Empirical Wavelet Transform-Based Power Quality Indices for Balanced and Unbalanced Three-Phase Systems

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Abstract: Three-phase Power-Quality Indices (PQIs) can be used to quantify and hence evaluate the quality of the Electric Power Systems (EPS) waveforms. In this paper the three-phase power components definitions contained in the IEEE Standard 1459-2000 for balanced and unbalanced three-phase systems with non-sinusoidal situations are reformulated using the Empirical Wavelet Transform (EWT). EWT technique is applied on a balanced and unbalanced three-phase signals to estimate three-phase PQIs. This technique first estimates the frequency components and then adaptively tunes the wavelet and scaling function based on the boundaries to decompose the signal accurately and there by using EWT-based reformulated indices, three phase PQIs can be accurately estimated. It can be observed from the results that EWT-based PQIs for balanced and unbalanced three-phase supply are very close to the true values.

Key words: Empirical Wavelet Transform, Power Quality Indices, Balanced three-Phase system, un balanced three-Phase system,

1. Introduction

Power Quality (PQ) is defined as a combination of voltage quality and current quality [1]. Voltage quality can be defined based on how much deviations exist between the actual voltage (or current) and the ideal voltage (or current) waveforms. The ideal voltage (or current) should have sinusoidal wave shape with fixed magnitude and fixed frequency as their nominal values. Voltage and/or current deviations are considered as power quality disturbances. The PQ disturbances can be categorized into stationary and non stationary signals. The stationary signal is the one in which mean and variance do not vary with time while in the case of a nonstationary signal, either of them or both may vary with time. Since most PQ disturbances are noisy and nonstationary in nature, and advanced signal processing technique is required to accurately decompose the nonstationary power signal and determine the location of frequency components in time as well.

The traditional Power Quality Indices definitions can be found in the IEEE standard 1459-2000[2,3] based on the frequency domain approach using the Fourier Transform (FT). Frequency domain approaches suffers from high computational burden and loss of time information.

Reference [4] presents an interesting review of mostly used signal-processing techniques for the estimation of PQ indices and algorithms for classifying power disturbances. The Discrete Wavelet Transform (DWT) has been frequently used for evaluating the quality of nonstationary power signals [4].

Discrete Wavelet Transform (DWT) has been successfully used for solving PQ problems in the electric power system network [4]-[6]. Wavelet packet Transform (WPT) is a generalization of the wavelet transform that can provide a time-frequency representation of any nonstationary waveforms without loosing any time or frequency related information. WPT was proven to be very effective to analyze many power quality disturbances and accurately measure many electrical quantities. The main advantage of using WPT over other wavelet transforms is that it can provide uniform frequency bands therefore identifying more frequency components especially at high frequency in [7].

The literature survey reveals that the analysis of a signal with the discrete wavelet transform (DWT) or the Wavelet Packet Transform (WPT) requires proper selection of mother wavelet, decomposition levels, and sampling frequency. The selection of these parameters along with a suitable choice of a mother wavelet differs for the signals containing different frequency components and this limits the application of DWT and WPT to analyze real-time nonstationary signals.

To overcome these drawbacks, various adaptive techniques have been proposed, such as the S-transform and recursive Newton-type algorithm, to assess the PQ indices for stationary and nonstationary signals. In the literature, parametric high resolution
methods, such as the prony, ESPRIT, and root-MUSIC methods [8] have been proposed to calculate the PQ indices. Recent contributions have extended the concepts to hybrid methods using DFT and parametric methods [9]-[12].

Recently a new approach, empirical wavelet transform (EWT), has been proposed to build a family of adaptive wavelets capable of extracting different components of a signal. This method has an advantage of adaptability according to the analyzed signal and can isolate the different modes of the signal. An attempt has been made in this paper to utilize the self-adaptiveness of the EWT in estimating the PQIs. The accurate frequency estimation and adaptive wavelets make this technique well suited to analyze the stationary and highly distorted non-stationary signals.

The remainder of this paper is organized as follows: Section II presents a review of EWT technique required to analyze the signal. Then, the EWT-based PQ indices for three-phase system are listed in Section III. In order to investigate the effectiveness of this method, EWT-based three-phase PQ indices are calculated in section IV. Finally, the conclusion is given in Section V.

2. Empirical Wavelet Transform

EWT is a recently proposed method to adaptively detect the different modes of the signal and consequently construct the empirical wavelets to represent the signal by different modes detected. Empirical wavelets means constructing a set of wavelets adapted to the processed signal, i.e. in Fourier domain means constructing a set of band-pass filters [13, 15]. Adaptation here lies in detecting filter supports according to the information located in the processed signal. Modes can be viewed as the principal components (referred to as amplitude modulated and frequency modulated (AM-FM) components) of the signal which represents the signal completely [13, 15].

The adaptability in this transform is provided by the segmentation of Fourier axis is done in away so as to separate different portions of the spectrum which correspond to modes that are centered around a specific frequency and of compact support. To find such boundaries we find the local maxima’s in the Fourier spectrum [13].

Consider a real signal \( x(n) \), which is sampled at a frequency of \( f_s \). First apply the FFT to the discrete signal \( x(n) \) to find the frequency spectrum \( X(\omega) \) and then obtain the set of maxima \( \delta = \{\delta_n\}_{n=1,2,\ldots,N} \) in the Fourier spectrum by means of the magnitude and frequency distance thresholds and infer their corresponding frequency \( \Omega_n \). Here \( N \) is the number of frequency components assessed using FFT. Now with this set of frequencies \( \Omega = \{\Omega_n\}_{n=1,2,\ldots,N} \), corresponding to FFT, the Fourier spectrum \( [0, f_s]\) is segmented into \( N \) segments, where each segment is defined as \( \Lambda_n = [\omega_{n-1}, \omega_n] \). Assuming \( \omega_0 = 0 \) and \( \omega_N = f_s / 2 \), the boundaries \( \omega_n \) are obtained as given in (1)

\[
\omega_n = \frac{\Omega_n + \Omega_{n+1}}{2} \text{ for } 1 \leq n \leq N - 1.
\]

Where \( \Omega_n \) and \( \Omega_{n+1} \) are the frequencies, and \( \omega_n \) is their corresponding boundary. Then, the Fourier segments will be \( [0, \omega_0], [\omega_0, \omega_1], \ldots, [\omega_{N-1}, f_s / 2] \).

After attaining the set of bounds \( \omega = \{\omega_n\}_{n=1,2,\ldots,N} \), we designate \( \omega_n \) to be the limits between each segment \( \{\omega_0 = 0 \text{and } \omega_N = \pi\} \). Each segment is denoted \( \Lambda_n = [\omega_{n-1}, \omega_n] \), then it is easy to see that \( \bigcup_{n=1}^N \Lambda_n = [0, \pi] \) centered about each \( \omega_n \), we define a transition phase \( T_{n} \) of width \( 2\tau_{n} \). A bank of \( N \) wavelet filters, consist of one low-pass filter and \( N-1 \) band-pass filters are defined based on the well-detected boundaries[13]-[15].

The empirical wavelets are defined as bandpass filters on each \( \Lambda_n \) and based on \( \Lambda_n \), a wavelet tight frame be able to defined, to do so, we use the notion used in the building of both Little wood-Paley and Meyer’s wavelets [13], a wavelet tight frame

\[
B = \left\{ \phi_n(t) \right\}_{n=1}^{N-1} \quad \left\{ \psi_n(t) \right\}_{n=1}^{N-1}
\]

is defined. And \( \forall n > 0 \), their Fourier transforms, i.e. the empirical scaling function \( \hat{\phi}_n(\omega) \) and the empirical wavelets \( \hat{\psi}_n(\omega) \) by expressions of (2) and (3), respectively.

\[
\hat{\phi}_n(\omega) = \begin{cases} \cos \left( \frac{\pi}{\tau_n} \omega \right) & \text{if } |\omega| \leq \omega_n - \tau_n \\ 0 & \text{otherwise} \end{cases}
\]

\[
\hat{\psi}_n(\omega) = \begin{cases} \sin \left( \frac{\pi}{\tau_n} \omega \right) \left( |\omega| - \omega_n + \tau_n \right) & \text{if } \omega_n - \tau_n \leq |\omega| \leq \omega_n + \tau_n \\ 0 & \text{otherwise} \end{cases}
\]
and

\[
\psi_n(x) = \begin{cases} 
\frac{\sqrt{2}}{2^{n/2}} \varphi(2^n x - k), & \text{if } |x| < 2^{n/2}, \\
\frac{1}{\sqrt{2}} \varphi(x - 2^{-n/2} k), & \text{if } 2^{n/2} \leq |x| < 2^{(n+1)/2}, \\
0, & \text{otherwise}
\end{cases}
\] (3)

The function \( \beta(x) \) is an arbitrary function such that

\[
\beta(x) = \begin{cases} 
0; \text{ for } x \leq 0 \\
\beta(x) \neq 0; \text{ for } x \geq 1
\end{cases}
\] (4)

Many functions satisfy these properties, the most used in the literature [11] is

\[
\beta(x) = \xi^4 \left( 35 - 84x + 70x^2 - 20x^3 \right)
\] (5)

Regarding the choice of \( \tau_n \), several choices are possible. The simplest is to select \( \tau_n \) proportional to \( \omega_n \), where \( 0 < \gamma < 1 \). To meet the requirement of tight frame [13], the parameter \( \gamma \) must fulfill the following equation:

\[
\gamma < \min \left( \frac{\omega_n^2 - \omega_{n+1}}{\omega_n^2 + \omega_{n+1}} \right)
\] (6)

Therefore, \( \forall n > 0, (2) \) and (3) simplify to (7) and (8)

\[
\psi_n(x) = \begin{cases} 
\frac{1}{\sqrt{2}} \varphi\left(2^n x - \frac{k}{2^n} \right), & \text{if } \left|\frac{k}{2^n}\right| < 1, \\
0, & \text{otherwise}
\end{cases}
\] (7)

and

\[
\psi_n(x) = \begin{cases} 
\frac{1}{\sqrt{2}} \varphi\left(2^n x - \frac{k}{2^n} \right), & \text{if } \left|\frac{k}{2^n}\right| < 1, \\
0, & \text{otherwise}
\end{cases}
\] (8)

We can now define the Empirical Wavelet Transform (EWT), \( W_f(n,t) \) in the similar way as for the classic wavelet transform. By performing the inner product operation between the applied signal \( f \), and with empirical wavelets we can obtain the detail coefficients as given in (9) and (10) respectively [13] - [15].

\[
W_f(n,t) = \int f(t) \psi_n(t) dt = \text{IFT} \left( f(u) \psi_n(u) \right)
\] (9)

By performing the inner product operation between the applied signal \( f \), and with empirical scaling functions we can obtain the approximation coefficients as given

\[
W_j(n,t) = \int f(t) \phi_n(t) dt = \text{IFT} \left( f(u) \phi_n(u) \right)
\] (10)

Where \( \phi_n(o) \) and \( \psi_n(o) \) are defined by (7) and (8), respectively. The reconstruction is obtained by

\[
f(t) = W_f(n,t) \phi_n(t) + \sum_{n=1}^{N} W_f(n,t) \psi_n(t)
\] (11)

\[
f(t) = \text{IFT} \left( W_f(n,u) \phi_n(u) + \sum_{n=1}^{N} W_f(n,u) \psi_n(u) \right)
\] (12)

Resulting this formalism, the empirical mode \( f_k \), as defined

\[
f_0(t) = W_f(0,t) \phi_1(t)
\] (13)

\[
f_k(t) = W_f(k,t) \psi_k(t)
\] (14)

Where * represents the convolution

3. EWT Based Three Phase Power Quality Indices

This section presents the brief review of PQ indices recommended in [1] and [2] for three-phase system and the EWT based reformulated indices. Consider the three-phase 3- wire system having non sinusoidal periodic phase voltages and line currents containing the fundamental and harmonic components as

\[
v_n(t) = \sum_{n=1}^{H_{\max}} V_{an} \sin(2 \pi f_{an} t - \theta_{an})
\] (15)

and

\[
i_n(t) = \sum_{n=1}^{H_{\max}} I_{an} \sin(2 \pi f_{an} t - \varphi_{an})
\] (16)

Where \( x \) corresponds to R, S and T phases, \( t \) is the time and \( H_{\max} \) is the maximum frequency component present in the signal \( v_{an}, f_{an}, \vartheta_{an} \) are the amplitude, frequency and phase angle of the \( n^{th} \) component of \( x \) - phase voltage signal respectively. \( I_{an}, f_{an}, \varphi_{an} \) are the amplitude, frequency and phase angle of the \( n^{th} \) component of \( x \) -phase current signal respectively. Now, for this set of three phase voltage and currents, EWT is applied and the corresponding mono component signal coefficients of line voltage and line currents are obtained. The approximate coefficients are
obtained by inner product of the input Signal with the scaling function \( \Phi_{x,1,k} \) and the detail coefficients are obtained by inner product of the input signal with the empirical wavelets \( \psi_{x,1,k} \) as explained in detail in section 2 of this paper.

\[
W_{x,1,k} = \langle v_x(k), \Phi_{x,1,k} \rangle; \quad W_{x,1,k} = \langle i_x(k), \Phi_{x,1,k} \rangle \quad (17)
\]

\[
W_{x,j,k} = \langle v_x(k), \psi_{x,j,k} \rangle; \quad W_{x,j,k} = \langle i_x(k), \psi_{x,j,k} \rangle \quad (18)
\]

Where \( j \) is the harmonic level and \( x \) corresponds to the phase.

3.1. Effective RMS calculations

The RMS value calculation of the non-sinusoidal voltage using EWT can be extended to three phase systems by considering any phase \( x \) of the three phases\( (R, S \) or \( T) \) as defined below

\[
V_{x,\text{rms}} = \sqrt{V_{x,1}^2 + V_{x,H}^2} = \sqrt{\frac{1}{N} \sum_{k=1}^{N} W_{x,1,k}^2 + \sum_{j=1}^{N} W_{x,j,k}^2} \quad (19)
\]

Where \( V_{x,1} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} W_{x,1,k} \) and \( V_{x,j} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} W_{x,j,k} \)

The RMS value of the non-sinusoidal current is defined as

\[
I_{x,\text{rms}} = \sqrt{I_{x,1}^2 + I_{x,H}^2} = \sqrt{\frac{1}{N} \sum_{k=1}^{N} W_{x,1,k}^2 + \sum_{j=1}^{N} W_{x,j,k}^2} \quad (20)
\]

Where \( I_{x,1} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} W_{x,1,k} \) and \( I_{x,j} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} W_{x,j,k} \)

Here \( V_{x,1} \) and \( I_{x,1} \) are the RMS values of the fundamental frequency component of the phase-\( x \) voltage and current, respectively. \( V_{x,j} \) and \( I_{x,j} \) are the sets of RMS values of the phase-\( x \) voltage and current of each frequency components higher than one. Also \( W_{x,1,k} \) and \( W_{x,1,k} \) are the EWT coefficients of phase-\( x \) voltage and current of first component at sample \( k \) while \( W_{x,j,k} \) and \( W_{x,j,k} \) are the EWT coefficients of phase-\( x \) voltage and current of any frequency level \( j \), higher than fundamental at sample \( k \). \( N \) corresponds to the total number of samples of the signal.

The effective RMS values for three phase voltages and currents, recommended in [3], can be calculated as

\[
V_e = \sqrt{V_{e1}^2 + V_{eH}^2} \quad I_e = \sqrt{I_{e1}^2 + I_{eH}^2} \quad (21)
\]

Where the fundamental effective RMS values for three phase voltage and current are

\[
V_{e1} = \sqrt{\frac{V_{R1}^2 + V_{S1}^2 + V_{T1}^2}{9}}; \quad I_{e1} = \sqrt{\frac{I_{R1}^2 + I_{S1}^2 + I_{T1}^2}{9}}
\]

The Non-fundamental effective RMS values for three phase voltage and current are

\[
V_{eH} = \sqrt{\frac{V_{R2}^2 + V_{S2}^2 + V_{T2}^2}{9}}; \quad I_{eH} = \sqrt{\frac{I_{R2}^2 + I_{S2}^2 + I_{T2}^2}{9}}
\]

Where \( V_{R1}, V_{S1}, V_{T1} \) and \( V_{R2}, V_{S2}, V_{T2} \) are the line-to-line voltage for phases labeled \( R, S, T \) . The subscript “1” refers to the fundamental component (50Hz) while subscript \( H \) refers to the nonfundamental harmonic components. \( I_{R1}, I_{S1}, I_{T1} \) and \( I_{R2}, I_{S2}, I_{T2} \) are the line currents for phases \( R, S, T \).

3.2 Equivalent Total Harmonic Distortion (THD)

The EWT based equivalent total harmonic distortion of voltage and current are defined as

\[
V_{\text{ethd}} = \frac{V_{eH}}{V_{e1}} \quad I_{\text{ethd}} = \frac{I_{eH}}{I_{e1}} \quad (22)
\]

3.3. Active Power

Fortescue [16] is credited with introducing the concept of symmetrical components which states that any unbalanced three-phase vectors can be decomposed into a set of balanced three-phase vectors. This set consists of positive-sequence components having the same phase sequences \( (R, S, T) \), negative -sequence components having the same phase sequences \( (R, T, S) \), and zero-sequence components having equal magnitudes with the same phase.

The concept of symmetrical components can be defined in the time-frequency domain by applying time shift instead of phase shift for the phase voltage and current. In order to get the sequence components in the wavelet domain, the time shift is applied to the voltage and currents of phases \( S \) and \( T \). First applying time advance \( (k + m) \) to the coefficients, where \( m \) represent the time corresponding to the angle \( 120^\circ \) therefore we obtain the advanced versions of the current phases \( S \) and \( T \) to be \( i_{s2}, i_{t2} \) and shifted versions of the voltages of the same phases to be \( v_{s2} \) and \( v_{t2} \) respectively.

Second applying time delay \( (k - m) \) to the coefficients,
therefore we obtain the delayed versions of the current phases $S$ and $T$ to be $i_S$, $i_T$ and shifted versions of the voltages of the same phases to be $v_S$ and $v_T$ respectively. Using the unshifted phase $R$ and shifted phases $S$ and $T$ the sequence components in the wavelet domain can be obtained as follows:

$$i^0 = \frac{1}{3}[i_R + i_S + i_T], \quad v^0 = \frac{1}{3}[v_R + v_S + v_T]$$

$$i^* = \frac{1}{3}[i_R + i_S + i_T], \quad v^* = \frac{1}{3}[v_R + v_S + v_T]$$

$$i^- = \frac{1}{3}[i_R + i_S + i_T], \quad v^- = \frac{1}{3}[v_R + v_S + v_T].$$

The fundamental positive sequence active power $P_1$ defined using the symmetrical components is

$$P_1^* = 3\left(\frac{1}{T_0}\int v^*i^* dt\right)$$

Where $v^*$, $i^*$ are the instantaneous positive sequence voltage and current in the wavelet domain [8]

The harmonic active power of a phase $x$ is defined as

$$P_{x,H} = \sum_{j=1} W_x W_{x,j} X_{x,j}$$

Where $P_{x,H}$ is the sum of the harmonic active powers for phase $x$.

The total harmonic active power of three phases is

$$P_H = P_{R,H} + P_{S,H} + P_{T,H}$$

The total active power of three phases is defined as

$$P = P_1^* + P_H$$

### 3.4. Apparent Power

The fundamental positive sequence apparent power based on the symmetrical components approach is defined as

$$S_1^* = 3V_1 I_1^*$$

The fundamental effective apparent power is defined as

$$S_{1I} = 3V_{1I} I_{1I}$$

The fundamental unbalanced power is

$$S_{1I} = \sqrt{S_{1A}^2 - S_{1I}^2}$$

The current distortion power and voltage distortion power can be defined as

$$D_{st} = 3V_{st} I_{st}$$

The harmonic apparent power is given by

$$S_{st} = 3V_{st} I_{st}$$

The non-fundamental effective apparent power is defined as

$$S_{nI} = \sqrt{D_{st}^2 + D_{st}^2 + S_{st}^2}$$

The total effective apparent power is

$$S_e = \sqrt{S_{1A}^2 + S_{1I}^2}$$

The non-active power $N$ is defined as

$$N = \sqrt{S_e^2 - P_e^2}$$

### 3.5. Reactive Power

The fundamental positive sequence reactive power is defined as

$$Q_1^* = \sqrt{(S_1^*)^2 - (P_1^*)^2}$$

### 3.6. Power Factor

The fundamental positive sequence power factor is defined as

$$PF_1^* = \frac{P_1^*}{S_1^*}$$

The total three phase power factor is defined as

$$PF = \frac{P}{S_e}$$

### 3.7. Harmonic Pollution

The Harmonic pollution is defined as the ratio of the non fundamental effective apparent power to the fundamental effective apparent power

$$HP = \frac{S_{nI}}{S_{1I}}$$

### 3.8. Load unbalance

The load unbalance is the ratio of fundamental apparent power to the fundamental positive sequence power that can measure the system unbalance. It is given as

$$LU = \frac{S_{1I}}{S_{1I}}$$

## 4. NUMERICAL EXAMPLES

This section contains Four numerical examples that are solved using the definitions contained in the IEEE Standard and the definitions based on the EWT. First,
the true values are computed as per the IEEE standard definitions and then the EWT is applied to decompose the signal into mono frequency components. By applying EWT-based reformulated indices, on to these mono components we can accurately estimate the three phase PQIs. The percentage difference has been calculated for all the indices using the mathematical expressions shown below by

\[
\text{% difference} = \left| \frac{\text{Indices}_{\text{true}} - \text{Indices}_{\text{calculated}}}{\text{Indices}_{\text{true}}} \right| \times 100
\]  

(39)

4.1. Balanced Three-Phase System

The first numerical example considers balanced three-phase load supplied from nonsinusoidal balanced three-phase source as shown in Fig.1. The time domain equations for the three-phase source are [10]

\[
v_R(t) = 100 \sin(\omega_t) + 20 \sin(\omega_t + 5 \sin(\omega_t))
\]  

(40)

\[
v_S(t) = 100 \sin(\omega_t - 120^\circ) + 20 \sin(\omega_t + 5 \sin(\omega_t - 120^\circ))
\]  

(41)

\[
v_T(t) = 100 \sin(\omega_t + 120^\circ) + 20 \sin(\omega_t + 5 \sin(\omega_t + 120^\circ))
\]  

(42)

Here \(\omega_t\), \(\omega_3\), and \(\omega_5\) are the angular frequency at the fundamental \((f_1 = 60Hz)\), third harmonic \((f_3 = 180Hz)\) and fifth harmonic \((f_5 = 300Hz)\), respectively.

Balanced three-phase line voltages \(v_R(t), v_S(t), v_T(t)\), is applied to balanced three-phase load as shown in Fig.1. From which we can calculate line currents \(i_R(t), i_S(t), i_T(t)\), and the calculated line currents are shown in equation (43) to (45).

For this set of three-phase line voltages and line currents after being sampled with a sampling frequency \((f_s = 7680Hz)\), EWT is applied and the corresponding mono-component signal coefficients of line voltage and line currents are obtained. It can be clearly observed from the filtered signals shown in Fig.2, that EWT technique is able to accurately estimate the frequency components, which are present in the signal and provide its mono-components. It can also be noticed that no two frequency components are combined in a single mode. In all these Figures mode 2 corresponds to fundamental component and mode 3 and mode 4 are corresponding to harmonics of the signals. By using EWT-based reformulated indices, on to these fundamental components and harmonics we can accurately estimate the three phase PQIs. Table1 compares the PQIs obtained from the IEEE standard definitions with those obtained from the EWT. As the Results in Table 1 show, the percentage differences are too small in the case of effective RMS and equivalent total harmonic distortion. We also observe that, the percentage differences for active power, apparent power, distortion power, and reactive power are considerably small; however for power factor, pollution factor and load unbalance indicates zero percentage differences which proves the accuracy of the EWT-based definitions.

![Fig.1 Balanced three-phase load supplied from nonsinusoidal balanced three phase source.](image)

![Fig.2 Modes extracted by the EWT for \(v_R(t)\)](image)
Fig. 3. Modes extracted by the EWT for \( i_R(t) \)

Fig. 4. Modes extracted by the EWT for \( v_s(t) \)

Fig. 5. Modes extracted by the EWT for \( i_s(t) \)

Fig. 6. Modes extracted by the EWT for \( v_f(t) \)

Fig. 7. Modes extracted by the EWT for \( i_f(t) \)

Table 1 PQIs for Balanced Three-Phase Signals

<table>
<thead>
<tr>
<th>Indices</th>
<th>IEEE standard Definitions</th>
<th>EWT based Indices</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Voltage RMS</td>
<td>( V_{el} = 70.700 ) ( V_{el} = 70.6245 ) ( 0.106 )</td>
<td>( V_{el} = 72.180 ) ( V_{el} = 72.1123 ) ( 0.093 )</td>
<td></td>
</tr>
<tr>
<td>Effective Current RMS</td>
<td>( I_{el} = 23.900 ) ( I_{el} = 23.900 ) ( 0.000 )</td>
<td>( I_{el} = 3.100 ) ( I_{el} = 3.111 ) ( 0.354 )</td>
<td></td>
</tr>
<tr>
<td>THD</td>
<td>( V_{thd} = 0.206 ) ( V_{thd} = 0.206 ) ( 0.000 )</td>
<td>( I_{thd} = 0.130 ) ( I_{thd} = 0.130 ) ( 0.000 )</td>
<td></td>
</tr>
<tr>
<td>Distortion Index</td>
<td>( DIN_{el} = 0.201 ) ( DIN_{el} = 0.202 ) ( 0.497 )</td>
<td>( DIN_{el} = 0.1283 ) ( DIN_{el} = 0.129 ) ( 0.545 )</td>
<td></td>
</tr>
<tr>
<td>Active Power</td>
<td>( P = 4586 ) ( P' = 4591.35 ) ( 0.116 )</td>
<td>( P = 4674 ) ( P = 4674.85 ) ( 0.018 )</td>
<td></td>
</tr>
<tr>
<td>Apparent Power</td>
<td>( S_e = 5069.19 ) ( S'_e = 5065.169 ) ( 0.079 )</td>
<td>( S_e = 5069.19 ) ( S'_e = 5065.169 ) ( 0.079 )</td>
<td></td>
</tr>
<tr>
<td>Total Apparent Power</td>
<td>( S_e = 5218.6 ) ( S'_e = 5215.48 ) ( 0.059 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reactive Power</td>
<td>( Q_e = 2159.6 ) ( Q'_e = 2159.86 ) ( 0.012 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distortion Power</td>
<td>( D_e = 657.51 ) ( D_e = 659.145 ) ( 0.248 )</td>
<td>( D_e = 1043.95 ) ( D_e = 1045.145 ) ( 0.114 )</td>
<td></td>
</tr>
<tr>
<td>Power Factor</td>
<td>( PF_e = 0.904 ) ( PF'_e = 0.904 ) ( 0.000 )</td>
<td>( PF_e = 0.896 ) ( PF_e = 0.896 ) ( 0.000 )</td>
<td></td>
</tr>
<tr>
<td>Harmonic Pollution</td>
<td>( HP = 0.245 ) ( HP = 0.245 ) ( 0.000 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load Unbalance</td>
<td>( LU = 0.000 ) ( LU = 0.000 ) ( 0.000 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.2. Unbalanced Three-Phase System

The second numerical example [10] considers an unbalanced three-phase load supplied from the same nonsinusoidal balanced three-phase source used in the previous example shown in Fig. 8.

Fig. 8. Unbalanced three-phase load supplied from nonsinusoidal balanced three-phase source.

Fig. 9. Modes extracted by the EWT for $v_R(t)$

Fig. 10. Modes extracted by the EWT for $i_R(t)$

Here, $\omega_1$, $\omega_3$, and $\omega_5$ are the angular frequency at the fundamental ($f_s = 60\,\text{Hz}$), third harmonic ($f_s = 180\,\text{Hz}$) and fifth harmonic ($f_s = 300\,\text{Hz}$), respectively. Balanced three-phase line voltages $v_R(t)$, $v_S(t)$, $v_T(t)$, is applied to unbalanced three-phase load as shown as in Fig. 8. From which we can calculate line currents $i_R(t)$, $i_S(t)$, $i_T(t)$, and the calculated line currents are shown in equation (49) to (51).

\begin{align*}
v_R(t) &= 100 \sin(\omega_1 t) + 20 \sin(\omega_2 t) + 5 \sin(\omega_4 t) \quad (46) \\
v_S(t) &= 100 \sin(\omega_1 t - 120^\circ) + 20 \sin(\omega_2 t - 120^\circ) + 5 \sin(\omega_4 t - 120^\circ) \quad (47) \\
v_T(t) &= 100 \sin(\omega_1 t + 120^\circ) + 20 \sin(\omega_2 t + 120^\circ) + 5 \sin(\omega_4 t + 120^\circ) \quad (48)
\end{align*}

For this set of three-phase line voltages and line currents after being sampled with a sampling frequency ($f_s = 7680\,\text{Hz}$), EWT is applied and the corresponding monocomponent signal coefficients of line voltage and line currents are obtained. It can be clearly observed from the filtered signals shown in Fig. 9 to Fig. 16 that EWT technique is able to accurately estimate the frequency components, which are present in the signal and provide its monocomponents, it can also be noticed that no two frequency components are combined in a single mode. In all these Figures mode 2 corresponds to fundamental component and mode 3 and mode 4 are corresponding to harmonics of the signals. By using EWT-based reformulated indices, on to these fundamental components and harmonics we can accurately estimate the three phase PQIs. Table 2 compares the PQIs obtained from the IEEE standard definitions with those obtained from the EWT. As the Results in Table 2 show, the percentage differences are too small in the case of effective RMS and equivalent total harmonic distortion. We also observe that from the results shown in Table 2, the percentage differences for active power, apparent power, distortion power, and reactive power are considerably small; the fundamental unbalanced power $S_{U1}$ has a nonzero value, the percentage difference for power factor is zero.

For this set of three-phase line voltages and line currents after being sampled with a sampling frequency ($f_s = 7680\,\text{Hz}$), EWT is applied and the corresponding monocomponent signal coefficients of line voltage and line currents are obtained. It can be clearly observed from the filtered signals shown in Fig. 9 to Fig. 16 that EWT technique is able to accurately estimate the frequency components, which are present in the signal and provide its monocomponents, it can also be noticed that no two frequency components are combined in a single mode. In all these Figures mode 2 corresponds to fundamental component and mode 3 and mode 4 are corresponding to harmonics of the signals. By using EWT-based reformulated indices, on to these fundamental components and harmonics we can accurately estimate the three phase PQIs. Table 2 compares the PQIs obtained from the IEEE standard definitions with those obtained from the EWT. As the Results in Table 2 show, the percentage differences are too small in the case of effective RMS and equivalent total harmonic distortion. We also observe that from the results shown in Table 2, the percentage differences for active power, apparent power, distortion power, and reactive power are considerably small; the fundamental unbalanced power $S_{U1}$ has a nonzero value, the percentage difference for power factor is zero.
4.3. Balanced Distorted Signals:

This example considers balanced three-phase voltage and balanced current signals containing four frequency components as shown in Table 3 and 4, respectively.

Table 3 Parameters of the Three-Phase Voltage signal

<table>
<thead>
<tr>
<th>Time(S)</th>
<th>Phase</th>
<th>Amplitude, Frequency and Phase angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1</td>
<td>R</td>
<td>100,50, 30, 50, 0, 0</td>
</tr>
<tr>
<td>0-1</td>
<td>Y</td>
<td>100,50, 30, 50, 0, 0</td>
</tr>
<tr>
<td>0-1</td>
<td>B</td>
<td>100,50, 30, 50, 0, 0</td>
</tr>
</tbody>
</table>

Table 2 PQIs for Unbalanced Three-Phase Signals

<table>
<thead>
<tr>
<th>Indices</th>
<th>IEEE standard Definitions</th>
<th>EWT based Indices</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Voltage RMS</td>
<td>$V_{vt}$ = 70.700</td>
<td>$V_{vt}$ = 70.624</td>
<td>0.106</td>
</tr>
<tr>
<td>Effective Current RMS</td>
<td>$I_{vt}$ = 20.670</td>
<td>$I_{vt}$ = 20.652</td>
<td>0.087</td>
</tr>
<tr>
<td>THD</td>
<td>$V_{theta}$ = 0.206</td>
<td>$V_{theta}$ = 0.206</td>
<td>0.00</td>
</tr>
<tr>
<td>Distortion Index</td>
<td>$DIN_{vt}$ = 0.0235</td>
<td>$DIN_{vt}$ = 0.020</td>
<td>0.737</td>
</tr>
<tr>
<td>Active Power</td>
<td>$P^*$ = 3713</td>
<td>$P^*$ = 3717.34</td>
<td>0.116</td>
</tr>
<tr>
<td>Power Factor</td>
<td>$PF$ = 0.893</td>
<td>$PF$ = 0.846</td>
<td>0.000</td>
</tr>
<tr>
<td>Harmonic Pollution</td>
<td>$HP$ = 0.254</td>
<td>$HP$ = 0.255</td>
<td>0.393</td>
</tr>
<tr>
<td>Load Unbalance</td>
<td>$LU$ = 0.335</td>
<td>$LU$ = 0.333</td>
<td>1.782</td>
</tr>
</tbody>
</table>
Table 4 Parameters of the Three-Phase Current signal

<table>
<thead>
<tr>
<th>Time (S)</th>
<th>Phase</th>
<th>Amplitude, Frequency and Phase angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-1</td>
<td>R</td>
<td>28.62,50, 17.44°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.54,150 43.3° 57.5°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.28,250 65.62°</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>28.62,50, 137.44°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.54,150 163.3° 177.5°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.28,250 185.6°</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>28.62,50, 102.56°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.54,150 76.69° 62.48°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.28,250 54.38°</td>
</tr>
</tbody>
</table>

The unbalanced three-phase currents are obtained by applying balanced three-phase voltages to unbalanced three-phase load. Table 5 shows the PQIs for a balanced three-phase system calculated from the IEEE standard definitions and that estimated using the EWT. It can be seen that the deviation in the EWT-based PQIs is small.

4.4. Un balanced Distorted Signals:

This example considers balanced three-phase voltage and unbalanced current signals containing four frequency components as shown in Table 6 and 7, respectively[15]. The unbalanced three-phase currents are obtained by applying balanced three-phase voltages to unbalanced three-phase load. Table 8 shows the PQIs for an unbalanced three-phase system calculated from the IEEE standard definitions and that estimated using the EWT. It can be seen that the deviation in the EWT-based PQIs is small.

Table 5 PQIs for Balanced Three-Phase Signals

<table>
<thead>
<tr>
<th>Indices</th>
<th>IEEE standard Definitions</th>
<th>EWT based Indices</th>
<th>%difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Voltage RMS</td>
<td>V_{re} = 70.981</td>
<td>V_{re} = 70.607</td>
<td>0.426</td>
</tr>
<tr>
<td></td>
<td>V_{ef} = 22.378</td>
<td>V_{ef} = 22.16</td>
<td>0.974</td>
</tr>
<tr>
<td></td>
<td>V_{em} = 74.12</td>
<td>V_{em} = 74</td>
<td>0.161</td>
</tr>
<tr>
<td>Effective Current RMS</td>
<td>I_{re} = 20.302</td>
<td>I_{re} = 20.207</td>
<td>0.467</td>
</tr>
<tr>
<td></td>
<td>I_{ef} = 4.734</td>
<td>I_{ef} = 4.732</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>I_{em} = 20.87</td>
<td>I_{em} = 20.75</td>
<td>0.574</td>
</tr>
<tr>
<td>THD</td>
<td>V_{shd} = 0.318</td>
<td>V_{shd} = 0.313</td>
<td>1.572</td>
</tr>
<tr>
<td></td>
<td>I_{shd} = 0.2345</td>
<td>I_{shd} = 0.234</td>
<td>0.213</td>
</tr>
<tr>
<td>Distortion Index</td>
<td>DIN_{re} = 0.30</td>
<td>DIN_{re} = 0.299</td>
<td>0.333</td>
</tr>
<tr>
<td></td>
<td>DIN_{ef} = 0.229</td>
<td>DIN_{ef} = 0.228</td>
<td>0.436</td>
</tr>
<tr>
<td>Active Power</td>
<td>P_{1} = 4096.78</td>
<td>P_{1} = 4097.34</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>P_{2} = 223.31</td>
<td>P_{2} = 221.82</td>
<td>0.067</td>
</tr>
<tr>
<td></td>
<td>P = 4320.09</td>
<td>P = 4319.17</td>
<td>0.021</td>
</tr>
<tr>
<td>Apparent Power</td>
<td>S_{1} = 4297.78</td>
<td>S_{1} = 4299.14</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td>S_{2} = 4296.99</td>
<td>S_{2} = 4294.81</td>
<td>0.033</td>
</tr>
<tr>
<td>Total Apparent Power</td>
<td>S_{r} = 4609.43</td>
<td>S_{r} = 4607.65</td>
<td>0.038</td>
</tr>
<tr>
<td>Reactive Power</td>
<td>Q_{r} = 1301.3</td>
<td>Q_{r} = 1301.70</td>
<td>0.030</td>
</tr>
<tr>
<td>Distortion Power</td>
<td>D_{1} = 1002.46</td>
<td>D_{1} = 1002.38</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>D_{2} = 1345.43</td>
<td>D_{2} = 1343.42</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td>S_{1} = 314.92</td>
<td>S_{1} = 314.60</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>S_{2} = 1707.8</td>
<td>S_{2} = 1705.43</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>S_{1} = 1608.7</td>
<td>S_{1} = 1604.75</td>
<td>0.245</td>
</tr>
<tr>
<td>Power Factor</td>
<td>PF_{r} = 0.9538</td>
<td>PF_{r} = 0.953</td>
<td>0.003</td>
</tr>
<tr>
<td>Harmonic Pollution</td>
<td>HP = 0.398</td>
<td>HP = 0.398</td>
<td>0.175</td>
</tr>
<tr>
<td>Load Unbalance</td>
<td>LU = 0.000</td>
<td>LU = 0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

5. Conclusions

In this paper, the EWT is successfully applied to formulating the power components definitions contained in the IEEE standard 1459-2000 for balanced and unbalanced three-phase systems with non sinusoidal situations. This technique first estimates the frequency components and then adaptively tunes the wavelet and scaling function based on the boundaries to decompose the signal accurately. Four numerical examples considering balanced and unbalanced three-phase systems with non sinusoidal situations are solved using the IEEE Standard definitions and the EWT-
Table 8 PQIs for Unbalanced Three-Phase Signals

<table>
<thead>
<tr>
<th>Indices</th>
<th>IEEE standard Definitions</th>
<th>EWT based Indices</th>
<th>%difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Voltage RMS</td>
<td>$V_{e1} = 70.7107$</td>
<td>$V_{e1} = 70.605$</td>
<td>0.149</td>
</tr>
<tr>
<td></td>
<td>$V_{e2} = 22.2374$</td>
<td>$V_{e2} = 22.208$</td>
<td>0.132</td>
</tr>
<tr>
<td></td>
<td>$V_{e3} = 74.1249$</td>
<td>$V_{e3} = 74.022$</td>
<td>0.138</td>
</tr>
<tr>
<td>Effective Current RMS</td>
<td>$I_{e1} = 15.1107$</td>
<td>$I_{e1} = 15.099$</td>
<td>0.077</td>
</tr>
<tr>
<td></td>
<td>$I_{e2} = 3.3087$</td>
<td>$I_{e2} = 3.308$</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>$I_{e3} = 15.4687$</td>
<td>$I_{e3} = 15.458$</td>
<td>0.069</td>
</tr>
<tr>
<td>THD</td>
<td>$V_{thd} = 0.3145$</td>
<td>$V_{thd} = 0.3144$</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td>$I_{thd} = 0.2190$</td>
<td>$I_{thd} = 0.218$</td>
<td>0.456</td>
</tr>
<tr>
<td>Active Power</td>
<td>$DIN_{e1} = 0.300$</td>
<td>$DIN_{e1} = 0.30$</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>$DIN_{e2} = 0.2139$</td>
<td>$DIN_{e2} = 0.213$</td>
<td>0.420</td>
</tr>
<tr>
<td>Apparent Power</td>
<td>$P^* = 2713.15$</td>
<td>$P^* = 2715.7$</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td>$P^* = 146.584$</td>
<td>$P^* = 146.97$</td>
<td>0.263</td>
</tr>
<tr>
<td></td>
<td>$P = 2859.734$</td>
<td>$P = 2862.68$</td>
<td>0.103</td>
</tr>
<tr>
<td></td>
<td>$S^*_u = 3145.51$</td>
<td>$S^*_u = 3147.9$</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td>$S^*_u = 3205.464$</td>
<td>$S^*_u = 3205.98$</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>$S_{thd} = 617.113$</td>
<td>$S_{thd} = 608.8$</td>
<td>1.347</td>
</tr>
<tr>
<td>Total Apparent Power</td>
<td>$S = 3439.847$</td>
<td>$S = 3439.91$</td>
<td>0.001</td>
</tr>
<tr>
<td>Reactive Power</td>
<td>$Q^* = 1591.55$</td>
<td>$Q^* = 1591.93$</td>
<td>0.023</td>
</tr>
<tr>
<td>Distortion Power</td>
<td>$D_{el} = 701.8815$</td>
<td>$D_{el} = 701.52$</td>
<td>0.051</td>
</tr>
<tr>
<td></td>
<td>$D_{el} = 1008.068$</td>
<td>$D_{el} = 1005.98$</td>
<td>0.207</td>
</tr>
<tr>
<td></td>
<td>$S_{el} = 220.7307$</td>
<td>$S_{el} = 220.01$</td>
<td>0.326</td>
</tr>
<tr>
<td></td>
<td>$S_{el} = 1248.02$</td>
<td>$S_{el} = 1247.80$</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>$N = 1911.667$</td>
<td>$N = 1908.96$</td>
<td>0.141</td>
</tr>
<tr>
<td>Power Factor</td>
<td>$PF^* = 0.8657$</td>
<td>$PF^* = 0.8625$</td>
<td>0.369</td>
</tr>
<tr>
<td></td>
<td>$PF = 0.8314$</td>
<td>$PF = 0.834$</td>
<td>0.312</td>
</tr>
<tr>
<td>Harmonic Pollution</td>
<td>$HP = 0.389$</td>
<td>$HP = 0.389$</td>
<td>0.077</td>
</tr>
<tr>
<td>Load Unbalance</td>
<td>$LU = 0.1962$</td>
<td>$LU = 0.1947$</td>
<td>0.764</td>
</tr>
</tbody>
</table>


