ACCOUNTING OF DC-DC POWER CONVERTER DYNAMICS IN DC MOTOR VELOCITY ADAPTIVE CONTROL

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Abstract: This paper deals with the problem of velocity control of a DC motor taking account the dynamics of the DC-DC power converter, supposed here of Buck type. We will develop for the global system, constituted by combined DC-motor and Buck-converter, a model of fourth order. On the basis of this model a regulator is designed using the backstepping technique. The control purpose is, on one hand, asymptotic stability of the closed-loop system and, on the other hand, perfect tracking of the reference signal (the machine speed). Both non adaptive and adaptive versions are designed and shown to yield quite interesting performances. A theoretical analysis shows that both controllers meet their objectives. These results are confirmed by experimental results which, besides, show that the adaptive version deals better with load torque changes.

Key words: DC-motor, Buck converter, Backstepping, Lyapunov stability.

1. Introduction
DC machines are extensively used in many industrial applications such as servo control and tractions tasks due to their effectiveness, robustness and the traditional relative ease in the devising of appropriate feedback control schemes [4], specially those of the PI and PID types ([3], [5], [8]).

Therefore, the problem of velocity control of DC-motors has been extensively dealt in the specialized literature during the last years ([1], [2], [6],[7]). However, in the most works, the dynamics of DC power converter has generally been ignored when analyzing the resulting closed-loop systems. Indeed, the converter is generally considered transparent as being assimilated to the proportional gain. Moreover, this simplification can be tolerated for the high powers, but not for the low and the medium powers.

Such a simplification can only limit performances of the regulator and can even compromise the global stability of the system in closed loop. Indeed, a dynamics non taken in account in the design of the regulator can generate a reduction of the phase margin and in turn produces the instability of the system.

In this paper, we propose an adaptive control of DC motor velocity taking account the dynamics of DC-DC power converter of the Buck type. The converter input (duty ratio) is designed so that a smooth trajectory is followed by the motor angular velocity.

The proposed controller, based on the backstepping approach, is shown to achieve a good stability and perfect tracking objectives under constant but unknown load torque.

The paper is organized as follows: in section 2 the combined DC-motor and Buck converter model is described. Section 3 and Section 4 are devoted to the controller theoretical design; the controller stability and tracking performance are illustrated in section 5. A conclusion and reference list end the paper.

2. The combined DC Motor-Buck Converter Model
Consider a permanent magnet motor with its armature circuit loaded to a DC-DC power converter of Buck type as shown in Figure 1. Such a configuration constitutes a single quadrant angular velocity control configuration for a DC drive. The mathematical model of the composite system is given by:

Fig. 1: The combined DC-motor- “Buck” converter model
3. Non Adaptive Controller Design

The aim is to directly enforce the average angular velocity \( x_1 \) to track a given reference signal \( \omega_r(t) \), when the model parameters are perfectly known. The reference signal and its four first derivatives are assumed to be known, bounded, and piecewise continuous. Following closely the backstepping design [10], the controller is designed in four steps, since the control input appear after four derivatives of \( x_1 \). The control law is summarized in Table 1.

\[
\begin{align*}
\mu &= (JL_mLC / KE)(((b_1f / J - b_2L_m)/x_1) \\
&- (b_1K / J + b_4R_m / L_m + b_2 / (L_mC)x_2) \\
&+ (K / (JL_mC)) + b_4 / (L_mC)x_3 + b_2x_4 / (L_mC) + b_7T_e / J \\
&+ c_1^4 - 3c_1^2 - c_2^2 - 2c_1c_2 + 2) \xi_3 \\
&+ (-c_1^3 - c_2^2 - c_3 + 3c_1 + 4c_2 + 3c_3) \xi_2 \\
&+ (c_1^2 + c_2^2 + c_3 + c_1c_3 + c_2c_3 - 3) \xi_3 \\
&- (c_1 + c_2 + c_3) \xi_4 + \omega_r^{(4)} + c_4 \xi_4 
\end{align*}
\]

where
\[ b_1 = b_1f / J + b_2K / L_m \]
\[ b_4 = b_1K / J - b_2R_m / L_m + K / (JL_mC) \]

Error variables:
\[ z_1 = x_1 - \omega_r \]
\[ z_2 = K\omega_r / J - \alpha_1 \]
\[ z_3 = K\omega_r / (JL_mC) - \alpha_2 \]
\[ z_4 = K\omega_r / (JL_mC) - \alpha_3 \]

Stabilizing functions
\[ \alpha_1 = f_1 / J + T_e / J + \dot{\alpha}_1 - c_1z_1 \]
\[ \alpha_2 = (K^2 / (JL_m)) - (f / J)^2) \xi_1 \]
\[ + (KR_m / (JL_m) + fK / J) \xi_2 \]
\[ - JF / J^2 + c_2^2 - c_1z_2 - c_1\dot{z}_1 - 2z_1 - c_1z_2 \]
\[ + c_1 - c_1 + c_2 + c_3 + \dot{z}_3 + \ddot{\omega}_r \]

where
\[ b_1 = K^2 / (JL_m) - (f / J)^2 \]
\[ b_2 = KR_m / (JL_m) + fK / J \]

Control law:
\[ \mu = (JL_mLC / KE)((b_1f / J - b_2L_m)/x_1) \\
- (b_1K / J + b_4R_m / L_m + b_2 / (L_mC)x_2) \\
+ (K / (JL_mC)) + b_4 / (L_mC)x_3 + b_2x_4 / (L_mC) + b_7T_e / J \\
+ c_1^4 - 3c_1^2 - c_2^2 - 2c_1c_2 + 2) \xi_3 \\
+ (-c_1^3 - c_2^2 - c_3 + 3c_1 + 4c_2 + 3c_3) \xi_2 \\
+ (c_1^2 + c_2^2 + c_3 + c_1c_3 + c_2c_3 - 3) \xi_3 \\
- (c_1 + c_2 + c_3) \xi_4 + \omega_r^{(4)} + c_4 \xi_4 \]

where
\[ b_1 = b_1f / J + b_2K / L_m \]
\[ b_4 = b_1K / J - b_2R_m / L_m + K / (JL_mC) \]
Proposition 1
Consider the closed loop system consisting of the subsystem (2) and the control law (13). If the angular velocity reference \( \omega_r \) and its four first derivatives are known and bounded, then the closed loop undergoes, in the \((z_1, z_2, z_3, z_4)\)-coordinates, the following equation:

\[
\begin{bmatrix}
    z_1 \\
    z_2 \\
    z_3 \\
    z_4
\end{bmatrix} =
\begin{bmatrix}
    -c_1 & 1 & 0 & 0 \\
    -1 & -c_2 & 1 & 0 \\
    0 & -1 & -c_3 & 1 \\
    0 & 0 & 0 & -1 - c_4
\end{bmatrix}
\begin{bmatrix}
    z_1 \\
    z_2 \\
    z_3 \\
    z_4
\end{bmatrix}
\]

(16)

where the design parameters \((c_1, c_2, c_3, c_4)\) are positives and freely chosen. Consequently, the error system (16) is globally asymptotically stable. It follows that:

- All signals in closed loop are bounded,
- The estimation error \( z_{\sim} = x_1 - \omega_r \) vanishes. □

Proof. Let us consider the following Lyapunov function candidate:

\[
V = \sum_{i=1}^{4} 0.5z_i^2
\]

(17)

its time derivative, using Table 1, is given by

\[
\dot{V} = -\sum_{i=1}^{4} c_i z_i^2 \leq 0
\]

(18)

This shows that the \((z_1, z_2, z_3, z_4)\)-system is globally asymptotically stable. From Table 1 we see that, since the \(c_i\)'s are smooth function, the state variables \(x_i\) can be expressed as a smooth function of \(z_1, \cdots, z_4\). Hence, the state vector \(x(t)\) is globally asymptotically stable. Furthermore, from (17) and (18), we can see that \(V \to 0\) as \(i \to \infty\), hence, all variable errors \(z_i, i=1\ldots,4\), vanish asymptotically. This ends the proof of Proposition 1.

4. Adaptive Controller Version
Controllers of section 3 guarantee perform well only when the model (2) is perfectly known. This particularly means that the load torque \(T_L\) is constant and time-invariant. When this is not the case, the controllers may still provide an acceptable behavior, in particular, the tracking error \(z_1\) is bounded but not vanishing. Therefore adaptive versions of the above controllers turn out to be interesting alternatives.

To cope with such a model uncertainty the new controller will be given a learning capacity. More specifically, the controller to be designed should involve an on-line estimation of the unknown parameter

\[
\theta = T_L / J
\]

(19)

The obtained estimate is denoted \(\hat{\theta}\), it follows that

\[
\theta = \hat{\theta} + \bar{\theta}
\]

(20)

where \(\bar{\theta}\) is the estimation error.

Following the tuning functions backstepping design [10], then the control law and the parameter update law, designed in four steps are summarized in table 2.

<table>
<thead>
<tr>
<th>Error variables</th>
<th>(z_1 = x_1 - \omega_r)</th>
<th>(z_2 = Kx_2 / J - \alpha_1)</th>
<th>(z_3 = Kx_3 / (JL_m) - \alpha_2)</th>
<th>(z_4 = Kx_4 / (JL_m C) - \alpha_3)</th>
</tr>
</thead>
</table>

(21) (22) (23) (24)

Stabilizing functions

\[
\begin{align*}
\alpha_1 &= -c_1 z_1 + f s_1 / J - w_1 \hat{\theta} + \dot{\omega}_r \\
\alpha_2 &= -z_1 - c_2 z_2 - \psi_2 - w_1 \psi_2 \\
\alpha_3 &= -z_2 - c_3 z_3 - \psi_3 - \lambda_3 \psi_3 + \nu_3
\end{align*}
\]

(25) (26) (27)

Tuning functions

\[
\begin{align*}
\tau_1 &= w_1 z_1 \\
\tau_i &= w_1 z_i + w_i z_i, \quad i = 2, \ldots, 4
\end{align*}
\]

(28) (29)

Regressor constants

\[
\begin{align*}
w_1 &= -1 \\
w_2 &= c_1 w_1 + f / J \\
w_3 &= b_1 - w_3 w_3 - w_1 \psi_2 + w_2 a_3 \\
w_4 &= b_3 + a_4 w_1 + a_3 w_2 + a_6 w_3
\end{align*}
\]

(30) (31) (32) (33)

Adaptive control law

\[
\mu = \frac{J L_m C}{J E K} \left[ z_3 - c_4 z_4 - \psi_4 - \lambda_4 \psi_4 + \nu_4 \right]
\]

(34)

Parameter update law

\[
\dot{\hat{\theta}} = \gamma W z
\]

(35)

where \(z = [z_1, z_2, z_3, z_4]^T\) and \(W = [w_1, w_2, w_3, w_4]\)
The functions and constants introduced in the above Table 2 have the following expressions

\[ \psi_1 = -b_1 x_1 - b_2 x_2 + f \dot{\theta} / J - c_1^2 z_1 + c_1 z_2 - \ddot{\omega}_r \]  

(36)

\[ \psi_2 = b_3 x_1 + b_4 x_2 - b_3 x_3 / L_m + b_5 \dot{\theta} + a_1 z_1 + a_2 z_2 + a_3 z_3 - \ddot{\omega}_r \]  

(37)

\[ \psi_3 = -(b_3 f / J + b_4 K / L_m) x_1 + (b_3 K / J - b_4 R_m / L_m + b_2 / (L_m C)) x_2 \]  

+ \( b_2 / L_m - K / (L_m C) \) x_3 - \( b_2 x_4 / (L_m C) - b_5 \dot{\theta} \)

\[ + \left[ c_1 a_4 - a_5 \left[ 1 + \omega_1^2 \right] - a_6 \lambda_3 \omega_1 \right] z_1 \]  

\[ + \left[ a_4 \left( c_2 + \omega_1 w_2 \right) - a_6 \left( c_3 + \lambda_3 \omega_2 \right) \right] z_2 \]  

\[ + \left[ a_5 \left( \sigma_23 + \omega_1 w_3 \right) - a_6 \left( c_3 + \lambda_3 \omega_3 \right) \right] z_3 \]  

\[ + a_6 z_4 - \omega_r^2 (4) \]  

(38)

\[ \lambda_3 = f / J + w_5 a_3 \]  

(39)

\[ \lambda_4 = b_1 + a_3 w_5 + a_6 \lambda_3 \]  

(40)

\[ b_1 = K^2 / (L_m C) - (f / J)^2 \]  

(41)

\[ b_2 = K R_m / (J L_m) + f K / J^2 \]  

(42)

\[ b_4 = -K / (J L_m) - b_1 K / J + b_2 R_m / L_m \]  

(43)

\[ a_1 = c_1 \left( c_1^2 - 1 - \omega_2^2 \right) - a_5 \left[ 1 + \omega_1^2 \right] \]  

(44)

\[ a_2 = -c_1^2 - 1 - \omega_2^2 + a_4 \left( c_2 + \omega_1 w_2 \right) \]  

(45)

\[ a_3 = c_1 + c_2 + \omega_1 w_2 \]  

(46)

\[ a_4 = c_1 + \lambda_3 \omega_1 \]  

(47)

\[ a_5 = 1 + a_2 + \lambda_3 \omega_2 + \sigma_23 \]  

(48)

\[ a_6 = c_3 + a_3 + \lambda_3 \omega_3 \]  

(49)

\[ \sigma_23 = w_1 \omega_2 \]  

(50)

\[ \sigma_24 = w_1 \omega_3 \]  

(51)

\[ \sigma_34 = \lambda_3 \omega_4 \]  

(52)

\[ \sigma_34 = \lambda_3 \omega_4 \]  

(53)

\[ \sigma_34 = \lambda_3 \omega_4 \]  

(54)

\[ \sigma_34 = \lambda_3 \omega_4 \]  

(55)

where \( c_1, c_2, c_3, c_4 \) and \( \gamma \) are positive design parameters freely chosen.

The main result of this subsection is summarized in the following proposition

**Proposition 2**

Consider the closed loop system (2) subject to uncertain load torque \( T_L \) and the controller composed of the adaptive control law (34) and the parameter update law (35). If the angular velocity reference \( \omega_r \) and its four first derivatives are known and bounded, then the closed loop undergoes, in the following equations:

\[ z = A_z z + W^T \hat{\theta} \]  

(56)

\[ \dot{\theta} = \gamma W z \]  

(57)

where \( A_z \) is a skew symmetric matrix defined as follows

\[
A_z = \begin{bmatrix}
-c_1 & 1 & 0 & 0 \\
-1 & -c_2 & 1 + \sigma_23 & \sigma_24 \\
0 & -1 - \sigma_23 & -c_3 & 1 + \sigma_34 \\
0 & -\sigma_24 & -1 - \sigma_34 & -c_4 \\
\end{bmatrix}
\]

(58)

Consequently, the error system (56) is globally asymptotically stable. It follows that:

i) All signals in closed loop are bounded,

ii) The tracking error \( z_1 = x_1 - \omega_r \) vanishes.

iii) The estimate parameter \( \hat{\theta} \) converges toward uncertain and constant parameter \( \theta \)

\( \square \)

**Proof.** Consider the following Lyapunov function candidate:

\[ V = 0.5 \left( z^T z + \hat{\theta}^2 / \gamma \right) \]  

(59)

its time derivative, using Table 2, is given by

\[ \dot{V} = -4 \sum_{i=1}^{4} c_i z_i^2 \leq 0 \]  

(60)

i) From (59) and (60) we can see that the equilibrium \( \{z, \hat{\theta}\} = \theta \) is globally asymptotically stable and in turn the state vector \( x(t) \) is globally asymptotically stable.

ii) From LaSalle’s Invariance Theorem [10], it further follows that the state \( \{z, \hat{\theta}\} \) converges to the largest invariant set of (56)-(57) contained in \( E = \{ z, \hat{\theta} \} \in IR^5 / z = 0 \}, that is, in the set where \( \dot{V} = 0 \). This means, in particular, that \( z(t) \) \( \to 0 \) as \( t \to \infty \).

iii) On the invariant set \( M \), we have \( z = 0 \) and \( \dot{z} = 0 \). Setting \( z = 0 \) and \( \dot{z} = 0 \) in (56) and (57) we obtain \( \dot{\theta} = 0 \) and \( W^T \hat{\theta} = 0 \). As the regressor vector is composed of constant parameters it follows that we get \( \hat{\theta} = 0 \) on \( M \), which implies that
M = \{(0,0)\} and, in particular, we have \( \dot{\theta} \to \theta \) as \( t \to \infty \).

We thus established the proof of Proposition 2.

5. Experimental Results

The backstepping no adaptive and adaptive controllers shown in Table 1 and Table 2 has been applied to the combined DC-Motor-“Buck” Converter according to the experimental setting of figure 2, where x denotes the state vector.

![Diagram](image)

Fig. 2: Experimental bench for DC-motor velocity control

Table 3 lists the numerical values for the parameters the combined system studied in this paper.

<table>
<thead>
<tr>
<th>TABLE 3: PARAMETERS OF THE COMBINED DC-MOTOR-BUCK CONVERTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = 20 \times 10^{-3} ) [H]</td>
</tr>
<tr>
<td>( K_e = 0.046 ) [V.s/rad]</td>
</tr>
<tr>
<td>( J = 7.06 \times 10^{-2} ) [kgm²]</td>
</tr>
<tr>
<td>( \omega = -2.63 \times 10^{-3} ) [H]</td>
</tr>
<tr>
<td>( E = 12 ) [V]</td>
</tr>
</tbody>
</table>

A. Non adaptive controller

Fig.3. illustrates the behavior of the controller (13) in presence of a constant reference \( \omega_r = 60 \) rad/s and a variable load torque. The value of the load torque \( T_L \) was step changed for a time period of 0.1 seconds to 50% value of its original one. The relevant design parameters have the following values: \( c_1 = 1 \times 10^3 \); \( c_2 = 1.5 \times 10^3 \); \( c_3 = 400 \); \( c_4 = 500 \). It is seen that motor velocity perfectly tracks its reference only when the load torque is equal to its nominal value. Besides, when the load torque deviate around its nominal value, it can be seen that the motor velocity deviate, in turn, around its reference.

B. Adaptive controller version

We consider now the adaptive controller (34)-(35). The reference signal value and the load torque variation are the same as in simulation A. The adaptive controller design parameters have the following values: \( c_1 = 600 \); \( c_2 = 700 \); \( c_3 = 400 \); \( c_4 = 500 \); \( \gamma = 1 \times 10^{-11} \). The corresponding performances are illustrated by Fig.4. This shows that, despite the load torque uncertainty, the controller behavior is quite satisfactory. It is worth noting that such a good behavior is preserved when facing different variations of the load torque. As can be expected, the estimation parameter \( \theta \) converges asymptotically to the uncertain value \( \theta \) as depicted in Fig.4.

Fig.5 shows that the motor velocity tracks perfectly its varying reference. The desired trajectory \( \omega_r \) is a signal switching between 40 and 50 rad/s.
C. Importance of converter dynamics

We are going to illustrate here the importance to take in account, when designing the controller, the dynamics of the converter. When the converter dynamic is not taken in account, the output voltage of the converter is \( V = \mu E \) which leads, after averaging, to the following second order model

\[
\dot{x}_1 = -(f/J)x_1 + (K/J)x_2 - T_L/J
\]
\[
\dot{x}_2 = -(K/L_m)x_1 - (R_m/L_m)x_2 + (E/L_m)\mu
\]

where we denote by \( x_1 \) and \( x_2 \), the average values of the angular velocity \( \omega \) and the dc motor armature circuit current \( i_a \), respectively.

Following closely the backstepping design procedure in [10], one gets the following regulator

\[
\mu = \left( JL_m/KE \right) \left( K^2/(JL_m) - f^2/J^2 \right) + \left( KR_m/(JL_m) + fK/J^2 \right) - fT_L/J^2 + \left[ c_1^2 - Ic_1 - c_2 \right] z_2 + \dot{\omega}_r
\]

where \( z_1 \) and \( z_2 \) are given by (4) and (5), respectively.

Regulator (62) has been applied successively to the system (2) and to the simplified model (61). The relevant design parameters have been given the following values: \( c_1 = 8 \times 10^3 \), \( c_2 = 10^4 \). Fig. 6 illustrates the tracking behaviour of two systems in presence of a constant reference \( \omega_r = 70\text{rad/s} \). As it can be seen from the figure, if the regulator (62) perfectly stabilises the simplified system (61), it is not the same way for the system (2). This clearly shows that neglecting the converter dynamics in the regulator design may lead to drastic deterioration of the closed-loop performances.

6. Conclusion

In this paper, we have dealt with the problem of DC-motor velocity control tacking account the dynamics of a switching power converter. The average controller design is elaborated via the use of the backstepping approach. In the case of perfectly known converter model, the control objective can be ensured using a backstepping non adaptive controller (13). In the case of unknown load torque, an adaptive version of the backstepping controller ((34)-(35)) has been developed to achieve the control objective. Simulation results illustrates that the adaptive controller provide excellent asymptotic stability, a perfect tracking behavior, and a good compensation of load torque changes.


