ROBUST FUZZY-PID CONTROL OF THREE-MOTOR DRIVE SYSTEM USING SIMULATED ANNEALING OPTIMIZATION
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Abstract: Multi-motor drive system is a multi-input multi-output (MIMO), nonlinear and strong-coupling system. In turn to improve synchronous performance of multi-motor system, this paper presents a synchronous control system for three-induction motors. It includes three robust Fuzzy-PID controllers based Simulated Annealing Optimization (SAO). The parameters of Fuzzy-PID controllers are adjusted for fulfilling the open loop control of multivariable system to reduce mutual coupling effect. The SAO technique is used to adjust the input and output scaling factor (SF’s) by minimizing both integral of absolute of errors (IAE) and integral of time multiplied absolute of errors (ITAE) as performance measures. To test the robustness of the proposed system, several sudden changes are presented. The results indicate that the control system can acquire the optimal parameters of the Fuzzy-PID controllers according to different running states of the system. Results of system performance endorse the proposed technique and emphasize its feasibility.

Keywords: three-motor drive system, Fuzzy-PID controllers, Simulated Annealing Optimization.

1. Introduction
Due to its high precision coordinated control performance, a multi-motor system has attracted more and more attentions in the drive applications such as urban rail transit, paper making, electric vehicle drive, and steel rolling [1-5]. The AC multi-motor drive system is a multi-input multi-output (MIMO), nonlinear and strong-coupling system [6, 7]. Thus, its accurate mathematical model is hard to obtain. Meanwhile, industrial production also requires the multi-motor drive control system to decouple the speed and tension, which increases the control difficulty.

Nowadays, common decoupling control methods mainly contain: improved control algorithm based on the traditional PID [8], cross-coupled control [9], feed forward control [10], optimal control [11], sliding mode control [12, 13], BP Neural Network [14], and fuzzy control [15]. To a certain degree, these methods have improved coordinated control performance. However, it should be noted that most of the existing methods depend on dynamic model of the motor and traditional single motor drive system.

Fuzzy-PID control, as one of the promising intelligent control techniques, is applied for multi-motor synchronous control purposes. The authors in [16] proposed a variable gain intelligent Fuzzy reasoning control scheme in the stretch tension and synchronization control system of a wide-fabric heating-shaping machine. In [17] a synchronization control strategy of multi-motor system based on PROFIBUS network was proposed using BP Neural Network and an adaptive double mode Fuzzy-PID arithmetic. Fuzzy-PID C-means clustering algorithm [18] was applied to two-motor variable frequency speed-regulating synchronous system as a multivariable nonlinear coupling system. It is used to cluster the data of input-output according to a satisfactory performance index in order to identify speed and tension for the two-motor synchronous system based on local model networks. Two different Fuzzy-PID decoupling controllers in the speed feedback and the tension feedback [19] introduced greatly improved performance. It should be noted that most of these researches consider only two-motor synchronous drive system.

There are several researches in the literature employed Simulated Annealing Optimization (SAO) in Fuzzy-PID controllers. The authors in [20] proposed a general technique for optimizing fuzzy models in fuzzy expert systems (FES’s) by simulated annealing (SA) and N-dimensional simplex method. In [21], Fuzzy-PID control is combined with BP Neural Network with a Genetic Algorithms (GA) and SAO to short-term load forecasting. An optimizing
Fuzzy Neural Networks for tuning PID controllers using an SAO was proposed, in [22], to solve the large-scale constrained optimization problems. The authors in [23] discuss the design of fuzzy control systems with a reduced parametric sensitivity using simulated-annealing algorithms.

This paper introduces a robust Fuzzy-PID control of three-motor drive system using SAO. The main objective is to minimize both the integral of absolute of errors (IAE) and the integral of time multiplied absolute of errors (ITAE) as performance measures. SAO technique is applied to adjust the parameters of Fuzzy-PID controllers for reducing the strong coupling influences. The rest of the paper is organized as follows; section 2 formulates the mathematical model of the three-motor drive system by considering the first motor as a main and the others as supplement motors. In section 3, the proposed Fuzzy-PID controller is applied to the three-motor system. Moreover, the SAO algorithm is presented to optimize the input and output scaling factor (SF’s) such that the overall performance index is minimized. Different simulation cases are studied and presented in section 4. The proposed system robustness is tested by applying several sudden changes. The results are compared with those obtained using PID controllers whose gains are optimized also by SAO to guarantee fair comparison. Conclusions, remarks and perspectives of applying the proposed Fuzzy-PID controller to multi-motor drive system are discussed in section 5.

2. Mathematical Model

Figure 1 shows the block diagram of three-induction motor drive system, in which motor 1 is the main motor and other two motors are supplement motors. Every motor and its inverter can be regarded as a modular cell. The belt-pulley is installed on the motor shaft, and motors are combined by transmission belt on the belt-pulley. When the motors run, the rotation of their rotors pulls the belt to operate coordinately.

According to Hooke law and by considering the amount of forward slip, the tensions between the two adjacent motors can be expressed as:

\[
F_{12} = K_1 \left( \frac{a_{i1}}{p_1} o_{i1} - \frac{a_{i2}}{p_2} o_{i2} \right) - F_{i1}
\]

\[
F_{13} = K_2 \left( \frac{a_{i1}}{p_1} o_{i1} - \frac{a_{i3}}{p_3} o_{i3} \right) + F_{i2}
\]

where:

- \( F_{12}, F_{23} \) tensions of the belt;
- \( K_1 = E/V_1 \) transfer coefficients;
- \( K_2 = E/V_2 \) time constants of tension variation;
- \( r_i \) radius of i belt-pulley (i =1,2,3);
- \( a_i \) number i speed ratio;
- \( \omega_{ri} \) electric angular speed of i motor;
- \( p_i \) pole-pairs number of i motor;
- \( A \) sectional area of the belt;
- \( E \) Young’s Modulus of the belt;
- \( L_1, L_2 \) distances between the two motors;
- \( V_1, V_2 \) line speed.

\[
\dot{\omega}_{11} = \frac{p_1}{J_1} \left[ \left( \omega_1 - \omega_{11} \right) \frac{L_1 r_{r2}}{T_{r1}} \phi_{12}^2 - \left( T_{r1} + r_1 F_{12} \right) \right]
\]

\[
\dot{\phi}_{11} = \frac{L_{r1} a_{i1}}{\tau_{r1}} - \frac{1}{\tau_{r1}} \phi_{11}
\]

\[
\dot{\omega}_{12} = \frac{p_2}{J_2} \left[ \left( \omega_2 - \omega_{12} \right) \frac{L_2 r_{r3}}{T_{r2}} \phi_{23}^2 - \left( T_{r2} - r_2 F_{12} + r_2 F_{23} \right) \right]
\]

\[
\dot{\phi}_{12} = \frac{L_{r2} a_{i2}}{\tau_{r2}} - \frac{1}{\tau_{r2}} \phi_{12}
\]

\[
\dot{\omega}_{13} = \frac{p_3}{J_3} \left[ \left( \omega_3 - \omega_{13} \right) \frac{L_3 r_{r3}}{T_{r3}} \phi_{3}^2 - \left( T_{r3} - r_3 F_{23} \right) \right]
\]

\[
\dot{\phi}_{13} = \frac{L_{r3} a_{i3}}{\tau_{r3}} - \frac{1}{\tau_{r3}} \phi_{13}
\]

In vector control technique, the rotor flux is kept constant. Then the three-motor model can be reduced to:

Fig. 1. The three-motor drive system.
\[
\dot{\omega}_1 = \frac{P_1}{J_1} [(\omega_1 - \omega_{1s}) - \frac{P_2 r_1}{L_{1s}} \phi_1^2 - (T_{t1} + r_1 F_{12})]
\]
\[
\dot{\omega}_2 = \frac{P_2}{J_2} [(\omega_2 - \omega_{2s}) - \frac{P_2 r_2}{L_{2s}} \phi_2^2 - (T_{t2} - r_2 F_{12} + r_2 F_{23})]
\]
\[
\dot{\omega}_3 = \frac{P_3}{J_3} [(\omega_3 - \omega_{3s}) - \frac{P_2 r_3}{L_{3s}} \phi_3^2 - (T_{t3} - r_3 F_{23})] 
\]

The mathematical model for the whole drive system is represented by:

\[
\dot{\omega}_1 = \frac{P_1}{J_1} [(\omega_1 - \omega_{1s}) - \frac{P_2 r_1}{L_{1s}} \phi_1^2 - (T_{t1} + r_1 F_{12})]
\]
\[
\dot{\omega}_2 = \frac{P_2}{J_2} [(\omega_2 - \omega_{2s}) - \frac{P_2 r_2}{L_{2s}} \phi_2^2 - (T_{t2} - r_2 F_{12} + r_2 F_{23})]
\]
\[
\dot{\omega}_3 = \frac{P_3}{J_3} [(\omega_3 - \omega_{3s}) - \frac{P_2 r_3}{L_{3s}} \phi_3^2 - (T_{t3} - r_3 F_{23})]
\]

\[
\dot{F}_{12} = \frac{K_1}{T_1} \left( \frac{r_{d1}}{p_1} \omega_1 - \frac{r_{a2}}{p_2} \omega_{2s} - \frac{r_{a2}}{p_2} \omega_2 - \frac{F_{12}}{T_1} \right)
\]
\[
\dot{F}_{23} = \frac{K_2}{T_2} \left( \frac{r_{d2}}{p_2} \omega_2 - \frac{r_{a3}}{p_3} \omega_{3s} - \frac{r_a}{p_3} \omega_3 - \frac{F_{23}}{T_2} \right) 
\]

where

\[J_1, J_2, J_3\]  
rotor inertia;

\[\tau_{t1}, \tau_{t2}, \tau_{t3}\]  
electromagnetic time constant;

\[T_{l1}, T_{l2}, T_{l3}\]  
load torque;

\[\phi_{1s}, \phi_{2s}, \phi_{3s}\]  
rotor flux;

\[L_{1s}, L_{2s}, L_{3s}\]  
rotor self-inductance;

\[i_{ds1}, i_{ds2}, i_{ds3}\]  
d-axis stator current;

\[\omega, \omega_e, \omega_h\]  
synchronous angular speed of stator’s.

From equation (5), we can find the result of coupling are existing between the speeds of the three motors, the tension \(F_{12}\) and the tension \(F_{23}\), they are strongly coupled.

Solving equation (5) by using Laplace transform,

\[
\Omega_1(s) = \frac{1}{s + \beta_1} \left[ \beta_1 \omega_h - \frac{P_1}{J_1} (T_{l1}(s) + r_1 F_{12}(s)) \right]
\]
\[
\Omega_2(s) = \frac{1}{s + \beta_2} \left[ \beta_2 \omega_h - \frac{P_2}{J_2} (T_{l2}(s) - r_2 F_{12}(s)) \right]
\]
\[
\Omega_3(s) = \frac{1}{s + \beta_3} \left[ \beta_3 \omega_h - \frac{P_3}{J_3} (T_{l3}(s) - r_3 F_{23}(s)) \right]
\]

\[
F_{12}(s) = \frac{K_1}{(T_{l1} + 1)} \left[ \frac{r_{d1}}{p_1} \Omega_1(s) - \frac{r_{a2}}{p_2} \Omega_2(s) \right]
\]
\[
F_{23}(s) = \frac{K_2}{(T_{l3} + 1)} \left[ \frac{r_{d2}}{p_2} \Omega_2(s) - \frac{r_{a3}}{p_3} \Omega_3(s) \right] 
\]

where,

\[
\beta_1 = \frac{P_1^2 r_1}{J_1 L_{1s}} \phi_1^2, \quad \beta_2 = \frac{P_2^2 r_2}{J_2 L_{2s}} \phi_2^2, \quad \text{and} \quad \beta_3 = \frac{P_3^2 r_3}{J_3 L_{3s}} \phi_3^2
\]

Figure (2.a) shows a Matlab/Simulink model of the equations (1-3) for the three-motor system, while figure (2.b) shows a Matlab/Simulink model of the equations (4-5) for the same system.

**Fig. 2.** Simulink three-motor model equations. (1-3).

**Fig. 3.** Simulink three-motor model equations. (4-5).

3. The Proposed System Design Tools

3.1 Controllers Design for Speed/Tension Loops

A Fuzzy-PID-SA0 controller is proposed to improve the synchronous performance of three AC induction motors control system. Three different Fuzzy-PID-SA0 controllers are designed, one for the speed and two for the tensions. In the proposed approach, the gains of a PID controller are primary guessed using Ziegler-Nichols approach, and then these gains are utilized to get the corresponding values for input and output scaling factors of the proposed Fuzzy-PID-SA0 controller. These values are employed to suggest the ranges of the Fuzzy-PID-SA0 tuning parameters (input and output SF’s) for the SAO algorithm. The optimization algorithm is then applied to obtain the best values for the input and output SF’s of the Fuzzy-PID-SA0 controllers.

Fig. (4) shows a schematic diagram for the overall control system while Fig. (5) shows the Fuzzy-PID-SA0 controller structure for the speed loop. The Fuzzy-PID-SA0 controllers for both tension loops are the same except for their input and output SF’s.
3.1.1 Membership Functions

All membership functions (MF’s) controller inputs, i.e. error (e) and change of error (\(\Delta e\)) and controller output for Fuzzy-PID-SAO (u) are defined on the normalized interval \([1, 1]\) since the arbitrary selection of the universe of discourse causes some complexity in the selection of the scaling factors. We use five symmetric triangles membership functions with 50% overlapping with neighboring for both error and change of error for the three Fuzzy-PID-SAO controllers. Five singletons output membership functions are applied for the control signal. The input and output MF’s are shown in Fig. 6.

3.1.2 Rule Base

Rule tables for Fuzzy-PID-SAO controllers are obtained from the analysis of the system behavior. The Fuzzy-PID-SAO controller works in such a way that relates the controlled system output with the reference input by a set of fuzzy rules. These rules are the most important part of the controller structure and they must be obtained correctly so that the relationship between input and output of the controlled system are represented properly. The set of fuzzy rules should be determined and organized in such a way that the reference set point is tracked with the best performance measures. These performance measures are: minimum overshoot, rise time, settling time, integral of absolute of errors, and integral of time multiplied absolute of errors. Table (1) shows the applied fuzzy rule base.

Table (1). The Fuzzy rule base utilized for the proposed Fuzzy-PID-SAO controller

<table>
<thead>
<tr>
<th>e</th>
<th>(\Delta e)</th>
<th>NB</th>
<th>NS</th>
<th>Z</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NS</td>
<td>NS</td>
<td>NS</td>
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<td>PS</td>
<td>PS</td>
<td>PB</td>
<td>PB</td>
<td></td>
</tr>
</tbody>
</table>

In this work, we select the fuzzy implication as product-max, where the algebraic product is utilized for AND-connective while the rule base is the outer AND-product of all input families. Center of area method is applied for the defuzzification process.

3.2 Simulated Annealing Optimization

Annealing is the process of heating the solid body to a high temperature and allowed it to cool slowly. Annealing causes the particles of the solid material to reach the minimum energy state. This is due to the fact that when the solid body is heated to the very high temperature, the particles of the solid body are allowed to move freely and when it is cooled slowly, the particles are able to arrange themselves so that the energy of the particles are made minimum. The mathematical equivalence of the thermodynamic annealing as described above is called simulated annealing [24].

The energy of the particle in thermodynamic annealing process is corresponding to the cost function to be minimized in optimization problems. Similarly, there is an analogy between the particles of the solid and the independent variables used in the minimization function. Initially the values assigned to the variables are randomly selected from a wide range of values. The cost function corresponding to the selected values are treated as the energy of the current state. Searching the values from the wide range of the values can be compared with the particles flowing in the solid body when it is kept in high temperature.
The next energy state of the particles is obtained when the solid body is slowly cooled. This is equivalent to randomly selecting next set of the values. When the solid body is slowly cooled, the particles of the body try to reach the lower energy state. But as the temperature is high, random flow of the particles still continues and hence there may be chance for the particles to reach higher energy state during this transition. Probability of reaching the higher energy state is inversely proportional to the temperature of the solid body at that instant. The values are randomly selected so that cost function of the currently selected random values is minimum compared with the previous cost function value. At the same time, the values corresponding to the higher cost function compared with the previous cost function are also selected with some probability. The probability depends upon the current simulated temperature ‘T’. If the temperature is large, probability of selecting the values corresponding to higher energy levels are more. This process of selecting the values randomly is repeated for a finite number of iterations. The values obtained after the finite number of iterations can be assumed as the values with lowest energy state.

Most studies have applied a single-error criterion for representing overall performance. However, it is difficult to conduct a performance assessment for optimal control based on response error signals due to the conflicting requirements between static and dynamic responses and which may lead to a poor design with respect to other performance indexes.

In this work, both integral of absolute of errors (IAE) and integral-of-time-multiplied absolute of errors (ITAE) are taken as individual indexes to cover both static and dynamic responses. IAE and ITAE reflect the transient and steady-state characteristics of the control system, respectively. They have a significant insight about the system response because the mere observations of response curves are not sufficient to evaluate the control system performance. It is noted that large errors lead to large values for IAE, while ITAE penalizes heavily errors that occur late in time.

The optimization problem is to estimate the best values for the input scaling factors (‘G_u’ and ‘G_Δec’) as well as the output scaling factor (G_Δ) of the Fuzzy-PID-SAO controllers such that the cost function f(G_u, G_Δec, G_Δ) is minimized where ‘G_u’ varies from ‘G_u min’ to ‘G_u max’, ‘G_Δec’ varies from ‘G_Δec min’ to ‘G_Δec max’, and ‘G_Δ’ varies from ‘G_Δ min’ to ‘G_Δ max’.

Suppose J_f is the overall performance index of the system.

$$J_f = f(G_u, G_\Delta e, G_\Delta) = w_1 \times IAE + w_2 \times ITAE$$  \hspace{1cm} (8)

Where w1 and w2 are weighting factors used in the case that indexes have large difference in their magnitudes; and it can also emphasize some specific performance indexes over others. It should be noted that the smaller the value of J_f, the better the performance.

Thus, the SAO can be summarized as follow:

**Step 1:** Initialize the value of the temperature ‘T’.

**Step 2:** Randomly select the current values for the variables ‘G_u’, ‘G_Δec’ and ‘G_Δ’ from their allowable ranges. Let them be ‘G_u min’, ‘G_Δec min’, and ‘G_Δ min’, respectively.

**Step 3:** Compute the corresponding cost function value f(G_u, G_Δec, G_Δ).

**Step 4:** Randomly select the next set of values for the variables ‘G_u’, ‘G_Δec’ and ‘G_Δ’ from their allowable ranges. Let them be ‘G_u new’, ‘G_Δec new’, and ‘G_Δ new’ respectively.

**Step 5:** Compute the corresponding cost function value f(G_u new, G_Δec new, G_Δ new).

**Step 6:** If f(G_u, G_Δec, G_Δ) <= f(G_u new, G_Δec new, G_Δ new), then the current values for the random variables G_u = G_u new, G_Δec = G_Δec new and G_Δ = G_Δ new.

**Step 7:** If f(G_u, G_Δec, G_Δ) > f(G_u new, G_Δec new, G_Δ new), then the current values for the random variables G_u = G_u new, G_Δec = G_Δec new and G_Δ = G_Δ new are assigned when

$$exp [(f(G_u, G_\Delta e, G_\Delta) - f(G_u new, G_\Delta e new, G_\Delta new)) / T] > rand$$

Note that when the temperature ‘T’ is less, the probability of selecting the new values as the current values is less.

**Step 8:** Reduce the temperature T = r X T, where r is a scaling factor varying from 0 to 1.

**Step 9:** Repeat **Step 3** to **Step 8** for n times until ‘T’ reduces to the particular percentage of initial value assigned to ‘T’.

4. Results

Both PID-SAO and Fuzzy-PID-SAO controllers are applied to the three-induction motors drive system. The SAO algorithm given in this paper is achieved by using Matlab software. Figures (4) and (5) show how SAO optimizer is utilized on-line to adjust the controllers tuning parameters. The SAO, on-line, optimizes the controller parameters of both the PID-SAO controllers (k_p, T_f and T_i) as well as the Fuzzy-PID-SAO controllers (input and output SF’s) to enhance the controller response. Hence, the maximum degree of system stability is obtained by
solving the minimize optimization problem via SAO. Ziegler-Nichols tuning method is utilized as an initial guess for the PID controller parameters.

In order to show the validity of the proposed Fuzzy-PID-SAO controllers, the performances of the proposed controllers are compared to the performances of the PID-SAO controllers in terms of integral absolute error (IAE) and integral time-multiplied absolute error (ITAE). To test the robustness of the proposed system, several simulation cases are considered to evaluate the performance of both PID-SAO and Fuzzy-PID-SAO controllers.

Case (1): A step sudden increase in speed reference from 300 to 500 r/min at the tenth second is given in Fig. 7-a. The speed Fuzzy-PID-SAO controller shows a faster response and better measure for IAE and ITAE compared to PID-SAO controller. In the other hand, the two-tension Fuzzy-PID-SAO controllers for F12 and F23 give robust and faster responses compared to PID-SAO controllers as shown in Fig. 7(b-c). Referring to the results shown in table (2), IAE measures for speed and tensions F12 and F23 Fuzzy-PID-SAO controllers have been reduced by 45.9%, 44.38% and 44.6%, respectively compared to PID-SAO controller. ITAE measures for speed and tensions F12 and F23 Fuzzy-PID-SAO controllers have been reduced by 47.21%, 44.3% and 44%, respectively compared to SAO-PID controller.

Case (2): A step sudden decrease in speed reference from 500 to 300 r/min at the tenth second is given in Fig. 8-a. Again, the speed and tension Fuzzy-PID-SAO controllers show faster responses and give better measures for IAE and ITAE compared to PID-SAO controller as shown in Figures 8(b-c). Table (2) shows that IAE measures for speed and tensions F12 and F23 Fuzzy-PID-SAO controllers have been reduced by 45.9%, 44.84% and 44.53%, respectively compared to PID-SAO controller. Moreover, ITAE measures for speed and tensions F12 and F23 Fuzzy-PID-SAO controllers have been reduced by 47.69%, 44.7% and 44.64%, respectively compared to SAO-PID controller.

Case (3): A step sudden change in tension F12 reference from 20 to 25 r/min at the tenth second is given in Fig. 9-a. The speed and two tensions Fuzzy-PID-SAO controllers show faster responses and give enhanced measures for IAE and ITAE compared to PID-SAO controller as shown in Figures 9(b-c).

Table (2) shows that IAE measures for speed and tensions F12 and F23 Fuzzy-PID-SAO controllers have been reduced by 45.88%, 44.5% and 44.4%, respectively compared to PID-SAO controller. In addition, ITAE measures for speed and tensions Fuzzy-PID-SAO controllers have been reduced by 72.8%, 44.5% and 44.4%, respectively compared to SAO-PID controller.

Case (4): A step sudden decrease in tension F23 reference from 30 to 25 r/min at the tenth second is given in Fig. 10-a. The speed and tension Fuzzy-PID-SAO controllers show faster responses and better measure for IAE and ITAE compared to PID-SAO controller as shown in Figures 10(b-c). Table (2) shows that IAE measures for speed and tensions F12 and F23 Fuzzy-PID-SAO controllers have been reduced by 45.88%, 40.53% and 36.64%, respectively compared to PID-SAO controller. Also, the ITAE measures for speed and tensions F12 and F23 Fuzzy-PID-SAO controllers have been reduced by 72.76%, 40.69% and 36.23%, respectively compared to SAO-PID controller.

<table>
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<th>ITAE</th>
<th>Response</th>
</tr>
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<td>458</td>
<td>Speed</td>
</tr>
<tr>
<td></td>
<td>Fuzzy-PID-SAO</td>
<td>58.93</td>
<td>241.8</td>
<td>F12</td>
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<td>3.38</td>
<td>2.98</td>
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</tr>
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<td>1.66</td>
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<td>3.25</td>
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<td>1.85</td>
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Fig. 7 - a Speed responses

Fig. 7 - b Tension (F12) responses

Fig. 7 - c Tension (F23) responses

Fig. 7. Speed and tension responses with a sudden increase in speed

Fig. 8 - a Speed responses

Fig. 8 - b Tension (F12) responses

Fig. 8 - c Tension (F23) responses

Figure (8-c) Tension (F23) responses

Fig. 8. Speed and tension responses with a sudden decrease in speed

Fig. 9 - a Speed responses

Fig. 9 - b Tension (F12) responses

Fig. 9 - c Tension (F23) responses

Fig. 9. Speed and tension responses with a sudden increase in tension (F12)

Fig. 10 - a Speed responses

Fig. 10 - b Tension (F12) responses

Fig. 10 - c Tension (F23) responses

Fig. 10. Speed and tension responses with a sudden decrease in tension (F23)
5. Conclusion
In this paper, a robust Fuzzy-PID control using Simulated Annealing Optimization (SAO) has been firstly introduced to multi-motor drive system. A mathematical model of the three-motor drive system is deduced by considering the first motor as a main and the others as supplement motors. The proposed Fuzzy-PID-SAO controller is applied to the three-motor system using SAO technique to adjust the input and output scaling factor (SF’s) by minimizing both integral of absolute of errors (IAE) and integral of time multiplied absolute of errors (ITAE) as performance measures. To test the robustness of the proposed controllers, several simulation cases are studied and presented. In all cases, the proposed controller introduces excellent, robust and faster performance compared to PID-SAO controller.

References