ASYMPTOTIC STABILITY OF A SPEED VECTOR CONTROL SYSTEM FOR AN INDUCTION MOTOR THAT CONSTAINS IN IT’S LOOP A GOPINATH OBSERVER

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Abstract: In this paper we analyze the asymptotic stability of a vector control system for a squirrel-cage induction motor that contains in its loop a Gopinath observer. The studied control system is based on the direct rotor flux orientation method (DFOC) and the stability study is based upon the linearization theorem around the equilibrium points of the control system, emphasizing the estimated variation domain of the rotor resistance for which the control system remains asymptotically stable when the prescribed speed of the control system is close to zero. The stability study is made in both the continual and discrete cases. The mathematical model of the vector regulating system is made using a value $d\lambda_c - q\lambda_c$ linked to stator current.

In order to mathematically describe the DFOC control system we will consider the following hypotheses: the static frequency converter (CSF) is assumed to contain a tension inverter; the static frequency converter is considered ideal so that the vector of the command measures is considered to be the entry vector of the induction motor; the dynamic measure transducers are considered ideal; the system and axis transformation blocks are considered dynamically ideal; the mathematical model of the vector control system will be written in an $d\lambda_c - q\lambda_c$ axis reference bounded to the stator current.

1. Introduction
This paper approaches a difficult problem within vector driving systems for induction motors namely the asymptotic stability study in a Lyapunov manner. The difficulty is brought by the mathematical model of the non-linear analyzed system that makes the Lyapunov stability analysis methods difficult to apply. The novelty of the paper consists in obtaining the method and the form of the linerized mathematical model on which the analysis of the asymptotic stability is made. The shape and structure of the mathematical model depends on the actual components within the analyzed control system and the way in which the state values are selected. The paper describes the analysis of the asymptotic stability of a vector control system for a squirrel-cage induction motor with contains in its loop a Gopinath observer. The analyzed control system is orientated according to the estimated rotor flux by the Gopinath observer.

We have realized the stability analysis for low speeds, emphasizing the influence of the identified rotor resistance within the stability of the control system. We have obtained the variation range of the identified rotor resistance for which the control system remains asymptotically stable.

2. The Mathematical Description of the Vector Control System
In order to mathematically describe the DFOC control system we will consider the following hypotheses:

- The static frequency converter (CSF) is assumed to contain a tension inverter.
- The static frequency converter is considered ideal so that the vector of the command measures is considered to be the entry vector of the induction motor.
- The dynamic measure transducers are considered ideal.
- The system and axis transformation blocks are considered dynamically ideal.
- The mathematical model of the vector control system will be written in an $d\lambda_c - q\lambda_c$ axis reference bounded to the stator current.

Based on these hypotheses the block diagram of the direct vector control system that contains a Gopinath type rotor flux estimator in its loop is presented within Figure 1 and the DFOC field orientation block within Figure 2.

Some of the equations that define the vector control system are given by the elements which compose the field orientation block and consist of:

- Couple PI regulator ($PI_{M_e}$) defined by the $K_M$ proportionality constant and the $T_M$ integration time;
• PI flux regulator (PI_\phi) defined by the K_\phi proportionality constant and the T_\phi integration time;
• mechanical angular speed PI regulator (PI_\omega) defined by the K_\omega proportionality constant and the T_\omega integration time;
• current PI regulator (PI_I) defined by the K_i proportionality constant and the T_i integration time;
• stator tensions decoupling block (C1Us);
• Flux analyzer (AF).

The input vector of the control system will be
\[ u = [u_1 \quad u_2 \quad u_3]^T \] (2)
where \( u_1 = y_{\phi r} \); \( u_2 = \omega_{\phi r} \); \( u_3 = M_\omega \).

Under these circumstances the 12 differential equations system that define the mathematical model of the vector regulating system can be written as follows
\[ \frac{dx(t)}{dt} = f(x(t), u(t)) \] (3)
where \( f(x, u) = \left[ f_1(x, u), \ldots, f_{12}(x, u) \right] \) and the \( f_i = f_i(x, u) \) functions are:

\[ f_1(x, u) = a_{11} \cdot x_1 + \left( z_p \cdot x_5 + a_{31} \cdot \frac{x_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} \right) \cdot x_2 + \frac{a_{13} \cdot x_3 + a_{14} \cdot z_p \cdot x_5 \cdot x_4 + b_{11} \cdot u_{\phi de}}{\sqrt{\lambda_1^2 + \lambda_2^2}} \] (4)

\[ f_2(x, u) = -a_3 \cdot x_3 + a_4 \cdot z_p \cdot x_5 \cdot x_4 + b_{11} \cdot u_{\phi de} \] (5)

\[ f_3(x, u) = a_{31} \cdot x_1 + a_{33} \cdot x_3 + a_{35} \cdot \frac{x_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} \cdot x_4 \] (6)

\[ f_4(x, u) = a_{31} \cdot x_2 - a_{31} \cdot \frac{x_2}{\sqrt{\lambda_1^2 + \lambda_2^2}} \cdot x_3 + a_{33} \cdot x_4 \] (7)

\[ f_5(x, u) = \frac{K_m}{T_\omega} \cdot \left( x_3 \cdot x_2 - x_4 \cdot x_1 - K_{m2} \cdot x_3 - K_{m3} \cdot u_3 \right) \] (8)

\[ f_6(x, u) = u_1 - \sqrt{\lambda_1^2 + \lambda_2^2} \] (9)

\[ f_7(x, u) = K_{M} \cdot \left( x_6 + K_{M} \cdot \left( u_2 - x_5 \right) - K_{M} \cdot x_2 \cdot \sqrt{\lambda_1^2 + \lambda_2^2} \right) \] (10)

\[ f_8(x, u) = u_2 - x_5 \] (11)

\[ f_9(x, u) = \frac{K_{M}}{T_\phi} \cdot \left( x_6 + \frac{2}{\sqrt{\lambda_1^2 + \lambda_2^2}} \right) - x_1 \] (12)

\[ f_{10}(x, u) = \frac{K_{M}}{T_\phi} \cdot \left( x_6 + K_{M} \cdot \left( u_2 - x_5 \right) - K_{M} \cdot x_2 \cdot \sqrt{\lambda_1^2 + \lambda_2^2} \right) \] (13)

The other equations that compose the mathematical model of the speed control system are those given by the relations that define the stator currents – rotor fluxes mathematical model of the induction motor as well as the equations that define the mathematical model of the observer for the Gopinath type rotor flux.

All these expressions can be put together as a 12 differential equations system with 12 unknown values. In order to offer a coherent presentation of this differential equations system, we have used the following notations:

The state vector of the control system will be
\[ x = [x_i]_{i=1,12} \] (1)
where \( x_1 = i_{\phi de} \); \( x_2 = i_{qr} \); \( x_3 = \omega_{dr} \); \( x_4 = \omega_{qr} \); \( x_5 = \omega_r \); \( x_11 = \omega_{dr} \); \( x_12 = \omega_{qr} \).
3. The Asymptotic Stability Study of the Control System

In order to realize the analysis of the asymptotic stability we will consider an induction motor that has the following characteristics:

- **electrical parameters**
  \[ R_s = 0.371 \, \Omega ; R_r = 0.415 \, \Omega ; L_s = 0.08694 \, [H] ; \]
  \[ L_r = 0.08762 \, [H] ; L_m = 0.08422 \, [H] . \]

- **mechanical parameters**
  \[ z_p = 2 ; J = 0.15 \, [kg \cdot m^2] ; F = 0.005 \, \left[ \frac{N \cdot m \cdot s}{rad} \right] . \]

On the other hand, following the automated regulators tuning within the speed control system we obtain the following constants:

\[ K_V = 501.3834 ; T_V = \frac{K_V}{2374.7} ; K_t = 5.9881 ; T_t = \frac{K_t}{754.4176} \]
\[ K_M = 10.1988 ; T_M = \frac{K_M}{1020} ; K_{\omega} = 10 ; T_{\omega} = \frac{K_{\omega}}{350} \]

Under these circumstances by imposing the entry vector of the control system to be of the following type

\[ u_1 = \psi_r = 0.69 \, [Wb] ; u_2 = \omega_e \, m = n_m \cdot \frac{\pi}{30} \, \left[ \frac{rad}{s} \right] ; \]
\[ u_3 = M_r = M_N = 93.269 \, [N \cdot m] \quad (16) \]

where \( n_m = 100 \cdot \frac{m}{rpm} \) with \( m = 0.15 \) and the proportionality coefficient between the self values of the motor and those of the observer being \( k = 0.3 \) by solving the non-linear system

\[ f(x, u) = 0 \quad (17) \]

using the Newton method having as the start point the vector:

\[ x^* = [0 \quad 0 \quad u_1 \quad u_2 \, m \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad u_0 \quad 0] \quad (18) \]

we obtain an equilibrium point \( x_m = b_m \) where:

\[ b_m = [b_m]_{i=1}^{12} . \]

From those stated above by the linearization of the system (3) around the equilibrium point \( x_m = b_m \) obtained for an \( u = [u_1 \ u_2 \ u_3]^T \) entry vector defined by (16) we obtain:

\[ \Delta x(t) = A_L \cdot \Delta x(t) + B_L \cdot \Delta u(t) \quad (19) \]

where \( A_L, B_L \) matrices are:

\[ A_L = \left[ \frac{\partial f_i}{\partial x_j} (b_m u^*) \right]_{i=1}^{12} \quad ; \quad B_L = \left[ \frac{\partial f_i}{\partial u_j} (b_m u^*) \right]_{j=1}^{12} \quad (20) \]

Next, in order to study the asymptotic stability of the equilibrium points \( x_m = b_m \) the self values of the \( A_L \) matrix will be analyzed, so that if they have a strictly negative real part the \( x_m = b_m \) equilibrium point is asymptotically stable for the linearized system (19).

Under these circumstances according to the linearization theorem [6] in a vicinity of the equilibrium...
point \( x_m = b_m \) the non-linear system (3) is asymptotically stable.

As the self values of the \( A_L \) matrix are presented within Figure 3, it results that the equilibrium points \( x_m = b_m \) of the linearized system (19) are asymptotically stable and according to the linearization theorem the equilibrium points \( x_m = b_m \) are asymptotically stable in certain vicinity for the non-linear system (3).

Also in figure 3 for a more in depth study of the asymptotic stability of the (3) system we emphasized self values of the control system that does not contain a Gopinath observer.

From those presented above one may notice that the self values consist of the self values of the control system that does not contain it as well as the self values of the Gopinath observer.

The only self values that modify when a new entry vector is imposed are the self values of the Gopinath observer.

One may notice that inclusion of the Gopinath observer does not alter the self values of the control system that does not contain it.

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Figure 3 The self values of the matrix \( A_L \); \( M_x = M_N \)

For a more detailed study of the asymptotic stability, next we will present the \( A_L \) matrix self values under identical testing conditions with those above with the difference that \( u = [u_1, u_2, u_3]^T \) defined by (16), namely for a speed of \( n_0 = 0 \text{ [rpm]} \).

Based on this study we can obtain information related to the range of variation of the estimated rotor resistance for which the non-linear system (3) remains asymptotically stable around the \( x_0 = h_0 \) equilibrium point.

As the self values of the \( A_L \) matrix are presented in Figure 5, it results that the non-linear system (3) around the equilibrium point \( x_0 = h_0 \) is asymptotically stable for an estimated rotor resistance in range of\( D_1 = \left\{ R_s^*; 0.91 \cdot R_s \leq R_s^* \leq 1.48 \cdot R_s \right\} \), becoming asymptotically unstable for an estimated rotor resistance in range of\( D_2 = \left\{ R_s^*; 1.48 \cdot R_s < R_s^* < 0.91 \cdot R_s \right\} \).

Figure 4 The self values of the matrix \( A_L \); \( M_x = 0 \)

Next, the influence of the estimated rotor resistance will be emphasized, when studying asymptotic stability of the equilibrium point \( x_0 = h_0 \) of the non-linear system (3) in a certain vicinity of this point.

The equilibrium point is obtained for an entry vector like \( u = [u_1, u_2, u_3]^T \) defined by (16), namely for a speed of \( n_0 = 0 \text{ [rpm]} \).

Based on this study we can obtain information related to the range of variation of the estimated rotor resistance for which the non-linear system (3) remains asymptotically stable around the \( x_0 = h_0 \) equilibrium point.

As the self values of the \( A_L \) matrix are presented in Figure 5, it results that the non-linear system (3) around the equilibrium point \( x_0 = h_0 \) is asymptotically stable for an estimated rotor resistance in range of\( D_1 = \left\{ R_s^*; 0.91 \cdot R_s \leq R_s^* \leq 1.48 \cdot R_s \right\} \), becoming asymptotically unstable for an estimated rotor resistance in range of\( D_2 = \left\{ R_s^*; 1.48 \cdot R_s < R_s^* < 0.91 \cdot R_s \right\} \).

Figure 5 \( A_L \) self values by rotor resistance for \( n_0 = 0 \text{ [rpm]} \) \( M_x = M_N \)
Next the influence of the estimated rotor resistance to the asymptotic stability of the non-linear system (3) will be presented, when the entry vector is \( u = [u_1 \quad u_{20} \quad u_3]^T \) defined by (16) with the difference that \( u_3 = M_r = M_N = 0 \) [N\( \cdot \)m]. The self values of the \( A_L \) matrix are presented in Figure 6.

As the self values of the \( A_L \) matrix are presented in Figure 6, it results that the non-linear system (3) around the equilibrium point \( x_0 = b_0 \) is asymptotically stable for an estimated rotor resistance in range of \( D_1 = \left\{ R^*_r ; 0 \leq R^*_r \leq 2.61 \cdot 10^4 \cdot R_r \right\} \) becoming asymptotically unstable for an estimated rotor resistance in range of \( D_2 = \left\{ R^*_r ; 2.61 \cdot 10^4 \cdot R_r < R^*_r < 0.1 \cdot R_r \right\} \).

On the other hand in case we realize the digitization of the linear system (19) we get:

\[
x(k+1) = F_L \cdot x(k) + H_L \cdot u(k)
\]

where: the \( F_L, H_L \) matrices are obtained from the \( A_L, B_L \) matrices using one of the two digitization types:

- simplified digitization:
  \[
  F_L = I_{12} + A_L \cdot T ; H_L = B_L \cdot T
  \]

- complete digitization:
  \[
  F_L = I_{12} + A_L \cdot T + A_L^2 \cdot \frac{T^2}{2} ; H_L = B_L \cdot T + A_L \cdot B_L \cdot \frac{T^2}{2}
  \]

where: \( T \) is the sampling time.

Proceeding in a similar manner the self values of the \( F_L \) matrix in case the entry vector is defined by (16) and the \( F_L \) matrix is obtained by using simplified digitization using a \( T = 53.3 \) [\( \mu \)sec] sampling time are graphically presented in Figure 7.

From the figure above one may notice that the \( x_m = b_m \) equilibrium points are asymptotically stable for the discrete system (20) in case the \( F_L \) matrix is obtained through using the simplified digitization method.

This conclusion also remains valid when the \( F_L \) matrix is obtained through the complete digitization method both where the resistant couple is present or absent.

Next we will present the influence of the estimated rotor resistance over the asymptotic stability of the discrete non-linear system when the entry vector is defined by (16). This study was conducted for a sampling time of \( T = 53.3 \) [\( \mu \)sec] in case of simplified digitization. After the study we obtained the information that the \( x_0 = b_0 \) equilibrium point remains asymptotically stable for the (21) linear system for a variation of identified rotor resistance ranging from \( D_1 = \left\{ R^*_r ; 0.91 \cdot R_r \leq R^*_r \leq 1.48 \cdot R_r \right\} \).

The \( x_0 = b_0 \) equilibrium point becomes asymptotically unstable when the variation of the identified rotor resistance varies from \( D_2 = \left\{ R^*_r ; 1.48 \cdot R_r < R^*_r < 0.91 \cdot R_r \right\} \).

When the study of the influence of identified rotor resistance variation is done for an entry vector of the \( u = [u_1 \quad u_{20} \quad u_3] \) type defined by (16) with the difference that \( u_3 = M_r = M_N = 0 \) [N\( \cdot \)m] then the stability domain for which the \( x_0 = b_0 \) equilibrium point is asymptotically stable is \( D_1 = \left\{ R^*_r \leq R^*_r \leq 30.6 \cdot 39 \cdot R_r \right\} \) and the instability domain is \( D_2 = \left\{ R^*_r ; 30.6 \cdot 39 \cdot R_r < R^*_r < R_r \right\} \).

On the other hand the domains that define the upper and lower limits of variation of identified rotor resistance...
for which the $x_0 = b_0$ equilibrium point remains asymptotically stable for the (21) discrete linear system diminishes with the increase of the sampling time.

4. Conclusions

This paper presents analytically in a single form the mathematical model of the speed vector-controlled system with a Gopinath observer, suitable for stability analysis. The mathematical model is written in an $d\lambda_e - q\lambda_e$ orientation value linked to the stator current vector. This mathematical model allows for both the internal asymptotic study as well as the external stability of the control system.

The study helped us to conclude that the analyzed control system is asymptotically stable for the imposed speed range $(0...1500) \text{ [rpm]}$ both, when a resistant couple is lacking or present as $M_r = M_N$. This conclusion is valid in both the continual and discrete cases where the sampling time is $T = 53.3 \text{ [µsec]}$.

From these presented above we can say that the Gopinath observer adds self values to the control system that does not contain it in its loop without modifying the other self values. This conclusion is only valid when the matrices that are the basis of projecting the Gopinath observer are the same size with those of the induction motor.

In this paper we determined the upper and lower variation limits for the identified rotor resistance for which the $x_0 = b_0$ equilibrium point remains asymptotically in both the discrete and continual cases.

The following information emphasized within Table 1 for the continual case and Table 2 for the discrete case.

**Table 1**

<table>
<thead>
<tr>
<th>Continual case</th>
<th>$M_r = M_N$</th>
<th>$M_r = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LI</td>
<td>$0.91 \cdot R_r$</td>
<td>$0.1 \cdot R_r$</td>
</tr>
<tr>
<td>LS</td>
<td>$1.48 \cdot R_r$</td>
<td>$2.61 \cdot 10^6 \cdot R_r$</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>Discrete case</th>
<th>$M_r = M_N$</th>
<th>$M_r = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LI</td>
<td>$0.91 \cdot R_r$</td>
<td>$0.1 \cdot R_r$</td>
</tr>
<tr>
<td>LS</td>
<td>$1.48 \cdot R_r$</td>
<td>$306.39 \cdot R_r$</td>
</tr>
</tbody>
</table>

Where, through LI we noted the lower limit and through LS we noted the upper limit of variation of identified rotor resistance.

From the tables above one may notice that the upper limit for the discrete case decreases compared to the upper limit of the continual case when the resistant couple is null. The upper limit stays the same in both the discrete and continual cases when $M_r = M_N$.

The lower limit for the discrete case is the same as the lower limit of the continual case in both the lack or presence of the resistant couple.

Also, from the tables above one may notice that the upper limit in case of lacking resistant couple increases compared to the upper limit in case of having resistant couple and the upper limit in case of lacking the resistant couple decreases compared to the lower limit in case of having resistant couple. This conclusion remains valid for both, the continual and discrete cases.

References


