GPC/Dead-beat Controller for High Rotor Flux Response of an Induction Motor Drive with Energy Saving

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Abstract—The objective of this paper is to present an analysis of how to get high performance of an induction motor drive. Firstly, an energy saving control in steady state is developed to improve high efficiency. The rotor flux yielding maximum driving efficiency is calculated on the basis of reference torque. Secondly, in order to ensure minimum loss without degradation of the dynamic response, a generalized predictive controller with Deadbeat tuning was developed which outputs quickly and stably the rotor flux of the induction motor. Simulation results demonstrates the performance of the proposed study.

Index terms—Induction motor, generalized predictive control, deadbeat tuning, energy saving control, vector control

I. INTRODUCTION

Induction motors are receiving wide attention in industrial applications due to their large speed capability, mechanical robustness, cheapness and ease of maintenance [1-8]. It has been shown in previous studies [1,2] that electrical machines consumes about 60 % of the total consumed electrical energy, thereof, integral horse power induction motors account for 96 % of the energy consumption. This means that around 56 % of the total electrical energy is consumed by induction motors. Also about 70 % of the energy loss is dissipated in motors with a rating below 52 kW, consequently, the tendency is to concentrate the energy saving studies on induction motor drives below 52 kW rated power.

The key to solving the problem of the energy saving in induction motors is to obtain the better balance between different types of motor losses. Recently, many studies on efficiency maximization techniques have been presented and they may be divided into two categories[4] [5]: Loss model controller (LMC) and Search controller (SC). Both the two methods minimized the motor losses but in different ways. The LMC method calculates the optimum of the objective function (which is an analytical expression representing the total losses of the machine). The fast determination of the optimum variables is the merit of this method, but it is sensitive to parameter variations. Hence if the approach is not based on an online estimation of the parameters, then it is likely that the LMC method may offer only sub-optimal solution if the parameters of the machine change (due to saturation, temperature variations, skin effect, etc.). The SC technique depends on the exact measurement of the input power and converge slowly to the true optimum variables. The advantage of this approach is no-dependence on parameter variations. However, some disadvantages appear in practice, such as continuous disturbance in the torque caused by stepwise changes of the control variable, slow convergence, because it has no idea about the optimal magnitude of variables at the beginning of search process, difficulties in tuning the algorithm for a given application and inaccuracy in efficiency optimization. For these reasons, this is not a good method in industrial drives.

An exhaustive analysis of the LMC controller can be found in the literature[1-8]. In [3], the authors calculated the total power loss of the induction motor only (the inverter loss is not taken into account) and derived an optimal flux level that maximises the motor efficiency. In [8], a simple model neglecting leakage inductance has been presented and the motor model consists only of resistors reflecting core loss, rotor and stator copper losses as a function of stator current components, then, a d-axis current level is derived that minimises the total loss. Consequently, the exact loss minimization can not be achieved, especially for high speed operation (especially in area of electrical vehicles EV applications). The authors in[7] used copper and core loss only to formulate the cost function of motor loss and adjusted the optimal rotor flux under state constraints of the inverter.

Over the last decade, generalized predictive control GPC has received an increasing attention in many control applications and it has shown to be an effective strategy for high performance applications compared to some conventional control methods, with good temporal and frequency properties (small overshoot, cancellation of disturbances, good stability and robustness margins) [9] [10][11][12]. GPC is a model based method which employs receding horizon approach in order to predict future outputs. An appropriate sequence of the control signals is then calculated to reduce the tracking error by minimizing a quadratic cost function. After which only the first element of control signals is applied on the system. The process is repeated for every sample of duration, so that a new information is updated at each sample interval.
In this paper, we investigate how to obtain simultaneously a fast rotor flux response by QPC-Deadbeat controller, and high efficiency suitable for industrial applications and electric vehicles. The validity of the proposed study, which carries out the performance of the drive system will be revealed via simulation.

II. ENERGY SAVING CONTROL

A. Motor loss model

1) Core loss

The stator core loss resistance $R_p$ is connected in series with the mutual inductance and it is determined from the classical experimental no-load test data as:[3]

$$R_p = a_1 f_s + a_2 f_s^2$$  

where $a_1=0.0599$ and $a_2=0.0032$ are the coefficients of hysteresis and eddy-current loss respectively.

2) Stator copper loss

$$P_c = R_s (i_{a_c}^2 + i_{r_c}^2)$$  

3) Rotor copper loss

$$P_r = R_r (i_{a_r}^2 + i_{r_r}^2) = \frac{R_r}{(1 + \sigma_r)} \cdot i_{r_r}^2$$

4) Stray load loss

The stray-load loss $P_{st}$ is taken into account by the inclusion of an equivalent stray loss resistance $R_{st}$ in series with the stator phase resistance[5][6]:

$$P_{st} = K_{st}(K_s f_s + K_r f_r) \cdot i_{s}^2 = R_{st} \cdot i_{s}^2$$

where $K_{st}$ is the stray load constant determined from load tests as it is explained by the IEEE standard 112 B method and $a_3=0.0042, a_4=0.00027$

B. Induction motor model

A squirrel cage induction motor model used under field oriented control FOC which tackles more accurately the behaviour of the machine can be expressed in the synchronously rotating $d$-$q$ reference frame as follows:[3]:

$$V_d = \left( R_s + R_r + \frac{\sigma R_e}{1 + \sigma_r} \right) i_d + \sigma L_s \frac{di_d}{dt} + \sigma L_{st} i_{s_r}$$
$$+ \left(1 - \sigma_r\right) L_s \frac{di_{s_r}}{dt} + R_r i_{s_r}$$

$$V_q = \left( R_s + R_r + \frac{\sigma R_e}{1 + \sigma_r} \right) i_q + \sigma L_s \frac{di_q}{dt} + \sigma L_{st} i_{s_q}$$
$$+ \left(1 - \sigma_r\right) L_s \frac{di_{s_q}}{dt} + R_r i_{s_q}$$

$$T = \frac{2}{L_s} \phi_d \cdot i_q$$

where: $\sigma_i = \frac{i_q}{L_i}, \sigma_i = \frac{i_d}{L_i}, \sigma = \frac{L_m}{L_i L_r}$

$X_{d}, X_{q}$: $d$- and $q$- axes stator voltages, currents or flux

$\sigma_1, \sigma_1$: synchronous and rotor speed.

C. Inverter loss model

The inverter loss calculation is based on measurement of the diode and transistor conduction voltages and on measurements of the following switching energies: diode turn-off, transistor turn-on and transistor turn-off.

The approximate inverter loss as function of stator current is given by[7]:

$$W_{inv} = a_1 \left( i_{a}^2 + i_{r}^2 \right) + a_2 \sqrt{i_{a}^2 + i_{r}^2}$$

$a_1=0.863$ and $a_2=-5.69$ are coefficients determined by the electrical characteristics of a switching element

D. State constraints

The current ratings of the inverter are usually larger than that of the machine, to provide high acceleration torque during transients. The inverter current is assumed to be 1.5 times the machine's current ratings, so that:

$$0 \leq \sqrt{i_{a}^2 + i_{r}^2} \leq I_{max}$$

$I_{max}=1.5 \times$ machine's current ratings.

Also a hard limit may be used on the rotor flux [3][8], so that:

$$0 \leq \phi_{r,min} \leq \phi_r \leq \phi_{r,max}$$

Relation (8) means that, when the machine starts at $t=0$, the speed is initially zero so the torque is also zero, and rotor flux should be equal $\phi_{r,min}$, then when the machine starts, it has to be magnetized and must deliver a high torque value.

D. Loss minimization strategy

The total power loss $W_{tot}$ of the drive system can be expressed by:

$$W_{tot} = W_{inv} + W_{m} = P_d + P_r + P_{st} + P_{rot}$$

The total loss of the drive system as a quadratic function of the controllable currents can be represent as:

$$W_{m} = \left( R_s + R_r + \sigma R_e + \frac{R_r}{(1 + \sigma_r)} \right) i_1^2 + \sigma \sqrt{i_{a}^2 + i_{r}^2}$$

where:

$$A(\omega) = \left( R_s + R_r + \sigma R_e + \frac{R_r}{(1 + \sigma_r)} \right)$$
$$B(\omega) = \left( R_s + R_r + \sigma R_e + \frac{R_r}{(1 + \sigma_r)} \right)$$
$$C = \sigma$$
Here, mechanical loss are not considered, then, the total power loss of the drive system can be expressed also as a quadratic functions of rotor flux and torque by:

\[
W_{sw} = \left( \frac{A(\alpha)}{L_{sw}^2} \right) \phi_{r}^2 + B(\alpha) \left( \frac{T}{k} \right) \phi_{r} + C \left( \frac{\phi_{r}^2}{L_{sw}} + \frac{T^2}{k} \phi_{r}^2 \right)
\]

(12)

The skin effect in rotor bars (determined from the rotor locked test) is taken into account by the following expression:

\[
R_s(f_r) = \frac{a_e f_r}{a_0} + a_{10}
\]

(13)

where \( f_r \) is rotor frequency, and the coefficients \( a_e = 3.55, a_0 = 11.2 \) and \( a_{10} = 2.63 \) are determined from blocked rotor test. Note that the change variation of the rotor resistance caused from temperature is not considered here.

Fig. 1 shows the plot of the total power loss of the drive system including the inverter loss at constant speed (\( f_r = 50 \text{ Hz} \)) versus rotor flux.

![Graph showing total power loss of the drive system](image)

**Fig. 1: Total power loss of the drive system**

E. Optimal solution

The steady state electromagnetic torque is given by:

\[
T = \left( p \frac{L_s^2}{L_r} \right) i_{r} \phi_{r} = k i_{r} i_{m}
\]

(14)

We consider the following problem:

**Minimize**

\[
W_{sw} = A(\alpha) i_{r}^2 + B(\alpha) i_{m}^2 + C \sqrt{i_{r}^2 + i_{m}^2}
\]

(15)

Subject to the equality constraint:

\[
T = k i_{r} i_{m} = 0
\]

(16)

The cost function is defined using Kuhn-Tucker theorem as:

\[
J(i_{r}, i_{m}) = W_{sw}(i_{r}, i_{m}) + \mu (k i_{r} i_{m} - T^*)
\]

(17)

where \( \mu \) is Lagrange multiplier[7] [8]

The optimal solution is given when:

\[
\frac{\partial J}{\partial i_{r}} = \frac{\partial J}{\partial i_{m}} = \frac{\partial J}{\partial \mu} = 0
\]

\[
\frac{\partial J}{\partial i_{r}} = 2 A i_{r} + \frac{C i_{m}}{\sqrt{i_{r}^2 + i_{m}^2}} + \mu k i_{m} = 0
\]

(18)

\[
\frac{\partial J}{\partial i_{m}} = 2 B i_{m} + \frac{C i_{r}}{\sqrt{i_{r}^2 + i_{m}^2}} + \mu k i_{r} = 0
\]

(19)

\[
\frac{\partial J}{\partial \mu} = k i_{r} i_{m} - T^* = 0
\]

By combination of the above equations and after some manipulations, we can get:

\[
2 \sqrt{i_{r}^2 + i_{m}^2} (A i_{r}^2 - B i_{m}^2) = C (i_{r}^2 - i_{m}^2)
\]

(20)

The resolution of (20) is cumbersome, using an alternative formulation in terms of instantaneous position of the stator current reference space vector with respect to the \( d \)-axis of the reference frame and using the inverter current constraint (7), then stator \( d-q \) axis currents can be written as:

\[
i_{d} = I_{max} \cos \theta, \quad i_{q} = I_{max} \sin \theta
\]

(21)

Substituting (21) into (20) we can obtain the appropriate currents pair providing the minimum loss of the drive system.

\[
i_{d}^* = \frac{I_{max}}{\sqrt{2}} \left( \frac{C + 2 B I_{max} + \alpha \sqrt{T^*}}{\sqrt{C + (A + B) I_{max}^2}} \right)
\]

\[
i_{q}^* = \frac{I_{max}}{\sqrt{2}} \left( \frac{C + 2 A I_{max} + \beta \sqrt{T^*}}{\sqrt{C + (A + B) I_{max}^2}} \right)
\]

(22)

Where

\[
\alpha = \sqrt{K_{sw}}, \quad \beta = \frac{1}{\sqrt{K_{sw}}}, \quad K_{sw} = \sqrt{C + 2 B I_{max}}, \quad I_{max}^* \text{ is the boundary inverter current (1.5 times machine’s current ratings)}
\]

The optimal rotor flux corresponding to the point of maximum efficiency at the steady state is then given by:

\[
\phi_{r}^* = L_{e} \alpha(\alpha) \sqrt{T^*}
\]

(23)

The appropriate rotor flux given by (23) for various load torques is plotted in Fig. 2. It is affected by motor speed because core loss and stray load loss are taken into account. One can see that the rotor flux decreases in order to reduce core loss and stray load loss, which increases
with frequency than other loss at high speeds. The total
loss of the drive system including inverter loss is depicted
in fig. 3. One can see that total loss is also affected by
frequency variations. Fig. 4 shows an efficiency map
versus torque of the drive system including inverter loss.
It is clear that the proposed method is superior to the
conventional field oriented control method which keeps
rotor flux constant at its rated value over a wide range of
load torques.

It has mentioned in previous studies[3][6][7], that the
core losses increases at high speed and at light load
torque regions, resulting in worsening the drive system
efficiency. Consequently, the torque and speed of the
motor becomes far from the reference values and causes
an efficiency drop. For this reason a high efficiency
control with fast rotor flux control is needed. Here, a
generalized predictive control GPC with dead-beat tuning
is used, providing high dynamic torque in transients.

The transfer function between the estimated rotor flux
and the flux current component \( i_{sb} \) can be given as:
\[
\frac{\phi^e(s)}{i_{sb}(s)} = \frac{L_m}{T_s(s+1)}
\]  

(24)
The \( Z \)-transform of the process transfer function can
be derived as:
\[
\phi^e(q^{-1}) \frac{b_m q^{-1} + \cdots + b_0 q^{-n_m}}{1 + a_0 q^{-1} + \cdots + a_m q^{-n_a}} \frac{q^{-1} B(q^{-1})}{A(q^{-1})}
\]  

(25)

Then, an identification of the process is then
performed using a recursive least square RLS estimator
providing the unknown parameters \((a_0, \ldots, a_m, b_0, \ldots, b_m)\).

The \( j \)-step ahead rotor flux prediction \( \hat{\phi}^e(t+j) \) over
the costing horizons \( N_{t} \leq j \leq N_{s} \) is given by:[7][8][11][12][13]
\[
\hat{\phi}^e(t+j) = \frac{G_j(q^{-1})}{\Delta\xi_h(t+j-1)} \frac{\Delta\xi_h(t+j-1)}{\text{feed forward}} + \frac{F_j(q^{-1})}{\Delta\xi_h(t-j)} \frac{\Delta\xi_h(t-j)}{\text{feed forward}}
\]  

(26)

where \( F_j, G_j, H_j \) are polynomials obtained by solving
Diophantine equations. The predicted rotor flux values
can be expressed under matrix form as:
\[
\hat{\phi}^e = G \hat{\xi}_h + F
\]  

(27)
where:
\[
G = \begin{bmatrix}
G_{0}^{N_{s}+1} & G_{1}^{N_{s}+1} & \cdots & G_{N_{s}}^{N_{s}+1} \\
G_{N_{s}+1}^{N_{s}+1} & G_{N_{s}+2}^{N_{s}+1} & \cdots & G_{N_{s}+N_{s}}^{N_{s}+1} \\
\vdots & \vdots & \ddots & \vdots \\
G_{N_{s}+N_{s}+1}^{N_{s}+N_{s}+1} & G_{N_{s}+N_{s}+2}^{N_{s}+N_{s}+1} & \cdots & G_{N_{s}+N_{s}+N_{s}+1}^{N_{s}+N_{s}+1}
\end{bmatrix}
\]  

(28)

To achieve optimal flux current values, the GPC uses
a quadratic cost function defined as:
\[
J_{GPC} = \sum_{j=N_{t}}^{N_{s}} \left[ \phi^e(t+j) - \hat{\phi}^e(t+j) \right]^2 + 2 \sum_{j=N_{t}}^{N_{s}} \left[ \Delta\xi_h(t+j-1) \right]^2
\]  

(29)
with the assumption
\[
\Delta\xi_h(t+j) = 0 \quad \text{for } j \geq N_{t}
\]  

(30)
where:
\( \phi^e(t+j) \): rotor flux reference at \( (t+j) \)
\( \hat{\phi}^e(t+j) \): predicted output values at \( (t+j) \)
$N_1, N_2:$ are the costing horizons
$N_c:$ is the control horizon
$\lambda:$ is the control weighting factor

Minimisation of (29) gives the optimal control values $\hat{i}_{m}$, only the first value is applied on the system:

$$\frac{\partial J_{GPC}}{\partial i_m} = 0$$

$$\hat{i}_m = M(\varphi^* - F)$$

$$M = \left[ G^T G + \lambda I_{m} \right]^{-1} G^T = \begin{pmatrix} M_1 \\ \vdots \\ M_{nc} \end{pmatrix}$$ (31)

Finally, the control signal to apply on the system is:

$$i_{m(t)}(t) = i_{m(t-1)}(t-1) + M(t, \varphi^* - F)$$ (32)

The procedure repeats for the next sample intervals likewise. The dead-beat tuning imply that the predicted rotor flux $\hat{\varphi}(t+j)$ tracks its reference value $\varphi^*(t+j)$ after one sampling period.

IV. STABILITY ANALYSIS

The advantage of R.S.T polynomial structure is that these modules (R-S and T) can be computed off-line, providing a very short real-time loop and offers the possibility to analyse the stability of the controlled open loop in the frequency domain.

![Diagram](image-url)

Fig. 5: GPC Equivalent polynomial RST controller

The polynomials R-S-T can be identified by:

$$R(q^{-1}) = M_1 F$$

$$S(q^{-1}) = 1 + q^{-1} M_2 H$$

$$T(q^{-1}) = M_3 [q^{n_1} \cdots q^{n_2}]$$ (33)

The polynomial structure offers the possibility to analyse the stability of the flux loop. General characteristics can be deduced from the studies of phase and gain margins in the Bode and Nichols planes.

V. SYSTEM CONFIGURATION

The configuration of the proposed control system is shown in Fig. 6. It can be divided into two parts: the first part deals with the transient state, when the appropriate rotor flux changes quickly (load torque or speed variations), the system switches over to fast rotor flux control, the reference flux current given by (32) is used so that $\hat{\varphi}$ tracks $\varphi^*$ after one sampling period. The use of a GPC-deadbeat controller provides high dynamic torque capability needed for a variable speed control with high productivity. In the second part, when rotor speed reaches its reference, there is no need to use fast rotor flux control, so relation (22) is used to obtain the reference flux current.

![Diagram](image-url)

Fig. 6: System configuration of the proposed drive system
VI. RESULTS AND DISCUSSIONS

The system performance is examined by extensive computer simulation under different conditions such as motor speed and load torque variations. The drive system nameplate data and parameters are shown in the Appendix. The chosen example of motor speed reference and load torque trajectories is given by fig. 7. The reference speed is about 150 r/d/s applied at t=0.5s. The motor initially operates under no-load conditions, at t=2s, the load torque is stepped to 4 N.m. As can be seen on fig. 8, it is clear that the optimal rotor flux follows its reference value accordingly to the speed and torque variations. The GPC controller makes so that the output rotor flux anticipates its reference value, and we can see on fig. 9 that the GPC-Deadbeat controller leads to by far the smallest drop in rotor flux compared to any other GPC tuning.

Fig. 7: Rotor speed and load torque trajectories

Fig. 8: Rotor flux response

Fig. 9: Simulated rotor flux trajectories between t=0.45s and t=0.75s

Fig. 10: Nichols diagram of the rotor flux loop

Fig. 11: Rotor speed evolution

Fig. 12: Response of torque to load impact condition

Fig. 13: Zoom on torque response between t=0.5s and t=0.75s

Fig. 10 illustrates the stability analysis in the Nichols plane. The gain margin, phase margin and regulation bandwidth providing good stability are respectively: 10dB, 40deg and 155 r/d/s.

The mechanical parameters (speed and torque) are plotted according to fig. 11, fig. 12 and fig. 13. On these figures, we can see that motor speed and torque follows their references without steady state errors nor overshoot.
Fig. 14 depicts the allowable current boundary of the inverter, which is limited to 1.5 times the machine rated current. These results show that the reference flux current \( i_{ref} \) is calculated by GPC-Deadbeat controller when the reference rotor flux is given. Therefore, the flux response has good transient characteristics without dynamics.

Stator core resistance, stray load resistance and rotor resistance variations versus time are plotted in Fig. 15. Rotor resistance variations is only due to skin effect in rotor bars, and the temperature effect is not considered here. One can see that all these resistances changes considerably to frequency variations.

![Stator current components versus time](image)

**Fig. 14: Stator current components versus time**

![Stator core resistance, stray load resistance and rotor resistance variations versus time](image)

**Fig. 15: Stator core resistance, stray load resistance and rotor resistance variations versus time**

Finally, the efficiency of the drive system for the whole cycle is plotted on figure 16, we can see that the inverter loss does not exceed 3% of the total drive system loss. These results demonstrate that the proposed method is superior than conventional methods because it saves more energy without degradation in torque response.

![Efficiency of the whole cycle](image)

**Fig. 16: Efficiency of the whole cycle**

**VII. CONCLUSIONS**

In this paper, an induction motor model under direct field oriented control frame including core loss, stray load loss and inverter loss have been presented. Based on this model, a loss minimization algorithm was proposed by calculating the optimal rotor flux level depending on the operating conditions such as speed and torque. To maintain high efficiency without degradation of the dynamic response, a GPC-Deadbeat controller which outputs quickly the rotor flux was proposed. A selective set of simulation results have been carried out showing the high performance of the system and the validity of the proposed method which appears to be a useful method, well adapted for large application of drive systems.

**Appendix**

**Motor data:** 1.1Kw, 1500 rpm, 3.5A, 220/380V, 1.14Wb, 7Nm, \( R_s=8.1 \Omega, R_o=3.2 \Omega, L_s=L_o=0.48 H, L_m=0.45H, \) inertia=0.005Kg. m².

**REFERENCES**


