Abstract: This paper proposes two approaches for transient stability analysis and they have the new ideas. These approaches use Athay's PEBS. The first method is called CTSA that it's complex simulation and Athay's PEBS. CTSA is more reliable than all existing methods that use unstable equilibrium point (UEP) as it doesn't use any convergence methods.

POMP's method follows the point of maximum potential energy on post-fault system trajectory that this point is approximated to Taylor's expansion second orders.

This paper presents a detailed implementation of a fast and accurate method for Available Transfer Capability (ATC) calculations. We use two new methods for termination criteria in ATC calculations.

A novel formulation of the ATC problem has been adopted based on full ac power flow solution with matrix operations to incorporate the effects of voltage limits, and voltage collapse. This program written by MATLAB and don't use any do loop, then this power flow program is fivefold faster than any program in MATLAB.

The ideas are demonstrated on 4, 7 and 30 bus IEEE.

Keywords: Direct Analysis of Transient Stability, PEBS, CTSA, POMP and ATC

1. Introduction

This paper proposes two approaches for transient stability analysis and they have the new ideas. These approaches use Athay's PEBS. The first method is called CTSA that it's complex simulation and Athay's PEBS. CTSA is more reliable than all existing methods that use unstable equilibrium point (UEP) as it doesn't use any convergence methods.

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1. Introduction

Order 889 mandated each control area to compute ATC and post them on a communication system called the Open Access Same-Time Information System (OASIS). Approaches of computing ATC can be divided into the following groups: STATIC METHODS and DYNAMIC METHODS. STATIC Methods can be divided into OPF (linear & nonlinear optimization) and SENSITIVITY (linearization) [1] and CPF: (Continuation Power Flow) [2]. In OPF, for each transaction, the generations and loads are increased until allowable transmission between two areas reaches maximum. In CPF, for each transaction, the generation and load are increased until one line reaches its MVA limit or other static terminated conditions.

Termination criteria in Static methods are: Transmission line flow constraints (Thermal and Static stability), Diverging DC load flow, Diverging AC load flow (including Voltage Collapse) and Or Voltage limits on each bus (0.95 < Vi < 1.05).

Advantages are Simplicity, Transparency, Flexibility and Rapidity (velocity or celerity). Disadvantages and defects are Conservative Solutions, Inaccurate Solutions, and Inability to account for non-monotonic systems and not considering all constraints. DYNAMIC Method is MAT (transient stability constrained Maximum Allowable Transfer). This MAT method consists of screening a large number of contingencies and scrutinizing the dangerous ones [3,4]. Termination criteria are Transient Instability and Voltage Instability (Dynamic) and Static termination conditions. Advantages are Robustness and Accuracy. Disadvantage and defect is: The “Potentially Harmful (Dangerous)” contingencies in base case (first stable operating point) and ATC case (when system reaches terminated criteria) are not necessarily the same and Not considering voltage stability and transient stability simultaneously and Slow response. Stable operating points in base case and ATC case are different. Then we use contingency ranking in stressed stable operating point.

We improved FCTTC (First Contingency Total Transfer Capability) determination in [5] and [6] with transient and voltage stability. In new paper, we improve ATC with transient stability termination criteria.

Although transient stability analysis approaches have extended by energy function over 20 years [7-9], these approaches don’t responsible practical problems, because stager engineers believe that these approaches have deficiencies as follows: inability in solution of large problems, unireliability solutions and long time computations. Therefore, a robust method is needed to analyze transient stability quickly and accurately. Although the Athay's PEBS is one of the best methods in computation of critical time, this method has deficiencies as follows: firstly, the post-fault system is not used in computation of Vc directly, secondly the fault-on system is integrated twice and thirdly, sometimes the solutions is not reliable.

The above disadvantages are remedied in the present approaches.
In this paper, firstly we present two new methods of transient stability. Finally we present ATC with these new methods.

2. CTSA

Athay's PEBS approach integrate till PEBS’s criterion \( \{f(\theta) \ (\theta - \theta^*)\} \) become positive. The potential energy of this point is considered as \( V_{cr} [10,11] \) (fig. 1).

The CTSA approach uses this fact that PEBS’s criterion has a global minimum in integration trajectory on fault-on system. The experimental results and most simulation results imply that this point is near the critical clearing point. This fact is basis of this method. Sometimes this global minimum point occurs before actual \( t_{cr} \) and sometimes after it. In each case, a special approach is used for determination of \( V_{cr} \) as follows:

![Fig. 1. Sustained fault and critical trajectory [13].](image)

First case: time of reach to global minimum point of PEBS's criterion in integration trajectory of fault-on system is more than actual \( t_{cr} \). In this case, the post-fault system is integrated till reach to global minimum point, then system trajectory has passed from PEBS's boundary certainly. In other words, the system is unstable. So, the potential energy is computed in the point of intersection of post-fault trajectory with PEBS. This energy is too near to actual critical energy that in this case is considered as \( V_{cr} \). After computation of \( V_{cr} \), the post-fault system can be integrated forward or backward by comparison between \( V_{cr} \) and energy of global minimum point of PEBS's criterion. These two energies are equal together. Now we can determine \( t_{cr} \).

Second case: time of reach to global minimum point of PEBS's criterion in integration trajectory of fault-on system is less than actual \( t_{cr} \). In this case system is stable and post-fault trajectory would not meet PEBS's boundary but maximum potential energy can be computed in this trajectory. Since system is stable \( V_{cr} \) and potential energy at maximum point are different, therefore potential energy must be determined exactly. Figure (1) shows this.

Post-fault system trajectories are similar with different clearing times. Then a line from point \( \theta^* \) (stable equilibrium point of post-fault system) to point of maximum potential energy in post-fault trajectory is drawn. This line cross PEBS in point A. Potential energy in A is considered \( V_{cr} \).

It can be integrated forward or backward by comparison between \( V_{cr} \) and energy of global minimum point of PEBS's criterion. These two energies are equal together. The Time of \( V_{cr} \) is considered as \( t_{cr} \).

CTSA approach is more accurate and faster than Athay's PEBS. It's accurate because it uses post-fault system and it's faster because \( t_{cr} \) is near to global minimum time of PEBS's criterion.

Theory is shown CTSA ability with above reason. CTSA is tested for two systems, 4 and 7 machines. The results show CTSA efficiency.

2-1. Algorithm of computation critical clearing time

CTSA approach is summarized as follow:

1. Pre-fault, fault-on and post-fault systems are determined.
2. \( \theta^* \) and \( \theta^s \) are computed for pre-fault and post-fault system, respectively.
3. Pre-fault system is integrated till PEBS's criterion is minimized.
4. Post-fault system is integrated till potential energy is maximized. Initial conditions of this step are final conditions of step 3. In this step, PEBS's criterion is computed. If this criterion is zero or positive then potential energy is \( V_{cr} \) and go to six, else go to five.
5. A line from point \( \theta^* \) to point of maximum potential energy is drawn. This line cross PEBS in A. Potential energy in A is considered as \( V_{cr} \).
6. It can be integrated forward or backward by comparison between \( V_{cr} \) and energy of global minimum point of PEBS's criterion. These two energies are equal together. Time is considered as \( t_{cr} \).

3. POMP approach

Consider figure (1) [13]. In this figure trajectories system (solid line) and PEBS (dotted line) are shown for three machines system. Generator No. 3 is reference. Suppose fault clears at \( t_{cl} \), then the pre-fault and post-fault systems are the same. The SEP of post-fault system is considered as origin of coordinates. Three fault-on system trajectories are shown. First trajectory is sustained fault trajectory. This trajectory begins from \((0,0)\) and passes the PEBS directly. It passes near Controlling Unstable Equilibrium Point (CUEP) that it's
a saddle point. Second trajectory is called critical trajectory. Fault-on system trajectory begins from (0,0) or SEP and continues till critical time. After that post-fault system is integrated. This trajectory returns toward SEP after meeting PEBS tangentially. This trajectory is called critical trajectory and energy of tangent point is called \( V_{cr} \). Third trajectory is similar to critical trajectory except that fault-clearing time is less than \( t_{cr} \). This trajectory does not meet PEBS and returns to SEP. Assume the system is loss less and doesn't any energy exchange with other areas. Fault clearing time of critical trajectory system has potential and kinetic energy. Kinetic energy changes to potential energy after fault clearing and post-fault trajectory goes toward SEP. At tangent point of critical trajectory and PEBS, kinetic energy become zero and total kinetic energy has changed to potential energy. Potential energy is maximum at point A. Third trajectory reaches to point B. Kinetic energy is minimum and potential energy is maximum at B. These points (A and B) are important in transient stability analysis. These points are called Point of Maximum Potential energy and minimum kinetic energy (POMP).

The importance of POMP is discussed in this section. Assume that POMP is computable even if approximately. PEBS's criterion is computed for POMP in second step of Athay's approach. Now this specifies that POMP passed from stable boundary or not. POMP algorithm is summarized in following steps:

1. Fault-on system is integrated for one step time (\( \Delta t \)).
2. Point of maximum potential energy, POMP is computed for post-fault system trajectory.
3. POMP is tested. If it's negative, system is stable and go to 1 else system is unstable and \( t_{cr} \) and \( V_{cr} \) are specified.

This approach is more accurate than PEBS approach since it uses point of maximum potential energy. Also it is faster since if sign of PEBS's criterion change then \( t_{cr} \) and \( V_{cr} \) are specified. So POMP approach doesn't require integrating post-fault system or sustained fault systems.

3-1. POMP determination method

Determination of POMP is time consuming, then POMP is estimated. Consider figure (2), system trajectories (post-fault and fault-on) are shown for a stable system. Also POMP has been specified. The maximum magnitude of angles \( \delta_{13} \) and \( \delta_{23} \) are A1 and A2, respectively. Generator No. 3 is reference. A1 and A2 don't occur simultaneously. A2 will occur after A1. Consider point C, this point defined by set of maximum angles.

\[
\text{C = Point (A1,A2)}
\]

This point (C) is an approximation of POMP then it is called Estimate of POMP (EPOMP) (fig.2). This point is found using Taylor's expansion series up to the second order for simplicity. We have:

\[
\dot{\theta}_i = \omega_i^0 + \omega_i^0 \omega_i \dot{\theta}_i + \frac{1}{2} f_i (\theta_i) \dot{\theta}_i^2
\]

(1)

\( \omega_i^0 \) and \( \theta_i^0 \) are final conditions of fault-on system and initial conditions of post-fault system. Angles maximum are obtained with differentiation of above formula.

\[
\dot{\theta}_i = \omega_i^0 + \frac{1}{M_i} f_i (\theta_i). i \Rightarrow \omega_i = -\frac{\omega_i^0 M_i}{f_i (\theta_i)}
\]

(2)

\( t_i \) is time of maximum \( \theta_i^0 \). \( f_i(\theta_i) \) is negative if \( \theta_i \) is in domain of attraction.

Maximum \( \theta_i \) is computable now.

\[
\theta_i^{\max} = \theta_i^0 + (\omega_i^0)^2 \frac{M_i}{2f_i(\theta_i)} i = 1, 2, ..., n-1
\]

(3)

EPOMP (Estimate of POMP) is computed from (4).

\[
\text{EPOMP = point (} \theta_1^{\max}, ..., \theta_{n-1}^{\max}) \tag{4}
\]

Transient stability is determined more accurate and faster than PEBS using 4.

3.2. POMP algorithm

POMP approach summarized as follow:

1. Pre-fault, fault-on and post-fault systems are determined.
2. \( \theta^0 \) and \( \dot{\theta}^0 \) are computed for pre-fault and post-fault system, respectively. (Stable Equilibrium Points)
3. Fault-on trajectory is determined for a step time using a fast integration approach (e.g. Taylor's expansion series up to the six order). These points (\( \theta_n, \omega_n \)) are initial conditions for 4.
4. The maximum of \( \theta_i \) is determined using 4 formula.
5. \( f(\theta^{\max}) (\theta^{\max} - \theta^0) \) criterion is computed. If it's negative then go to 3 and else go to 6.
6. \( V_{pre} = V_{cr} = V_{pre}^{\max} \) and \( V_i = V_{cr}(\text{max}) \) are computed. \( V_{pre} \) is potential energy in POMP and it is considered as critical energy (\( V_{cr} \)). \( V_i \) is total energy in step 3.
7. If \( V_{cr} > V_i \) then fault-on system is integrated forward (\( t = t + \Delta t \)) and else it's integrated backward (\( t = t - \Delta t \)). If \( V_{cr} = V_i \) then time is critical (\( t_{cr} \)).
4. Numerical results

CTSA and POMP methods have been tested for two test systems. Simulation results for two systems, 4 and 7 machines (CIGRE), show the comparison of CTSA, POMP and Athay's PEBS approaches. Transient stability and critical clearing time are studied for several faults. Classical model of generator and energy function at center of inertia reference frame (COI) used [10,11].

Clearing fault means fault elimination, so pre-fault and post-fault systems are equal.

4-1. Test system No. 1 (4 machines)

This test system has 4 generators, 7 buses and 8 lines. Single line diagram of test system is shown in figure (3). System parameters and operating points are shown in figure (3) and table (1) [12]. Symmetric three phase short circuits are applied in lines 7-3, 3-4 and 7-5 at point %K from beginning of line.

Table 1. System parameters and operating points of 4 machines [12].

<table>
<thead>
<tr>
<th>NO</th>
<th>X'D(PU)</th>
<th>M(S)</th>
<th>D(S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.004</td>
<td>100</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>1.5</td>
<td>0.003</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>3.0</td>
<td>0.006</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>2.0</td>
<td>0.004</td>
</tr>
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</table>

Table 1. (Continue)

<table>
<thead>
<tr>
<th>GEN.</th>
<th>LOAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUS</td>
<td>PM</td>
</tr>
<tr>
<td>2</td>
<td>.4</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Fig. 2. Fault-on and post-fault system trajectories and EPOMP [13].

Fig. 3. Single line diagram of 4 machines [12].

4-1-2. Computation of critical clearing time

Critical clearing time has been computed by 4 methods (simulation, Athay’s PEBS, CTSA and POMP) and 7 short circuits. Short circuit at 25% from bus 7 in line 7-3 is analyzed. Actual $t_{cr}$ is determined 0.33 second by simulation method that it's shown in figure (4). Critical generator is number 2 and system become unstable at first swings if fault cleared at time 0.34 second (and greater than it does). (fig.5). Generator 2 is diverged from other generators (fig.5). $t_{cr}$ is computed 0.34 second in Athay's PEBS approach. Time of computation of $t_{cr}$ is determined for comparison PEBS and POMP approaches. POMP is 37% faster than Athay's PEBS and more accurate than Athay's PEBS in this fault. $t_{cr}$ is computed 0.34 second by CTSA and CTSA is 37% faster than Athay's PEBS and more accurate than Athay's PEBS in this fault. Other faults are shown in table (2). (T1 is CPU time)

Fig. 4. Fault cleared in 0.33 sec. Short circuit at 25% from bus 7 in lines 7-3 is applied [13].
Fig. 5. Fault cleared in 0.34 sec. Short circuit at 25% from bus 7 in lines 7-3 is applied [13].

Fig. 6. Test system No. 2 (7 machines, CIGRE)

Table 2. \( t_{cr} \) and time of calculation by 4 methods (simulation, PEBS, POMP and CTSA) for 4 machines [13].

<table>
<thead>
<tr>
<th>FAULT</th>
<th>SIMULATION</th>
<th>PEBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line</td>
<td>Percent</td>
<td>Stable</td>
</tr>
<tr>
<td>3-7</td>
<td>5</td>
<td>0.2</td>
</tr>
<tr>
<td>3-7</td>
<td>25</td>
<td>0.33</td>
</tr>
<tr>
<td>3-7</td>
<td>50</td>
<td>0.38</td>
</tr>
<tr>
<td>3-7</td>
<td>95</td>
<td>0.29</td>
</tr>
<tr>
<td>4-3</td>
<td>5</td>
<td>0.32</td>
</tr>
<tr>
<td>4-3</td>
<td>95</td>
<td>0.46</td>
</tr>
<tr>
<td>5-7</td>
<td>50</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Table 2. (Continue)

<table>
<thead>
<tr>
<th>Bus</th>
<th>( X_T )</th>
<th>M (MW)</th>
<th>( P_M ) (MW)</th>
<th>E (p.u.)</th>
<th>( \delta ) (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.4</td>
<td>6.02</td>
<td>217</td>
<td>1.106</td>
<td>7.9</td>
</tr>
<tr>
<td>2</td>
<td>11.8</td>
<td>4.11</td>
<td>120</td>
<td>1.156</td>
<td>-0.2</td>
</tr>
<tr>
<td>3</td>
<td>6.2</td>
<td>7.59</td>
<td>256</td>
<td>1.098</td>
<td>6.5</td>
</tr>
<tr>
<td>4</td>
<td>4.9</td>
<td>9.54</td>
<td>300</td>
<td>1.110</td>
<td>3.9</td>
</tr>
<tr>
<td>5</td>
<td>7.4</td>
<td>6.02</td>
<td>230</td>
<td>1.118</td>
<td>7.0</td>
</tr>
<tr>
<td>6</td>
<td>7.1</td>
<td>6.77</td>
<td>160</td>
<td>1.039</td>
<td>3.6</td>
</tr>
<tr>
<td>7</td>
<td>8.7</td>
<td>5.68</td>
<td>174</td>
<td>1.054</td>
<td>7.9</td>
</tr>
</tbody>
</table>

Table 4. \( t_{cr} \) and time of calculation by 4 methods (simulation, PEBS, POMP and CTSA) for 7 machines [13].

<table>
<thead>
<tr>
<th>Fault</th>
<th>SIMULATION</th>
<th>PEBS</th>
<th>POMP</th>
<th>CTSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td>CF</td>
<td>S</td>
<td>US</td>
<td>( t_{cr} )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>.35</td>
<td>.36</td>
<td>.34</td>
</tr>
</tbody>
</table>

5. Numerical Results

Tables 2 and 4 are shown that POMP approach decreases 35% time of calculation and CTSA approach decreases 30%, averagely. These two methods specify \( t_{cr} \) more accurate than Athay's PEBS.

These approaches are one of the most reliable and fastest methods in transient stability analysis [13,14,15,16].

4-2. Test system No. 2 (7 machine, CIGRE)

Single line diagram of test system is shown in figure (6). System data is shown in table (3). Critical clearing time has been computed by 4 methods (simulation, Athay’s PEBS, CTSA and POMP) and 6 short circuits and results are shown in table (4). Time of computation is determined. This example also confirms that POMP and CTSA are faster than Athay's PEBS and more accurate than Athay's PEBS. Clearing fault means fault elimination, so pre-fault and post-fault systems are the same too.
6. ATC Results

We use two new methods (CTSA and POMP) for termination criteria in ATC calculations. A novel formulation of the ATC problem has been adopted based on full ac power flow solution with matrix operations to incorporate the effects of voltage limits, and voltage collapse. This program written by MATLAB and don't use any do loop, then this power flow program is fivefold faster than any program in MATLAB. The ideas are demonstrated on 30 bus IEEE. Single line diagram of test system is shown in figure (7). Symmetric three phase short circuits are applied in all lines. Critical clearing time has been computed by one method (CTSA or POMP). Clearing fault means fault elimination, so pre-fault and post-fault are equal.

In order to implement this method the following steps needed to be followed:

1-Obtain a basecase the nose of the normal case or very stressed case close to it. (Ex. Repetitive load flow)
2-Implement the contingencies (Short circuit in all lines)
3-Critical clearing time \(T_{cr}\) is computed.
4-\(T_{cr}\) used as the basis of filtering the contingencies.
5-Number \(n\) of the most critical contingencies are selected. \(n=20\) in this paper for IEEE 30 bus
6-Loads and Generations are decreased in direction of ATC.
7-New stable equilibrium point is determined with CPF (Continuation Power Flow).
8-Repeat the process until dangerous contingency be stable \(T_{cr}\) is determine

To assess the performance of the screening and ranking methods, the ATC were first obtained using a repetitive load flow method and simulation for transient stability. This ranking is used as a reference for measuring the accuracy of other techniques. Table (5) shows the ATC between some bus by two methods, new and accurate method. Similar results were also obtained for the other systems. Celerity and accurately new method is very well.

<table>
<thead>
<tr>
<th>BUS</th>
<th>NEW METHOD</th>
<th>ACCURATE METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ATC(p.u.)</td>
<td>CPU time(p.u.)</td>
</tr>
<tr>
<td>3</td>
<td>0.40</td>
<td>0.11</td>
</tr>
<tr>
<td>16</td>
<td>0.35</td>
<td>0.10</td>
</tr>
<tr>
<td>18</td>
<td>0.34</td>
<td>0.10</td>
</tr>
<tr>
<td>23</td>
<td>0.35</td>
<td>0.09</td>
</tr>
<tr>
<td>24</td>
<td>0.30</td>
<td>0.09</td>
</tr>
<tr>
<td>30</td>
<td>0.18</td>
<td>0.09</td>
</tr>
</tbody>
</table>

7. Conclusion

The proposed method able to calculate ATC: SET-TO-SET: where a set may be an interchange area or any other arbitrary set of buses, BUS-TO-BUS: bilateral contracts, SET-TO-BUS: multiple suppliers or an area (or a GENCO) to an individual customer, BUS-TO-SET: single supplier to a customer with multiple locations. The method has advantages such as: simplicity, robustness, flexibility, accurately and celerity.

References


