A SIMULATED ANNEALING METHOD FOR SOLVING MULTI-
AREA UNIT COMMITMENT PROBLEM IN DEREGULATED 
ENVIRONMENT WITH IMPORT AND EXPORT CONSTRAINTS

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Abstract: This paper presents a new approach to solve
the multi-area unit commitment problem (MAUC) in
deregulated environment using a simulated annealing
method. The objective of this paper is to maximizing or
minimizing the operating cost among generating
companies (GENCO’S) in multiple areas that are
interconnected via tie-lines. Joint operation of generation
resources can result in significant operational cost
savings. The simulated annealing method is used to solve
multi area unit commitment problem, allocated
generation for each area and find the operating cost of
generation for each hour. Power transfer between the
areas through tie lines depends upon the operating cost of
generation at each hour and tie line transfer limits. The
tie line transfer limits were considered as a set of
constraints during the optimization process to ensure
system security and reliability. The overall algorithm can
be implemented on an IBM-PC, which can process a
fairly large system in a reasonable period of time.
Experimental results shows the application of this method
have the potential to solve multi area power generation
scheduling in deregulated electricity market with import
and export constraints.

Key words: Unit Commitment, Dynamic Programming
Method, Simulated Annealing, Deregulated Environment.

Nomenclature

\( F(P^K_{gi}) \) \hspace{1cm} \text{Production cost of unit i in area K}

\( P^K_{gi} \) \hspace{1cm} \text{Power generation of unit i in area K}

\( a^K_i, b^K_i, c^K_i \) \hspace{1cm} \text{Cost function parameters of unit I in}

\( X^{off}_{ij} \) \hspace{1cm} \text{Time duration for which unit i have been off at jth hour.}

\( P^K_{ij} \) \hspace{1cm} \text{Power generation of unit i in area K at jth hour}

\( t^K_{ij} \) \hspace{1cm} \text{Commitment state.(1 for ON, 0 for OFF)}

\( P^K_{gi} \) \hspace{1cm} \text{Power generation of unit i in area K at jth hour}

\( D^K_j \) \hspace{1cm} \text{Total system demand of area K at jth hour}

\( R^K_j \) \hspace{1cm} \text{Spinning reserve of area K at jth hour}

\( E^K_j \) \hspace{1cm} \text{Total import power to area K at jth hour}

\( p^K_{jmax} \) \hspace{1cm} \text{Maximum power generation in area K at jth hour}

\( p^K_{jmin} \) \hspace{1cm} \text{Minimum power generation in area K at jth hour}

\( T^{on}_i \) \hspace{1cm} \text{Minimum up time of unit i}

\( T^{off}_i \) \hspace{1cm} \text{Minimum down time of unit i}

\( I^K_{jmax} \) \hspace{1cm} \text{Maximum total import power in area K at jth hour}

\( W_j \) \hspace{1cm} \text{Net power exchange with outside system}

\( \lambda_{sys} \) \hspace{1cm} \text{Marginal cost of supplying the last}

\hspace{1cm} \text{incremental energy to meet entire}

\hspace{1cm} \text{system demand.}

\( P^K_{g_{i,j}} \) \hspace{1cm} \text{Maximum power generation at area K at ith hour}

\( P^K_{g_{i,j}} \) \hspace{1cm} \text{Minimum power generation at area k at ith hour}

1. Introduction

An important goal behind the restructuring of the
electric industry is to bring more choice. In the way
individual loads supply their needs, permitting them
to buy either from a centralized spot market or
directly from generators or marketers through pre-
arranged bi-lateral contracts. Joint operation of
generation resources in an interconnected multi area
system result in operational cost savings. Multi area joint operation can be implemented at two levels. At the first level an independent unit commitment made for each area and the commitment units are then jointly economically dispatched to satisfy the multi-area energy requirements. At the second level, both the capacity commitment decision and the energy dispatch decision are made jointly for all the participating area in the interconnected multi-area system[1].

In multi-area, several generation areas are interconnected by tie-lines, the objective is to achieve the most economic generation to meet out the local demand without violating tie-line capacity limits constraints [2]. The main goal of this paper is to develop a multi area generation scheduling scheme that can provide proper unit commitment in each area and effectively preserve the tie-line constraints.

Power pools are operated in a co-ordinate manner so as to minimize the pool cost. It is possible by transmitting power from a utility, which had cheaper sources of generation to another utility having costlier generation sources. The total reduction in system cost shared by the participating utilities [3]. The exchange of energy between two utilities having significant difference in their marginal operating costs. The utility with the higher operating cost receives power from the utility with low operating cost. This arrangement usually on an hour-to-hour basis and is conducted by the two system operators. In the competitive environment, customer request for high service reliability and lower electricity prices. Thus, it is an important to maximize own profit with high reliability and minimize overall operating cost.

The proposed Lagrangian Relaxation (LR)-Dynamic Programming (DP) method [4] is efficiently and effectively implemented to solve the unit commitment problem. The proposed LR total production costs over the scheduled time horizon are less than conventional methods especially for the larger number of generating units. Developed algorithms provide optimal unit commitment and also optimal MW values for energy, spinning reserve and non-spin. Presented algorithm and analysis could be beneficial to GENCO with big number of generators to maximize the profit and bid in competitive electricity market [5].

This method is also suitable to be implemented under a parallel computer system. The solution speed can be thus further improved. There is no obvious limitation on the size of the problem that must be addressed, for its data structure is such that must be addressed, for its data structure is such that the search space is reduced to a minimum; No relaxation of constraints is required; instead, populations of feasible solutions are produced at each generation and throughout the evolution process; multiple near optimal solutions to the problem involving multiple constraints and conflicting objectives can be obtained in a reasonable time with the use of heuristics; It works only with feasible solutions generated based on heuristics, thus avoiding the computational burden entailed by the Genetic Algorithm (GA) methods which first generate all feasible solutions and then purge the infeasible ones[6]. The flexibility in the demand constraint both in terms of possibility of buying and selling in the market gives better indication of the likely future scenarios so that better bidding strategy can be made [7]. More improvements could be made to the proposed algorithm in order to increase the speed of convergence of the algorithm [8] and its execution time by improving the gradient method, and by adjusting adequately the penalty weight factor.

Simulated annealing (SA) is the powerful general-purpose stochastic search technique to solve hard-constrained optimization problems. Though it takes long time, it has many strong features such as, it is easy to implement, requires little expert knowledge and it is not memory intensive. Further it can start with any initial solution and improves on it to find optimal solution with a high probability. The SA optimization technique begin with a randomly generated solution and then making successive random modifications, until a stopping criterion, is satisfied [9].

2. Problem Formulation

The cost curve of each thermal unit is in quadratic form [2]

\[ F(P_{gi}^k) = a_i^k (P_{gi}^k)^2 + b_i^k (P_{gi}^k) + c_i^k \quad \text{Rs/hr, } i=1...N_a \]  

(1)

The incremental production cost is therefore

\[ \lambda = 2 a_i^k g_i^k + b_i^k \]

(2)
The start-up cost of each thermal unit is an exponential function of the time that the unit has been off

\[ S(X_{i,j}^{\text{off}}) = A_i + B_i (1 - e^{-X_{i,j}^{\text{off}} / T_i}) \]  

(4)

The objective function for the multi-area unit commitment is to minimize the entire power pool generation cost as follows [2].

\[
\min \sum_{k} \sum_{i,j} \sum_{t} \left[ I_{i,j,t}^k \left( F^k_i \left( P_{i,j}^k \right) + I_{i,j-1} \left( 1 - I_{i,j-1} \right) S_i \left( X_{i,j}^{\text{off}} \right) \right) \right]
\]

(5)

To decompose the problem in above equation (5), it is rewritten as

\[
\min \sum_{j} \left[ F \left( P_{i,j} \right) \right]
\]

(6)

\[
F \left( P_{i,j} \right) = \sum_{k} F^k \left( P_{i,j}^k \right)
\]

(7)

Subject to the constraints of equations (9), (11) and (14–18). Each \( P^k(P_{i,j}) \) for \( K=1 \ldots N_A \) is represented in the form of schedule table, which is the solution of mixed variable optimization problem

\[
\min \sum_{i,j} \sum_{t} \left[ I_{i,j,t}^k \left( F^k_i \left( P_{i,j}^k \right) + I_{i,j-1} \left( 1 - I_{i,j-1} \right) S_i \left( X_{i,j}^{\text{off}} \right) \right) \right]
\]

(8)

Subject to following constraints are met for optimization.

1) System power balance constraint

\[
\sum_k P_{g,j}^k = \sum_k D_j^k
\]

(9)

Sum of real power generated by each thermal unit must be sufficient enough to meet the sum of total demand of each area while neglecting transmission losses.

2) Spinning reserve constraint in each area

\[
\sum_i P_{g,i}^k \geq D_j^k + R_j^k + E_j - L_j
\]

(10)

3) Generation limits of each unit

\[
P_{g,j}^{k_{\text{max}}} \leq P_{g,j}^k \leq P_{g,j}^{k_{\text{min}}}
\]

(11)

i=1…….N,  j=1……t,  k=1……N_A

4) Thermal units generally have minimum up time \( T_{on}^k \) and down time \( T_{off}^k \) constraints, therefore

\[
\left( X_{i,j-1,j} - T_{on}^k \right) \left( I_{i,j-1,j} - I_{i,j} \right) \geq 0
\]

(12)

\[
\left( X_{i,j-1,j} - T_{off}^k \right) \left( I_{i,j-1,j} - I_{i,j} \right) \geq 0
\]

(13)

5) In each area, power generation limits caused by tie-line constraints are as follows

Upper limits

\[
\sum_i P_{g,i}^k \leq D_j^k + E_j^k
\]\n
(14)

Lower limits

\[
\sum_i P_{g,i}^k \geq D_j^k - L_j^k
\]

(15)

Import/Export balance

\[
\sum_i E_j^k - \sum_k L_j^k + W_j = 0
\]

(16)

6) Area generation limits

\[
\sum_i P_{g,i}^k \leq \sum_k P_{g,i}^{k_{\text{max}}} - R_j^k ; k=1\ldots N_A
\]

(17)
\[ \sum_{i} P_{g_{i}, j}^k \geq \sum_{i} P_{g_{i}, mn}^k \quad j=1, \ldots, t \]  

(18)

The objective is to select \( \lambda_{sys} \) at every hour to minimize the operation cost.

\[ P_{g_{j}}^k = D_{j}^k + E_{j}^k - L_{j}^k \]  

(19)

where \( P_{g_{j}}^k = \sum_{i=1}^{N_k} P_{k_{i}, j}^k \)  

(20)

Since the local demand \( D_{j} \) is determined in accordance with the economic dispatch within the pool, changes of \( P_{g_{j}}^k \) will cause the spinning reserve constraints of equations (10) to change accordingly and redefine equation (8). Units may operate in one of the following modes when commitment schedule and unit generation limits are encountered [10].

a) Coordinate mode: The output of unit \( i \) is determined by the system incremental cost

\[ \lambda_{\min, i} \leq \lambda_{sys} \leq \lambda_{\max, i} \]  

(21)

b) Minimum mode: unit \( i \) generation is at its minimum level

\[ \lambda_{\min, i} \leq \lambda_{sys} \]  

(22)

c) Maximum mode: unit \( i \) generation is at its maximum level

\[ \lambda_{\max, i} \leq \lambda_{sys} \]  

(23)

d) Shut down mode: unit \( i \) is not in operation, \( P_{i} = 0 \)

Besides limitations on individual unit generations, in a multi-area system, the tie-line constraints in equations (12), (13) and (15) are to be preserved. The operation of each area could be generalized into one of the modes as follows.

(i) Area coordinate mode

\[ \lambda^k = \lambda_{sys} \]

\[ D_{j}^k - L_{\max}^k \leq \sum_{i} P_{g_{i}, j}^k \leq D_{j}^k + E_{\max}^k \]  

(24)

(or)

\[ \begin{align*}
- L_{\max}^k \leq \sum_{i} P_{g_{i}, j}^k - D_{j}^k \leq E_{\max}^k
\end{align*} \]

(25)

(ii) Limited expert mode

When the generating cost in one area is lower than the cost in the remaining areas of the system, that area may generate its upper limits according to equations (14) or (17) therefore

\[ \lambda^k \leq \lambda_{sys} \]  

(26)

For area \( k \), area \( \lambda^k \) is the optimal equal incremental cost which satisfies the generation requirement.

(iii) Limited import mode

An area may reach its lower generation limit according to equation (15) or (18) in this case because of higher generation cost

\[ \lambda^k \geq \lambda_{sys} \]  

(27)

The proper generation schedule will result in dispatching the power flow by satisfying the tie-line constraints and minimizing the system generation cost.

3. Tie Line Constraints

To illustrate the tie-line flow in a multi-area system, the four area system given in Fig.1 is studied [10].

![Fig.1: Multi-area connection and tie-line limitations](image)

An economically efficient area may generate more power than the local demand, and the excessive
power will be exported to other areas through the tie-lines [11]. For example assume area 1 has the excessive power the tie-line flows would have directions from area1 to other areas, and the maximum power generation for area1 would be the local demand in area1 plus the sum of all the tie-line capacities connected to area1.

If we fix the area 1 generation to its maximum level than the maximum power generation in area 2 could be calculated in a similar way to area 1. Since tie-line C_{12} imports power at its maximum capacity, this amount should be subtracted from the generation limit of area2. According to power balance equation (9), some areas must have a power generation deficiency and requires generation imports. The minimum generation limits in these areas is the local demand minus all the connected tie-line capacities. If any of these tie-lines is connected to an area with higher deficiencies, then the power flow directions should be reserved.

4. Simulated Annealing

Simulated annealing (SA) is the powerful general purpose stochastic search technique to solve hard-constrained optimization problems. This SA procedure has been successfully applied to a range of combinational problems in electrical engineering including power systems.

4.1 Physical concepts of simulated annealing

Annealing, in physical terms, refers to the process of heating up a solid to a high temperature followed by slow cooling achieved by decreasing the temperature of the environment in steps. At each step the temperature is maintained constant for a period of time sufficient for the solid to reach thermal equilibrium [12-15]. At equilibrium, the solid could have many configurations; each corresponding to different spins of the electrons and to specific energy levels. At equilibrium, the probability of a given configuration, \( P_{\text{config}} \) is given by Boltzmann distribution:

\[
P_{\text{config}} = K \exp(-E_{\text{config}}/C_p)
\]

(28)

Where, \( E_{\text{config}} \) is the energy of the given configuration and \( K \) is a constant. Metropolis et al. proposed a Monte Carlo method to simulate the process of reaching thermal equilibrium at a fixed temperature \( C_p \). In this method, a randomly generated perturbation of the current configuration of the solid is applied so that a trial configuration is obtained. Let \( E_c < E_t \), till a lower energy level has been reached and the trial solution has to be altered. If \( E_t > E_c \), then the trial configuration is accepted as the current configuration with probability given below:

\[
\exp \left[ (E_c - E_t) / C_p \right]
\]

(29)

where \( C_p \) is the control parameter of the cooling schedule. The process continues where a transition to a configuration of higher energy level is not necessarily rejected. Eventually thermal equilibrium is achieved after a large number of perturbations, where the probability of a configuration approaches Boltzmann Distribution. By gradually decreasing \( C_p \) and repeating Metropolis simulation, new lower energy levels become achievable. As \( C_p \) approaches zero, the least energy configurations will have a positive probability of occurring.

4.2 Applications of simulated annealing to combinational optimization problems

By making an analogy between the annealing process and the optimization problem, a great class of combinational optimization problems can be solved following the same procedure of transition from equilibrium state to another, reaching the minimum energy of the system. This analogy can be stated as:

- Solutions in the combinational optimization problems are equivalent to states (configurations) of the physical system.
- The cost of the solution is equivalent to the energy of a state.
- Demand as a control parameter is introduced to play the role of temperature in the annealing process.

In applying the SAA to solve the combinational optimization problems, the basic idea is to choose a feasible solution at random and then get a neighbour to this solution. A move to this neighbor is performed if it has a better lower objective value or, in case the neighbour has a higher objective function value, if \( \exp(-\Delta E / C_p ) > U(0,1) \), where \( \Delta E \) is the increase in objective value in the neighbour. The effect of decreasing \( C_p \) is that the probability of accepting an increase in the objective function value is decreased during the search. The most important part in the SAA is to have a good rule for finding a
diversified and intensified neighbourhood so that a large amount of the solution space can be explored. Another important part is how to choose the initial value of $C_p$ and how $C_p$ should decrease during the search. SA means a simulation of the annealing process of metal. If the temperature is lowered carefully from a high temperature in the annealing process, the method will produce the crystal at 0K. Kirkpatrick [13] developed an algorithm that finds the near-optimal solution by substituting the random movement of solution for the fluctuation of particle of system in the annealing process and by making the objective function value correspond to the energy of the system which decrease with the decent of temperature.

4.3 Simulated annealing algorithm for MAUCP

Simulated annealing algorithm for multi-area unit commitment problem is given in Fig. 2.

4.4 Repair mechanism

A repair mechanism to restore the feasibility of the constraints is applied and described as follows:

- Pick at random one of the OFF units at one of the violated hours.
- Apply the rules in section 4.4 to switch the selected units from OFF to ON keeping the feasibility of the down time constraints.
- Check for the reserve constraints at this hour. Otherwise repeat the process at the same hour for another unit.

4.5 Implementation

The training and identification part as implemented in the SA technique is employed here and considered as a process involving random recommitment constraint verification. C++ program is developed for simulated annealing algorithm and it is used to solve multi-area unit commitment problem with import and export constraints.

5. Numerical Results

A NTPS (Neyveli Thermal Power Station) in India with seven generating units, each with a capacity of 210 MW, has been considered with different generation at each area for time period of 24h is considered as a case study. The test system consists of four areas, and each area has 7 thermal generating units. Load demand profile for each area is different and is given in Fig. 3. The hourly operating cost of four areas is given in Table 2. Cost in each iteration at 11am and 4pm are given in Table 1 and Table 3 respectively. Tie-line flow pattern of 11 am and 4 pm is given in Fig. 4 and Fig. 5 respectively. Fig. 7-10 shows the sum of the total number of units switched ON during every hour for SA method of unit 1 to unit 4 respectively. The total operating cost between DP(Dynamic Programming), LR(Lagrangian Relaxation), TS(Tabu Search) and SA(Simulated Annealing) method for each area of 7 units (NTPS) are given in Table 4. The total operating cost for each area is given in Fig.6. The total operating cost (in lakhs) includes fuel cost, labour and maintenance cost. The total operating cost is calculated by the following computing process. (i) Enter the number of units and time period for each area (ii) Status of the units in each area is found (iii) Cost co-efficient are calculated (iv) The total fuel cost at each hour for each unit in all area are found (v) The start up cost are calculated (vi) The total operating cost is calculated (vii) Based on the total operating cost of each area for 24 hours, the power can be imported or exported from each area by following tie-line constraints.

6. Conclusion

This paper presents SA method for solving multi-area unit commitment problem in deregulated environment with import and export constraints. In comparison with the results produced by the techniques DP[14][17], LR[15][17], TS[16][17], The SA method obviously displays satisfactory performance. Test results have demonstrated that the proposed method of solving multi-area unit commitment problem with import and export constraints reduces the total operating cost of the plant. An effective tie-line constraint checking procedure is implemented in this paper. This method provides more accurate solution for multi-area unit commitment problem in deregulated environment.
Fig. 2: Flowchart for SA for MAUC

Table 1: Cost in each iteration at 11 am

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
<th>Area 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19692.308</td>
<td>26902.978</td>
<td>21335.345</td>
<td>25009.121</td>
</tr>
<tr>
<td>2</td>
<td>19692.308</td>
<td>26902.980</td>
<td>21335.345</td>
<td>25009.121</td>
</tr>
</tbody>
</table>

Table 3: Cost in each iteration at 4 pm

<table>
<thead>
<tr>
<th>Iteration</th>
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<th>Area 3</th>
<th>Area 4</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>18518.982</td>
<td>26738.298</td>
<td>20827.009</td>
<td>25260.671</td>
</tr>
<tr>
<td>2</td>
<td>18518.982</td>
<td>26738.296</td>
<td>20827.009</td>
<td>25260.671</td>
</tr>
</tbody>
</table>

Fig. 4: Tie-Line flow pattern at 11 am

Fig. 5: Tie-Line flow pattern at 4 pm
Table 2: Hourly cost of each area

<table>
<thead>
<tr>
<th>HOUR</th>
<th>Total Cost</th>
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<tr>
<td>2</td>
<td>20770.756</td>
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<td>3</td>
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<td>4</td>
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<td>7</td>
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<td>21</td>
<td>21203.484</td>
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Table 4: Comparison of operating cost for 7 units

<table>
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<th>system</th>
<th>Method</th>
<th>Total operating cost (pu)</th>
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<tr>
<td></td>
<td></td>
<td>Area 1</td>
</tr>
<tr>
<td>7 Unit (NTPS)</td>
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</tr>
<tr>
<td></td>
<td>LR</td>
<td>0.978</td>
</tr>
<tr>
<td></td>
<td>TS</td>
<td>0.955</td>
</tr>
<tr>
<td></td>
<td>SA</td>
<td>0.947</td>
</tr>
</tbody>
</table>

Fig 6: Total Operating Cost Vs Each Area

Fig 7: No. of units switched on during every hour of area 1 for SA Method

Fig 8: No. of units switched on during every hour of area 2 for SA method

Fig 9: No. of units switched on during every hour of area 3 for SA method
7. References