EMISSION CONSTRAINED OPTIMAL POWER FLOW USING EFFICIENT
MULTIOBJECTIVE FUZZY OPTIMIZATION METHOD

Samir SAYAH, Abdellatif HAMOUDA and Khaled ZEHAR

Department of Electrical Engineering, University of Ferhat Abbas, Setif 19000, Algeria
samir_pg04@yahoo.fr, a_hamouda1@yahoo.fr, khaledzehar@yahoo.fr

Abstract — For electric power generation and dispatching problems, cost is not any more the only
criterion to be met. Environmental considerations have become one of the major management concerns. The
harmful ecological effects caused by the emission of particulate and gaseous pollutants like sulfur dioxide
(SO$_2$) and nitrogen oxides (NO$_x$), can be reduced by adequate distribution of load between the plants of a
power system. However, this leads to a noticeable increase in the operating cost of the plants.

In order to eliminate this conflict, and to study the trade-off relation between fuel cost and emissions, an
approach to solve this multiobjective environmental/economic load dispatch problem, based on an efficient
multiobjective fuzzy optimization technique, is proposed. To show the effectiveness of the
proposed solution method, it is applied to the IEEE 30-
bus benchmark test system and compared with some
recently published approaches, including linear
programming, genetic algorithm and evolutionary
algorithm. The obtained results reveal the performance
of the proposed method for dealing with the
multiobjective nature of power dispatch problem.

Keywords—Economic emission dispatch, Optimal power
flow, Fuzzy sets, Multiobjective fuzzy optimization.

1. INTRODUCTION

The economic dispatch (ED) or optimal power flow
(OPF) problem is to determine the optimal combination
of power outputs for all generating units, which
minimizes the total fuel cost, while satisfying load
demand and operational constraints [1].

Under the strict governmental regulations on
environmental protection, the conventional operation at
minimum fuel cost can no longer be the only basis for
dispatching electric power. Therefore, it is mandatory
for electric utilities to reduce pollution from power
plants either by design or by operational strategies.
Especially, emissions contribution of fossil-fired electric
power plants which use coal, oil, gas or combinations as
the primary energy resource cannot be neglected. The
most important emissions considered in the power
generation industry due to their effects on the
environment are sulfur dioxide (SO$_2$) and nitrogen
oxides (NO$_x$). It is obvious that trade-off among fuel cost
and emission objectives is impossible because of their
differences in nature.

Unfortunately, conventional optimization techniques
are not suitable to obtain the optimal solution which
simultaneously optimizes a variety of objectives. One
conceivable approach using conventional approach
methods is to convert a multiobjective problem into a
single objective problem by assigning distinct weights to
each objective, thereby allowing for relative importance
among goals [1, 2]. However, this artifice is not totally
satisfactory since different objectives cannot be
evaluated under a common measure and there is no
rational basis of determining adequate weights.

When permissible limit of emission are clearly
specified in a power system under study, this quantity
could be incorporated into the OPF as operational
constraint [3]. However, in system planning studies,
these limits posed on emission would be very
ambiguous, thus making such treatment difficult.
Furthermore, operation indices mentioned herein are in
conflicting trade-off relations, successful optimization
cannot be attained through any of conventional
optimization approaches.

Recently, intelligent computing techniques like
genetic algorithm, simulated annealing, evolutionary
programming and neural network have been applied to
solve the combined economic emission dispatch (EED)
problem [4-6].

In this paper, a fuzzy formulation of the EED problem
is presented and converted into a crisp optimization
problem. An efficient successive linear programming
(SLP) method is then used to solve the new problem.
Numerical test results on the IEEE 30-bus system show
that the developed fuzzy economic emission dispatch
(FEED) method could give the best compromise solution between fuel cost and emission. Comparison results demonstrate the superiority of the FEED for dealing with the multiobjective nature of power dispatch problem.

2. CRISP PROBLEM FORMULATION

The economic emission dispatch (EED) problem is to minimize simultaneously two conflicting objective functions, fuel cost and emission, while satisfying several equality and inequality constraints. Generally, the problem is formulated as follows:

2.1. Problem objectives

a. Minimization of fuel cost

The fuel cost curve is considered to be approximated by a quadratic function of generator power outputs \( P_{gi} \). The total \$/h fuel cost \( f(x) \) of the entire power system is expressed by the sum of the quadratic cost model for each generator \([1]\), as follows:

\[
f(x) = \sum_{i=1}^{ng} a_i + b_i P_{gi} + c_i P_{gi}^2
\]

where \( ng \) is the number of thermal units, \( P_{gi} \) is the active power generation at unit \( i \) and \( a_i, b_i, \) and \( c_i \) are the cost coefficients of generating unit \( i \).

b. Minimization of emission

The amount of pollutants generated from a fossil based generating unit depends on the amount of power generated by that unit. The total ton/h emission \( e(x) \) of these pollutants can be expressed as \([4]\):

\[
e(x) = \sum_{i=1}^{ng} 10^{-2}(\alpha_i + \beta_i P_{gi} + \gamma_i P_{gi}^2) + \omega_i \exp(\eta_i P_{gi})
\]

where \( \alpha_i, \beta_i, \gamma_i, \omega_i, \) and \( \eta_i \) are the emission coefficients of generator \( i \).

2.2. Constraints

a. Equality constraints

The equality constraints are represented by the power balance constraint, where the total power generation must cover the total power demand and the power loss. This implies solving the load flow problem, which has equality constraints on active and reactive power at each bus as follows \([7]\):

\[
P_i = P_{gi} - P_{di} = \sum_{j=1}^{n} V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij})
\]

\[
Q_i = Q_{gi} - Q_{di} = \sum_{j=1}^{n} V_i V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij})
\]

where, \( \theta_{ij} = \theta_i - \theta_j \), \( n \) is the number of buses, \( P_i \) and \( Q_i \) are respectively, the injected active and reactive power at bus \( i \). \( P_{di} \) and \( Q_{di} \) are respectively the active and reactive power demand at bus \( i \). \( V_i \) and \( \theta_i \) are respectively the bus voltage magnitude and angle at bus \( i \). \( G_{ij} \) and \( B_{ij} \) are the conductance and susceptance of the \((i,j)\) element in the admittance matrix.

b. Inequality constraints

The inequality constraints reflect the limits on physical devices in the power system as well as the limits created to ensure system security, which are:

Upper and lower bounds on the active and reactive generations:

\[
P_{gi\min} \leq P_{gi} \leq P_{gi\max}
\]

\[
Q_{gi\min} \leq Q_{gi} \leq Q_{gi\max}
\]

Upper and lower bounds on the tap ratio \( (t) \) and phase shifting \( (a) \) of variable transformers:

\[
t_{ij\min} \leq t_{ij} \leq t_{ij\max}
\]

\[
\alpha_{ij\min} \leq \alpha_{ij} \leq \alpha_{ij\max}
\]

Upper limit on the active power flow \( (P_{ij}) \) of line \( i-j \):

\[
|P_{ij}| \leq P_{ij\max}
\]

Upper and lower bounds on the bus voltage magnitude:

\[
V_{i\min} \leq V_i \leq V_{i\max}
\]

2.3. Problem formulation

Aggregating the objectives and constraints, the problem can be mathematically formulated as a nonlinear constrained multiobjective optimization problem, as follows \([4]\):

Minimize \( \{f(x), e(x)\} \) \hspace{1cm} (8)

Subject to: \( g(x) = 0 \) \hspace{1cm} (9)

\( h(x) \leq 0 \) \hspace{1cm} (10)

where \( f(x) \) and \( e(x) \) are the objective functions, \( g(x) \) and \( h(x) \) are respectively the set of equality and inequality constraints. \( x \) is the vector of control and state variables. The control variables are generator active and reactive power outputs, bus voltages, shunt capacitors/reactors and transformers tap-setting. The state variables are voltage and angle of load buses.

3. MULTIOBJECTIVE FUZZY OPTIMIZATION

In fuzzy optimization, the objective may not be optimized exactly, and constraints can be satisfied to varying degrees. This is opposed to crisp optimization where an optimal solution is sought satisfying all the constraints crisply. Most methods reported in the literature transform a fuzzy problem into a crisp one by using the symmetric approach of Bellman and Zadeh \([8]\). The basic idea is that the objective function should be essentially smaller than or equal to some “aspiration level” and this can be regarded as a constraint. Bellman and Zadeh treat this “objective function” and other constraints symmetrically, and define fuzzy optimization as maximizing the minimum degree of satisfaction among all the constraints. In the same manner, in multiobjective fuzzy optimization the objective functions and constraints are treated symmetrically, and
the goal is to maximize the minimum degree of satisfaction among all the objectives and constraints.

3.1. Fuzzy problem formulation

The fuzzy set theory has been developed to model inexact or imprecise objects in optimization problems [9, 10]. Enforcement of soft constraints does not need to be exact; furthermore minimization of the objective functions should not be rigid. Therefore, the fuzzy set theory can be applied to the EED problem to more accurately model practical considerations. Based on the fuzzy set theory, the fuzzy multiobjective economic emission dispatch problem can be written as:

Minimize \[ \sum_{j=1}^{n} \mu_j(\lambda) \]
Subject to:
1. \( g(x) = 0 \) \hspace{1cm} (11)
2. \( h(x) \leq 0 \) \hspace{1cm} (12)
3. \( e(x) \leq e_o \) \hspace{1cm} (13)

where \((\leq)\) denotes a fuzzy inequality relation.

Equation (11) states that the objective is to minimize \(f(x)\) and \(e(x)\) so that they will not exceed the desired values \(c_o\) and \(e_o\) “too much”. Equations (12-13) state that the hard constraints \(g(x)\) and \(h(x)\) must be enforced exactly.

The membership function of the fuzzy inequality in (11) is depicts in Fig. 1 and given by:

\[ \mu_f(f(x)) = \begin{cases} 1 & f(x) \leq c_o \\ \frac{f(x) - c_o}{\delta_{c_o}} & c_o < f(x) \leq c_o + \delta_{c_o} \\ 0 & f(x) > c_o + \delta_{c_o} \end{cases} \]

\[ \mu_e(e(x)) = \begin{cases} 1 & e(x) \leq e_o \\ \frac{e_o - e(x)}{\delta_{e_o}} & e_o < e(x) \leq e_o + \delta_{e_o} \\ 0 & e(x) > e_o + \delta_{e_o} \end{cases} \]

The cost \(c_o+\delta_{c_o}\) and emission \(e_o+\delta_{e_o}\) in (14) and (15) are the highest acceptable cost and emission. Usually, these values are calculated from the load flow solution which represents the current non-optimized operating state. Then, the lowest cost \(c_o\) and the lowest emission \(e_o\) are determined by the user desired maximum cost \(\delta_{c_o}\) and emission reductions \(\delta_{e_o}\). Selection of these parameters may be subjective and dependent on specific operational practices.

3.2. Solution methodology

The solution of the multiobjective fuzzy optimization problem in (11-13) consists of minimizing two fuzzy objectives while enforcing the hard constraints exactly. The degree of satisfaction for fuzzy objectives can be represented by a membership variable \(\lambda\). The membership variable \(\lambda\) is defined as the minimum of all the membership functions of the fuzzy objectives, that is:

\[ \lambda = \min \left[ \mu_f(f(x)), \mu_e(e(x)) \right] \]

(16)

The problem becomes maximizing \(\lambda\) [9], that is:

Maximize \(\lambda\) \hspace{1cm} (17)
Subject to:
1. \( g(x) = 0 \) \hspace{1cm} (18)
2. \( \mu_f(f(x)) \geq \lambda \) \hspace{1cm} (19)
3. \( \mu_e(e(x)) \geq \lambda \) \hspace{1cm} (20)
4. \( h(x) \leq 0 \) \hspace{1cm} (21)
5. \( 0 \leq \lambda \leq 1 \) \hspace{1cm} (22)

Substituting the membership functions into the above equations (19 and 20) yields the following crisp optimization problem:

Minimize \(-\lambda\) \hspace{1cm} (23)
Subject to:
1. \( g(x) = 0 \) \hspace{1cm} (24)
2. \( f(x) + \delta_{c_o} \lambda \leq c_o + \delta_{c_o} \) \hspace{1cm} (25)
3. \( e(x) + \delta_{e_o} \lambda \leq e_o + \delta_{e_o} \) \hspace{1cm} (26)
4. \( h(x) \leq 0 \) \hspace{1cm} (27)
5. \( 0 \leq \lambda \leq 1 \) \hspace{1cm} (28)

The problem thus becomes maximizing a scalar value \(\lambda\) representing the degree of satisfaction such that the membership values of all constraints should be greater than or equal to this \(\lambda\).

A successive linear programming (SLP) based algorithm is used and adapted to solve the new problem given by (23-28). The basic steps required in the SLP algorithm are summarized as follows [1]:

Step 1. Solve the load flow problem to obtain a feasible solution that satisfies the power balance equality constraint.

Step 2. Linearize the fuzzy objectives and hard constraints in (25-27), around the load flow solution.

Step 3. Solve the LP problem and obtain optimal incremental control variables \(\Delta x\) and membership variable \(\Delta \lambda\).

Step 4. Update the control and membership variables:

\[ x^{(k+1)} = x^{(k)} + \Delta x \text{ and } \lambda^{(k+1)} = \lambda^{(k)} + \Delta \lambda \]

Step 5. Obtain the load flow solution with updated control variables.

Step 6. If \(\Delta x\) and \(\Delta \lambda\) in step 3 are bellow user defined tolerances, the solution converges. Otherwise, go to step 2.

4. APPLICATION EXAMPLE

The proposed fuzzy economic emission dispatch (FEED) method is examined with the standard IEEE 30-bus 6-generators test system on AMD Athlon (tm) XP 2000 computer, using MATLAB program coding. The detailed data of this system are given in [11]. This power system is interconnected by 41 transmission lines and the total system demand for the 21 load buses is 283.40 MW. Fuel cost and emission coefficients for this system are given in Table 1.
Table 1. Generator fuel cost and emission coefficients

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>G.1</th>
<th>G.2</th>
<th>G.3</th>
<th>G.4</th>
<th>G.5</th>
<th>G.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>10</td>
<td>10</td>
<td>20</td>
<td>10</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>b</td>
<td>200</td>
<td>150</td>
<td>180</td>
<td>100</td>
<td>180</td>
<td>150</td>
</tr>
<tr>
<td>c</td>
<td>100</td>
<td>120</td>
<td>40</td>
<td>60</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>α</td>
<td>4.091</td>
<td>2.543</td>
<td>4.258</td>
<td>5.426</td>
<td>4.258</td>
<td>6.131</td>
</tr>
<tr>
<td>β</td>
<td>-5.554</td>
<td>-6.047</td>
<td>-5.094</td>
<td>-3.550</td>
<td>-5.094</td>
<td>-5.555</td>
</tr>
<tr>
<td>ω</td>
<td>2.0e-04</td>
<td>5.0e-04</td>
<td>1.0e-06</td>
<td>2.0e-03</td>
<td>1.0e-06</td>
<td>1.0e-05</td>
</tr>
<tr>
<td>η</td>
<td>2.857</td>
<td>3.333</td>
<td>8.000</td>
<td>2.000</td>
<td>8.000</td>
<td>6.667</td>
</tr>
</tbody>
</table>

Table 2. The best solutions for cost and emission optimized individually

<table>
<thead>
<tr>
<th>Case 1 (Best cost)</th>
<th>Case 2 (Best emission)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_{g1} (MW)</td>
<td>11.081</td>
</tr>
<tr>
<td>P_{g2} (MW)</td>
<td>30.193</td>
</tr>
<tr>
<td>P_{g3} (MW)</td>
<td>54.560</td>
</tr>
<tr>
<td>P_{g4} (MW)</td>
<td>101.739</td>
</tr>
<tr>
<td>P_{g5} (MW)</td>
<td>52.406</td>
</tr>
<tr>
<td>P_{g6} (MW)</td>
<td>36.037</td>
</tr>
<tr>
<td>Cost ($/h)</td>
<td>605.93</td>
</tr>
<tr>
<td>Emission (ton/h)</td>
<td>0.22209</td>
</tr>
<tr>
<td>Power loss (MW)</td>
<td>2.616</td>
</tr>
<tr>
<td>Satisfaction degree λ</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The simulations were run for three different goals as follows:
Case 1: Minimize total fuel cost.
Case 2: Minimize total emission.
Case 3: Minimize fuel cost and emission simultaneously.

Fuel cost and emission objectives are optimized individually in order to test the equivalence of the fuzzy EED to the crisp EED. This step is also necessary for exploring the extremes points of Pareto – optimal solutions obtained by the proposed FEED algorithm. The cost and emission of the initial operating state based on the load flow without optimization are respectively 765.92 $/h and 0.23872 ton/h, which are used as the highest acceptable values of $c_o + \delta_{eo}$ and $e_o + \delta_{co}$.

4.1. Minimization of each objective individually

The minimum value of a single objective is obtained by giving full consideration to one of the objectives, and neglecting the other. For minimum fuel cost, the desired cost is set to $c_o=650.00$ $$/h. For minimum emission, the desired emission is set to $e_o=0.19000$ ton/h. The best results of cost and emission functions are reported in Table 2. It can be seen that the fuel cost and emission are conflicting objectives. Emission has maximum value when cost is minimum. Convergence of total fuel cost (case 1) and total emission (case 2) are shown in Fig. 2.

4.2. Minimization of objectives simultaneously

In multiobjective optimization, fuel cost and emission are minimized simultaneously, subject to the imposed constraints. The set of compromise solutions or Pareto-optimal set of the problem is computed according to the lowest membership value $\lambda$ of all the fuzzy objectives. The highest acceptable cost and emission are set respectively to their maximum values obtained in case 2 (644.80 $$/h) and case 1 (0.22009 ton/h). The desired emission is set to its minimum value of 0.19418 ton/h.

To obtain the Pareto-optimal solutions by the FEED based operator, 12 independent runs were made using 12 different desired costs. The compromise solutions obtained for different values of desired cost are reported in Table 5. The trade off relationship between fuel cost and emission is shown in Fig. 3. From Table 5, it can be seen that the fuzzy EED is equivalent to crisp methods. Fuel cost and emission obtained with FEED algorithm are reduced compared with those from literature. The profit in cost and reduction in emission with the proposed approach are significant.

Table 3. The comparison results of best fuel cost

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P_{g1} (MW)</td>
<td>15.00</td>
<td>11.68</td>
<td>12.45</td>
<td>10.86</td>
<td>11.081</td>
</tr>
<tr>
<td>P_{g2} (MW)</td>
<td>30.00</td>
<td>31.65</td>
<td>27.92</td>
<td>30.56</td>
<td>30.193</td>
</tr>
<tr>
<td>P_{g3} (MW)</td>
<td>55.00</td>
<td>54.41</td>
<td>62.84</td>
<td>58.18</td>
<td>54.560</td>
</tr>
<tr>
<td>P_{g4} (MW)</td>
<td>105.00</td>
<td>94.47</td>
<td>102.64</td>
<td>98.46</td>
<td>101.739</td>
</tr>
<tr>
<td>P_{g5} (MW)</td>
<td>46.00</td>
<td>54.98</td>
<td>46.93</td>
<td>52.88</td>
<td>52.406</td>
</tr>
<tr>
<td>P_{g6} (MW)</td>
<td>35.00</td>
<td>39.64</td>
<td>39.93</td>
<td>35.84</td>
<td>36.037</td>
</tr>
<tr>
<td>Cost ($/h)</td>
<td>606.31</td>
<td>608.24</td>
<td>608.14</td>
<td>607.80</td>
<td>605.93</td>
</tr>
<tr>
<td>Emission (ton/h)</td>
<td>0.22330</td>
<td>0.21664</td>
<td>0.22364</td>
<td>0.22015</td>
<td>0.22209</td>
</tr>
</tbody>
</table>
Table 4. The comparison results of emission

<table>
<thead>
<tr>
<th></th>
<th>LP</th>
<th>NSGA</th>
<th>NPGA</th>
<th>SPEA</th>
<th>FEED</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{g1}$ (MW)</td>
<td>40.00</td>
<td>41.13</td>
<td>39.23</td>
<td>40.43</td>
<td>41.336</td>
</tr>
<tr>
<td>$P_{g2}$ (MW)</td>
<td>45.00</td>
<td>45.91</td>
<td>47.00</td>
<td>45.25</td>
<td>46.658</td>
</tr>
<tr>
<td>$P_{g3}$ (MW)</td>
<td>55.00</td>
<td>51.17</td>
<td>55.65</td>
<td>55.25</td>
<td>53.922</td>
</tr>
<tr>
<td>$P_{g4}$ (MW)</td>
<td>40.00</td>
<td>37.24</td>
<td>36.95</td>
<td>40.79</td>
<td>38.458</td>
</tr>
<tr>
<td>$P_{g5}$ (MW)</td>
<td>55.00</td>
<td>58.10</td>
<td>55.99</td>
<td>54.68</td>
<td>54.204</td>
</tr>
<tr>
<td>$P_{g6}$ (MW)</td>
<td>50.00</td>
<td>53.04</td>
<td>51.63</td>
<td>50.05</td>
<td>51.429</td>
</tr>
</tbody>
</table>

Emission (ton/h) 0.19424 0.19432 0.19424 0.19422 0.19418
Cost ($/h) 639.60 647.25 645.98 642.60 644.80

seen that the fuel cost is reduced when the desired cost is decreasing, and the emission objectives are slightly over there desired minimum value. The fuzzy EED will balance the trade-off of cost and emission. It can be also observed that each simulation run is characterized with its own degree of satisfaction $\lambda$, reflecting the total satisfaction of fuzzy objectives. The operator’s compromise solutions are then obtained by interactive adjustment of different desired cost and emission, depending on the operator’s preference. If one solution is not accepted by the operator, increase or decrease the desired values until the solution is satisfied by the operator.

4.3. The best compromise solution

To extract the best compromise solution over the trade-off curve, the desired cost and emission are set respectively to their minimum values obtained in case 1 (605.93 $/h) and case 2 (0.19418 ton/h). The highest acceptable cost and emission are not changed. The best compromise solution is obtained with cost = 615.11 $/h, emission = 0.20075 ton/h and $\lambda = 0.76$. Convergence of total fuel cost and total emission of this last solution is shown in Fig. 4.

The best compromise solution obtained with FEED algorithm is compared with those using genetic algorithm (NSGA, NPGA and SPEA) [11]. The comparison results are grouped in Table 5. It is noted that the emission value obtained with FEED is comparable with that obtained by other methods.

Table 5. The compromise solutions found by FEED for different desired costs with minimum emission

<table>
<thead>
<tr>
<th>Desired cost $c_o$ ($/h)</th>
<th>650.00</th>
<th>640.00</th>
<th>630.00</th>
<th>620.00</th>
<th>610.00</th>
<th>600.00</th>
<th>590.00</th>
<th>580.00</th>
<th>570.00</th>
<th>560.00</th>
<th>550.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired minimum emission $e_o$ (ton/h)</td>
<td>0.19418</td>
<td>0.19418</td>
<td>0.19418</td>
<td>0.19418</td>
<td>0.19418</td>
<td>0.19418</td>
<td>0.19418</td>
<td>0.19418</td>
<td>0.19418</td>
<td>0.19418</td>
<td>0.19418</td>
</tr>
<tr>
<td>$P_{g1}$ (MW)</td>
<td>40.681</td>
<td>39.444</td>
<td>35.083</td>
<td>30.919</td>
<td>27.742</td>
<td>25.302</td>
<td>22.301</td>
<td>20.180</td>
<td>19.203</td>
<td>16.630</td>
<td>15.962</td>
</tr>
<tr>
<td>$P_{g2}$ (MW)</td>
<td>46.688</td>
<td>45.309</td>
<td>42.474</td>
<td>39.972</td>
<td>38.595</td>
<td>37.389</td>
<td>35.852</td>
<td>34.180</td>
<td>33.881</td>
<td>32.465</td>
<td>32.186</td>
</tr>
<tr>
<td>$P_{g3}$ (MW)</td>
<td>54.552</td>
<td>54.691</td>
<td>54.595</td>
<td>54.714</td>
<td>53.649</td>
<td>54.691</td>
<td>53.866</td>
<td>54.347</td>
<td>53.646</td>
<td>54.693</td>
<td>54.981</td>
</tr>
<tr>
<td>$P_{g4}$ (MW)</td>
<td>39.582</td>
<td>41.965</td>
<td>50.399</td>
<td>58.913</td>
<td>67.172</td>
<td>73.466</td>
<td>77.947</td>
<td>81.126</td>
<td>84.087</td>
<td>89.172</td>
<td>91.432</td>
</tr>
<tr>
<td>$P_{g5}$ (MW)</td>
<td>53.830</td>
<td>54.066</td>
<td>54.691</td>
<td>54.909</td>
<td>54.563</td>
<td>53.471</td>
<td>54.321</td>
<td>54.845</td>
<td>54.685</td>
<td>53.619</td>
<td>52.813</td>
</tr>
<tr>
<td>$P_{g6}$ (MW)</td>
<td>50.638</td>
<td>50.456</td>
<td>48.584</td>
<td>46.332</td>
<td>44.050</td>
<td>41.424</td>
<td>41.497</td>
<td>41.114</td>
<td>40.331</td>
<td>39.271</td>
<td>38.489</td>
</tr>
<tr>
<td>Cost ($/h)</td>
<td>643.27</td>
<td>640.14</td>
<td>630.55</td>
<td>622.59</td>
<td>616.87</td>
<td>615.10</td>
<td>610.79</td>
<td>609.30</td>
<td>608.43</td>
<td>607.01</td>
<td>606.60</td>
</tr>
<tr>
<td>Emission (ton/h)</td>
<td>0.19418</td>
<td>0.19425</td>
<td>0.19514</td>
<td>0.19705</td>
<td>0.19963</td>
<td>0.20232</td>
<td>0.20474</td>
<td>0.20682</td>
<td>0.20848</td>
<td>0.21200</td>
<td>0.21354</td>
</tr>
<tr>
<td>Power loss (MW)</td>
<td>2.572</td>
<td>2.530</td>
<td>2.426</td>
<td>2.359</td>
<td>2.371</td>
<td>2.344</td>
<td>2.385</td>
<td>2.391</td>
<td>2.432</td>
<td>2.449</td>
<td>2.463</td>
</tr>
<tr>
<td>Satisfaction degree $\lambda$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.97</td>
<td>0.90</td>
<td>0.80</td>
<td>0.71</td>
<td>0.62</td>
<td>0.55</td>
<td>0.49</td>
<td>0.36</td>
<td>0.30</td>
</tr>
</tbody>
</table>

It is also clear that the savings in fuel cost obtained with the proposed approach is revealed. The satisfaction degree of the objectives is very acceptable.
Table 6. Comparison of the best compromise solution

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{g1}$ (MW)</td>
<td>29.35</td>
<td>29.76</td>
<td>27.52</td>
<td>26.41</td>
</tr>
<tr>
<td>$P_{g2}$ (MW)</td>
<td>36.45</td>
<td>39.56</td>
<td>37.52</td>
<td>38.01</td>
</tr>
<tr>
<td>$P_{g3}$ (MW)</td>
<td>58.33</td>
<td>56.73</td>
<td>57.96</td>
<td>53.90</td>
</tr>
<tr>
<td>$P_{g4}$ (MW)</td>
<td>67.63</td>
<td>69.28</td>
<td>67.70</td>
<td>69.92</td>
</tr>
<tr>
<td>$P_{g5}$ (MW)</td>
<td>53.83</td>
<td>52.01</td>
<td>52.83</td>
<td>54.19</td>
</tr>
<tr>
<td>$P_{g6}$ (MW)</td>
<td>40.76</td>
<td>39.04</td>
<td>42.82</td>
<td>43.33</td>
</tr>
<tr>
<td>Cost ($/h)</td>
<td>617.80</td>
<td>617.79</td>
<td>617.57</td>
<td>615.11</td>
</tr>
<tr>
<td>Emission (ton/h)</td>
<td>0.20020</td>
<td>0.20040</td>
<td>0.20010</td>
<td>0.20075</td>
</tr>
</tbody>
</table>

5. CONCLUSION

In this paper, a multiobjective fuzzy optimization-based method is developed to determine the best compromise solution of the economic emission dispatch problem with fuzzy objectives. The problem is first converted to a crisp optimization problem, and then solved using an efficient iterative linear programming technique. Implementation of the proposed approach was based on fuzzy set theory to obtain the Pareto-optimal solutions. Then, the desired fuel cost and emission values are used to help the power system operator to obtain the appropriate dispatch solution.

The proposed method has been tested and validated on the standard IEEE 30-bus 6-generators test system. Considering the cases and comparative studies presented in this paper, FEED algorithm appears to be efficient in particular for its flexibility and its interesting financial profit. Numerical results show that the fuzzy optimization method appears to be a promising and efficient approach for dealing with the multiobjective nature of power dispatch problem.

REFERENCES


