

1. Introduction
Advanced Static Var Compensator (ASVC) is a shunt Flexible AC Transmission System (FACTS) devices that can regulate line voltage at the Point of Common Coupling (PCC), balance loads or compensate load reactive power by producing the desired amplitude and phase of the inverter output voltage. The AC system is connected to a DC capacitor (energy storage device) through the inverter [1]. The basic function of the ASVC installed in a power system is for the line voltage control. To achieve smooth control of the proposed system, two controllers were implemented. The first one is an AC voltage controller which regulates the reactive power exchange between the ASVC and the power system. A second controller installed also in the ASVC is the DC voltage controller which regulates the DC voltage across the DC capacitor of the ASVC [2]. The multilevel converter topology was used thoroughly in the ASVC. Alternatively, multilevel converters can also be used to eliminate the complex transformer array needed to suppress the harmonics. In comparison with other converter topologies, the multilevel structure can reduce voltage stress across the switches and provide more available vectors.

Fig.1. ASVC using three-level inverter
The main circuit configuration consists of a bridge inverter made up of twelve power GTO’s with antiparallel diodes which is connected to the three-phase supply through an impedance $L_s$ of small value, comprising an inductance $L_s$ and a resistor $R_s$ on the AC side. $L_s$ represents the leakage inductance of the transformer, and $R_s$ represents the inverter and transformer conduction losses.

![Fig.2. Per-phase fundamental equivalent circuit](image)

Fig.2. Per-phase fundamental equivalent circuit

Two capacitors are connected to the dc side of the converter. The structure of one leg of the inverter itself is made up of four pairs of diode-GTO forming a switch and two diodes allowing to have the zero level point of the inverter output voltage. The operation principles of the system can be explained by considering the per-phase fundamental equivalent circuit of the ASVC system as shown in Figure 2. In this figure, $v_{sa}$ is the ac mains voltage source. $i_{at}$ and $v_{oa}$ are the fundamentals components of current and output voltage of the inverter supply respectively.

2.2. Modelling of the system

The main circuit of the ASVC shown in Figure 1 is modeled in this section. The equivalent circuit is obtained by performing a circuit dq transformation method with the following assumptions:

a) The source voltages are balanced,

b) Line impedance and total loss of the inverter is represented by lumped resistor $R_s$,

c) Harmonic components generated by switching action are negligible.

Figure 4 shows a simplified equivalent circuit of the ASVC. Using matrix form, the mathematical model is given by:

$$L_s \frac{d}{dt} \begin{pmatrix} i_{sa} \\ i_{sb} \end{pmatrix} = \frac{2}{3} \begin{pmatrix} -R_s & 0 & 0 \\ 0 & -R_s & 0 \\ 0 & 0 & -R_s \end{pmatrix} \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} + \begin{pmatrix} v_{sa} - v_{oa} \\ v_{sb} - v_{ob} \\ v_{sc} - v_{oc} \end{pmatrix}$$ (1)

$$K = \begin{pmatrix} \cos(\omega t) & \cos(\omega t - 2\pi/3) & \cos(\omega t + 2\pi/3) \\ -\sin(\omega t) & -\sin(\omega t - 2\pi/3) & -\sin(\omega t + 2\pi/3) \end{pmatrix}$$

$$K^{-1} = K^T$$

$$X_{dq0} = KX_{abc}, \quad X_{abc} = K^{-1}X_{dq0}$$ (3)

![Fig.3. Phasor diagram for leading and lagging mode](image)

Fig.3. Phasor diagram for leading and lagging mode

2.2.1. Transform of Part A, B

The voltage and current relation about resistor $R_s$ is:

$$v_{s,dq0} = R_si_{s,abc} + v_{dq0}$$ (4)

DQ transforms of (4) becomes:

$$v_{s,dq0} = R_si_{s,abc} + v_{dq0}$$ (5)

Where

$$v_{s,dq0} = Kv_{s,abc} = \begin{pmatrix} V_c \\ 0 \end{pmatrix}$$ (6)

Where $v_c$ is the rms line voltage.

2.2.2. Transform of Part C

The voltage and current relation with inductor $L_s$ is:

$$L_s i_{s,abc} = v_{abc} - v_{o,abc}$$ (7)

And the circuit dq transform becomes:

$$L_s \frac{d}{dt} \begin{pmatrix} i_{sd} \\ i_{sq} \end{pmatrix} = \frac{2}{3} \begin{pmatrix} -R_s & \omega L_s & 0 \\ -\omega L_s & -R_s & 0 \\ 0 & 0 & -R_s \end{pmatrix} \begin{pmatrix} i_{sd} \\ i_{sq} \end{pmatrix} + \begin{pmatrix} v_{sd} - v_{qo} \\ v_{sq} - v_{qo} \end{pmatrix}$$ (8)

That is:

$$L_s \frac{d}{dt} \begin{pmatrix} i_{sd} \\ i_{sq} \end{pmatrix} = \begin{pmatrix} -R_s & \omega L_s \\ -\omega L_s & -R_s \end{pmatrix} \begin{pmatrix} i_{sd} \\ i_{sq} \end{pmatrix} + \begin{pmatrix} v_{sd} - v_{qo} \\ v_{sq} - v_{qo} \end{pmatrix}$$ (9)
2. 2.3. Transform of Part D, E

The commutation function is defined by equation (10):

\[
S = \begin{bmatrix}
S_a \\
S_b \\
S_c
\end{bmatrix} = \frac{2}{\sqrt{3}}\begin{bmatrix}
\sin(\omega t + \alpha) \\
\sin(\omega t - 2\pi/3 + \alpha) \\
\sin(\omega t + 2\pi/3 + \alpha)
\end{bmatrix}
\]

(10)

mis the modulation index in the Park axis.
\(\alpha\) is the Phase angle between source voltage and switching function.

\[
m = \frac{\sqrt{v^2_{sd} + v^2_{sq}}}{v_{dc}/2}, \quad \alpha = \tan^{-1}\left(\frac{v_{sq}}{v_{sd}}\right)
\]

(11)

Note that modulation index is given by

\[
IM = \frac{V_{o,crête}}{v_{dc}} = \frac{E}{\sqrt{3}}m
\]

(12)

The DC current and voltage is given by equation (13) and (14)

\[
2i_d = S^T K^{-1}i_{d,apso} = m(1 \quad 0 \quad 0)
\]

(13)

\[
v_{dc} = \frac{1}{c} \int i_d dt
\]

(14)

2. 2.4. Active and reactive power

Both powers are expressed by equation (15)

\[
\begin{align*}
p_c &= \frac{3}{2}v_{sd}i_{sd} + v_{sq}i_{sq} \\
Q_c &= \frac{3}{2}v_{sd}i_{sd} - v_{sd}i_{sd}
\end{align*}
\]

(15)

If \(\alpha\) is chosen equal to zero, the \(v_{sq}\), voltage is equal to zero and the reactive power becomes proportional to \(v_{sd}i_{sq}\). To control the reactive power \(Q_c\), it may be sufficient to control \(i_{sq}\), (equation 16).

\[
Q_c = -\frac{3}{2}v_{sd}i_{sq}
\]

(16)

The average rate of change of energy associated with the AC link and DC link is given by

\[
p = \frac{3}{2}v_{sd}i_{sd} + v_{sq}i_{sq} = CV_{dc}\frac{dv_{dc}}{dt}
\]

(17)

From (14) and (17),

\[
\frac{dv_{dc}}{dt} = \frac{2}{3}v_{sd}i_{sd} + v_{sq}i_{sq}
\]

(18)

Equation (18) leads to nonlinear system with regard to \(v_{dc}\). Combination of equation (9) and (18) describes a nonlinear model as:

\[
\frac{d}{dt}\begin{bmatrix}
i_{sd} \\
i_{sq}
\end{bmatrix} = \begin{bmatrix}
-\frac{R_s}{L_s}i_{sd} + \omega i_{sq} \\
-\omega i_{sd} - \frac{R_s}{L_s}i_{sq} \\
\frac{2}{3}Cv_{dc}(v_{sd}i_{sd} + v_{sq}i_{sq}) \\
\end{bmatrix} + \begin{bmatrix}
\frac{1}{L_s} \\
0 \\
0
\end{bmatrix}(v_{sd} - v_{od})
\]

(19)

This system is of the third order with two control inputs.

3. Application of the input/output linearization to the ASVC

The feedback linearization is a method that eliminates the non-linearity of the system so that the closed-loop system dynamics are reduced to a linear form. Thus, the controller for the linearized system can be designed with the well-known linear control theory[8]-[9].

For feedback linearization process, at first, a multi-input multi-output (MIMO) [10], system is expressed from equation (19) as

\[
\begin{bmatrix}
\dot{x} \\
y
\end{bmatrix} = \begin{bmatrix}
f_1(x) \\
f_2(x) \\
f_3(x)
\end{bmatrix} + \begin{bmatrix}
g_1 \\
g_2
\end{bmatrix}u
\]

(20)

Where

\[
f(x) = \begin{bmatrix}
f_1(x) \\
f_2(x) \\
f_3(x)
\end{bmatrix} = \begin{bmatrix}
\frac{1}{L_s}x_1 - \omega x_2 \\
\frac{1}{L_s}x_2 + \omega x_1 \\
\frac{2}{3}C\frac{v_{dc}}{x_1}x_1
\end{bmatrix}
\]

\[
g = \begin{bmatrix}
g_1 \\
g_2
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

\[
x = \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
i_{sd} \\
i_{sq} \\
v_{dc}
\end{bmatrix}, \quad u = \begin{bmatrix}
u_1 \\
u_2
\end{bmatrix} = \begin{bmatrix}
v_{sd} - v_{od} \\
v_{sq} - v_{od}
\end{bmatrix}
\]

Where:

- \(x\) is the state vector,
- \(u\) is the control input vector,
- \(g\) is the input matrix,
- \(f(x)\) is a nonlinear vector field.
3.1. Control of the outputs

The aim of the control strategy is to regulate the dc link voltage and to control the reactive power current component. Therefore, the output equation is given by

\[ y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_1(x) \\ h_2(x) \end{pmatrix} = \begin{pmatrix} i_{sq} \\ v_{dc} \end{pmatrix} \]

3.2. Relative degree

We calculate the relative degree \( r_i \) associated with each output variable, \( y_i \), which will be chosen so that it corresponds to the number of times to divert this output to make explicit one of the control variables.

\[
\begin{aligned}
d\frac{y_1}{dt} &= L_f^T h_1(x) + L_g L_f^{r-1} h_1(x)u \\
d\frac{y_2}{dt} &= L_f^T h_2(x) + L_g L_f^{r-1} h_2(x)u
\end{aligned}
\]

(21)

3.2.1. Relative degree of the reactive power

The Lie derivative to the reactive power is given by:

\[ h_1(x) = L_f h_1(x) + L_g h_1(x)u_1 + L_g h_2(x)u_2 \] (22)

From Equation (19) we can deduce that:

\[
\begin{aligned}
L_f h_1(x) &= \frac{-K}{L} x_2 - \omega x_1 = f_2(x) \\
L_g h_1(x) &= 0 \\
L_g h_2(x) &= \frac{1}{l_s} = g_2
\end{aligned}
\]

(23)

So equation (23) gives:

\[ y_1 = \dot{h}_1(x) = \frac{d i_{sq}}{dt} = f_2(x) + g_2 u_2 \] (24)

The input appears in the first derivative of the output \( i_{sq} \). We stop, so that the relative degree associated to the output \( y_1 \) is: \( r_1 = 1 \)

3.2.2. Relative degree of DC voltage

The Lie derivative for DC voltage is given by:

\[ \dot{h}_2(x) = L_f h_2(x) + L_g h_2(x)u_1 + L_g h_2(x)u_2 \] (25)

After simplification we get:

\[
\begin{aligned}
L_f h_2(x) &= \frac{3}{2c_x} v x_1 \\
L_g h_2(x) &= 0 \\
L_g h_2(x) &= 0
\end{aligned}
\]

(26)

The input does not appear in the derivative of the output \( y_2 \) so we have derived a second time \( y_2 \).

The development of equation (25) gives:

\[ \dot{h}_2(x) = L_f h_2(x) + L_g L_f h_2(x)u_1 + L_g L_f h_2(x)u_2 \] (27)

After simplification we get:

\[
\begin{aligned}
L_f^2 h_2(x) &= -\frac{3}{2c_x^2} v x_1 f_3 + \frac{3}{2c_x} v f_1 \\
L_g L_f h_2(x) &= \frac{3}{2c_x} v g_1 \\
L_g L_f h_2(x) &= 0
\end{aligned}
\]

(28)

So Equation (27) becomes:

\[ \dot{y}_2 = \dot{h}_2(x) = -\frac{3}{2c_x^2} v x_1 f_3 + \frac{3}{2c_x} v f_1 + \frac{3}{2c_x} v g_1 u_1 \] (29)

The input appears finally in the second derivative of the output \( v_{dc} \). So the relative degree associated with the output \( y_2 \) is \( r_2 = 2 \).

The matrix defining the relationship between physical inputs \( \begin{pmatrix} u_1 \end{pmatrix} \) and the derivatives of the outputs \( \begin{pmatrix} y(x) \end{pmatrix} \) is given by:

\[
\begin{aligned}
\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} &= \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \\
L_f h_1(x) &= L_f h_2(x) + \begin{pmatrix} L_g h_1(x) \\ L_g h_2(x) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}
\end{aligned}
\]

(30)

\[
\begin{aligned}
\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} &= \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = A(x) + D(x) \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}
\end{aligned}
\]

(31)

According to this formula, the decoupling matrix is given by:

\[ D(x) = \begin{pmatrix} 0 \\ \frac{3v g_1}{2c_x^2} g_2 \\ 0 \end{pmatrix} \] (32)

Controls of \( u_1 \) and \( u_2 \) can be determined if the decoupling matrix is not singular.

\[ \det (D(x)) = \frac{3v g_1 g_2}{2c_x^2} \neq 0 \] (33)

\[ \text{Det} (D(x)) \] is non-nil then the matrix \( D(x) \) is invertible.

We have:

\[ A(x) = \begin{pmatrix} 3 \\ 3f_3 \end{pmatrix} v f_1 - \begin{pmatrix} 3 \\ 3f_3 \end{pmatrix} v x_1 \] (34)

We then define the non-linear control:

\[ \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = D^{-1}(x) \begin{pmatrix} -A(x) + \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \end{pmatrix} \] (35)

3.3. Control loop

For tracking control, the closed-loop error equations are given as follows:
The gains calculated were \(k_{ij}\) equation (37) and asymptotic tracking control to the reference was obtained [11].

\[
\begin{bmatrix}
  k_{11} \\
  k_{12}
\end{bmatrix} = \begin{bmatrix}
  10^4 \\
  100
\end{bmatrix} \quad \begin{bmatrix}
  k_{21} \\
  k_{22} \\
  k_{23}
\end{bmatrix} = \begin{bmatrix}
  10^9 \\
  10^8 \\
  10^8
\end{bmatrix}
\] (37)

Consequently, the state feedback controlled inputs \(v_{12}\) could be finally computed as in (38) by:

\[
\begin{bmatrix}
  v_1 \\
  v_2
\end{bmatrix} = \begin{bmatrix}
  y_{1\text{ref}} + k_{11}e_1 + k_{12}\int e_1\,dt \\
  y_{2\text{ref}} + k_{21}e_1 + k_{22}\int e_2\,dt + k_{23}\int e_2\,dt
\end{bmatrix}
\] (38)

The proposed nonlinear control block diagram of the multilevel ASVC is shown in Figure 5.

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**Table I. VALUE OF CONTROLLER GAINS**

<table>
<thead>
<tr>
<th>Control loop</th>
<th>Poles values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_2 = i_{eq})</td>
<td>(\mu_1 = -0.5)</td>
</tr>
<tr>
<td>(x_3 = v_{dc})</td>
<td>(\mu_1 = -0.0005 + 0.3162i)</td>
</tr>
<tr>
<td>(\mu_2 = -0.0005 - 0.3162i)</td>
<td></td>
</tr>
</tbody>
</table>

Tracking errors was achieved by locating the desired poles on the left-half plane. Table I gives the values of the desired poles.

\[
\begin{align*}
  e_1 &= i_q^* - i_q \\
  e_2 &= v_{dc}^* - v_{dc}
\end{align*}
\]

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**4. Simulation Results and Discussions**

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**Fig. 5. Block diagram of the state feedback controlled system**

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**Fig. 6. Simulation results for reactive power with reference change from \(0\,\text{A}\) to \(50\,\text{A}\) capacitive.**

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**Fig. 7. Simulation results for reactive power with reference change from \(50\,\text{A}\) capacitive to \(50\,\text{A}\) inductive.**
Figures (6.a) and (7.a) show the waveforms of the reactive and active currents before and after the insertion of the compensator. As shown in Fig. 7.a, at first, the ASVC is connected to the load resistance. Hence, the compensator provides no reactive power. In other cases the ASVC is connected to a capacitive and inductive load, respectively then the compensator provides and absorbs reactive power ($Q = \pm 23kVar$).

The dynamic response of the reactive current $i_d$ (its reference $i_{d}^{r} = 50A$) is shown to be very fast. The active current $i_q$ is now a constant value which maintains the DC voltage unchanged. Decoupling of $i_d$ and $i_q$ is guaranteed.

Figures (6.b), (6.c), (7.b), and (7.c) show the responses of the DC voltage. The DC voltage follows its reference (700V) perfectly. However, it fluctuates around its reference by 1V; this is due to the phenomenon of balancing the two DC voltages. Figures (7.d) shows the simulated current and voltage waveforms to a step reference of reactive power.

5. Conclusion

A study and mathematical modeling of the dynamic performance analysis of an Advanced Static Var Compensator (ASVC) using three-level voltage source inverter has been presented in this paper. A nonlinear control strategy was proposed for the control of this compensator. The controller is based on two separate loops.

The inner loop decouples the current components in the ‘dq’ reference and uses the exact feedback linearization with integral control to make the currents track their references in a fast and satisfactory manner.

The outer and slower loop regulates the DC voltage level using nonlinear feedback with integral control. The simulation results included show that the proposed nonlinear control law can improve considerably the system performance operation in different aspects.

References


Appendix

System Parameters

System simulation parameters used for the proposed system are as follows:

Three-phase power grid phase voltage = 220V.
PWM switch frequency = 1kHz.

$R_s = 0.4\Omega$

$L_s = 3.10^{-3}\text{H}$

$C = 8800.10^{-6}\text{F}$

$\omega = 100\pi\text{rad/s}$.