BACKSTEPPING CONTROL OF THREE-PHASE FOUR-LEG SHUNT ACTIVE POWER FILTER

M. BOUZIDI
Department of Electronics and Communications, Faculty of New Technologies of Information and Communication University Kasdi Merbah, Ouargla, Algeria
Intelligent Control Electronic Power System laboratory (I.C.E.P.S), University of Djilali Liabes, Sidi Bel Abbès, Algeria
bouzidi.m28@gmail.com / bouzidi.mansour@uni-ouargla.dz

S. BOUAFIA  A. BENAÏSSA
Intelligent Control Electronic Power System laboratory (I.C.E.P.S), University of Djilali Liabes, Sidi Bel Abbès, Algeria
Bouafia.saber@gmail.com & aek_benaissa@yahoo.fr

A. BOUZIDI
Department of Electrical Engineering, Faculty of Technology, University of Hadj Lakhdar, Batna, Algeria
bouzidi.ali@yahoo.fr

S. BARKAT
Department of Electrical Engineering, Faculty of Engineering, University of M’sila, M’sila, Algeria
sa_barkat@yahoo.fr

Abstract: This paper deals with the application of a backstepping strategy to control a three-phase shunt active power filter. The APF consists of four-leg voltage source inverter bridge. The APF ensures full compensation for harmonic phase currents, harmonic neutral current, and reactive and unbalanced nonlinear load currents. It also regulates its self-sustaining dc bus voltage. Complete simulation of the resultant active filtering system validates the efficiency of the proposed backstepping control law. Compared to the traditional control, the used of backstepping control allows to exhibit excellent transient response during balanced and unbalanced load.

Key words: Backstepping control, Four-leg shunt active power filter, 3D-Space vector modulation, Harmonics elimination.

1. Introduction

The increasing use of nonlinear loads is provoking a serious problem of power quality to the ac mains. The nonlinear loads as those fed by single-phase and three-phase ac/dc power rectifiers draw, beside the active current, undesirable reactive and harmonic currents. The concept of using shunt active power filters (APF) in order to compensate for these reactive and harmonic currents caused by the locally connected nonlinear loads is a viable solution for power quality improvement [1-3].

In three-phase four-wire systems, there are often load unbalance and considerable amount of neutral currents that cause a voltage unbalance affecting the performance of the neighboring loads. To compensate for the undesired neutral current a classical shunt APF topology having split dc capacitor inverter with three legs can be used. However, the zero-sequence component in the APF compensation currents will flow through the dc-bus capacitors. This current gives rise to imbalance in the capacitor voltage sharing. Such imbalance in the dc voltages deteriorates the dynamic capability of the APF to follow fast changes in the current references. Therefore, the controller should regulate and equalize the dc-bus capacitor voltages to eliminate the imbalance problem to the detriment of a total cancellation of the neutral current. Hence, the topology of three-leg inverter with split capacitor alleviates the neutral current but it is not suitable for full compensation of current unbalance [1-3].

For full compensation capability a four-leg inverter connected as a shunt APF can be used [4]. In this configuration, as shown in Fig. 1, the compensated neutral current is provided through a fourth leg allowing a better controllability than the three-leg with split-capacitor configuration. The main advantage of the four-leg configuration is the ability to suppress the neutral current from the source without any drawback in the filtering performance [4].

To improve the performances Four-Leg SAPF (FL-SAPF), a nonlinear control strategy based on the backstepping control is proposed. It is well known that the backstepping technique is a control method to eliminate the nonlinearity of the system by using the inverse dynamics [5]. It has been applied to control
permanent magnet synchronous motor [6], induction motor [7], and PWM ac-dc converter [8], which gives good performances, robustness to disturbances as well as fast transient responses.

In this work a three-phase FL-SAPF is considered to compensate harmonic distortion, reactive power and current unbalance caused by the nonlinear load. The state space model of the APF, developed in the stationary αβφ frame, is used to obtain the desired closed loop dynamics and to generate the reference inverter voltage by applying a backstepping control strategy.

2. System Configuration

2.1. Power circuit description

Fig. 1 shows a basic diagram of a three-phase four-leg SAPF connected in parallel to a set of loads at the point of common coupling (PCC). The four leg SAPF consists of three parts, i.e. four-leg converter, dc bus capacitor and three filter inductors.

2.2. Mathematical model of four-leg PWM inverter

The differential equations describing the dynamic model of the inverter are defined in αβφ-reference frame, as given in equation (1).

\[
\begin{align*}
\frac{dv_{\alpha}}{dt} &= \frac{1}{L_{\alpha}} (v_{\alpha} - v_{\alpha} - R_{\alpha}i_{\alpha}) \\
\frac{dv_{\beta}}{dt} &= \frac{1}{L_{\beta}} (v_{\beta} - v_{\beta} - R_{\beta}i_{\beta}) \\
\frac{dv_{\phi}}{dt} &= \frac{1}{L_{\phi}} (v_{\phi} - v_{\phi} - R_{\phi}i_{\phi}) \\
\frac{dv_{d}}{dt} &= \frac{P_{dc}}{Cv_{d}}
\end{align*}
\]  

(1)

Where, \(v_{\alpha}, v_{\beta}\) and \(v_{\phi}\) are voltages of (PCC) in the α-β-φ coordinates, \(i_{\alpha}, i_{\beta}\) and \(i_{\phi}\) are the α-β-φ axis AC current of SAPF, and \(v_{\alpha}, v_{\beta}\) and \(v_{\phi}\) are the AC side voltages of four-leg SAPF.

1. Control Design

The basic operation of the proposed control method is shown in Fig. 1. The capacitor voltage is compared with its reference value, \(v_{\text{dcref}}\), in order to maintain the energy stored in the capacitor constant. The backstepping controller is applied to regulate the error between the capacitor voltage and its reference. The output of backstepping controller presents the reference of DC active power \(p_{dc}\), the compensating currents are derived based on instantaneous \(p-q\)
theory [5]. The instantaneous active and reactive power can be computed in terms of transformed current signals and PCC voltages [5]. The alternate value of active power is extracted using high-pass filter. The output signals from backstepping current controller are used for switching signals generation by a 3D-space vector modulator (3DSVM).

2.3. The p–q theory based control strategy

Instantaneous active and reactive powers of the nonlinear load are calculated as:

\[
\begin{bmatrix}
  p_t \\
  q_t
\end{bmatrix} = \begin{bmatrix}
  v_u & v_p \\
  -v_p & v_u
\end{bmatrix} \begin{bmatrix}
  i_{lu} \\
  i_{lp}
\end{bmatrix}
\]  

The instantaneous active and reactive powers include AC and DC values and can be expressed as follows:

\[
\begin{align*}
p_t &= p_l + \bar{p}_t \\
q_t &= q_l + \bar{q}_t
\end{align*}
\]  

DC values \((p_F, q_F)\) of the \(p_t\) and \(q_t\) are the average active and reactive power originating from the positive-sequence component of the nonlinear load current. AC values \((\bar{p}_t, \bar{q}_t)\) of the \(p_t\) and \(q_t\) are the ripple active and reactive powers.

For harmonic, reactive power compensation and balancing of unbalanced 3-phase load currents, all of the reactive power \((\bar{q}_l, \bar{q}_c)\) components and harmonic component \((\bar{p}_t)\) of active power are selected as compensation power references and the compensation currents reference are calculated as (4).

\[
\begin{bmatrix}
  i_{ra,ref} \\
  i_{qb,ref}
\end{bmatrix} = \frac{1}{v_u + v_p} \begin{bmatrix}
  v_u & -v_p \\
  v_p & v_u
\end{bmatrix} \begin{bmatrix}
  \bar{p}_t - p_{\alpha} \\
  \bar{q}_t
\end{bmatrix}
\]  

The signal \(p_{\alpha}\) is used as an average real power and is obtained from the dc voltage controller.

Since the zero-sequence current must be compensated, the reference of homopolar current is given as:

\[
i_{ra,ref} = i_{la}
\]  

2.4. Backstepping Controller Synthesis

From equations (1), it is obvious that the converter is a nonlinear and coupled system. So a nonlinear controller based on the backstepping method is developed in this section. The system (1) is subdivided in four subsystems as follows:

**Subsystem 1:**

The first subsystem is characterized by only one state \(x = v_{dc}\) and only one control input \(u = p_{\alpha}\).

\[
\frac{dv_{dc}}{dt} = \frac{p_{\alpha}}{Cv_{dc}}
\]  

The equation (6) can be written as follow:

\[
\begin{align*}
x_1 &= L_i h_i + L_f h_i u \\
y_1 &= h_i(x) = v_{dc}, \quad y_2 = v_{dc,ref}
\end{align*}
\]  

Where:

\[
x_1 = v_{dc}, \quad u = p_{\alpha}, \quad L_i h_i = 0, \quad L_f h_i = \frac{1}{Cv_{dc}}
\]  

**Subsystem 2:**

The second subsystem is also characterized by only one state \(x = i_{ra}\) and only one control input \(u = v_{ra}\).

\[
\frac{di_{ra}}{dt} = \frac{1}{L_p} (v_{ra} - v_u - R_i i_{ra})
\]  

The equation (8) can also be written as follow:

\[
\begin{align*}
x_2 &= L_i h_2 + \bar{r}_p a \\
y_2 &= h_2 = i_{ra}, \quad y_3 = i_{ra,ref}
\end{align*}
\]  

Where:

\[
x_2 = i_{ra}, \quad L_i h_2(x) = \frac{1}{L_p} (v_u - R_i i_{ra}), \quad \bar{r}_p a = \frac{1}{L_p} v_{ra}
\]  

**Subsystem 3:**

The third subsystem is characterized by one state \(x = i_{pb}\) and one control input \(u = v_{pb}\).

\[
\frac{di_{pb}}{dt} = \frac{1}{L_p} (v_{pb} - v_u - R_i i_{pb})
\]  

The equation (10) can also be written as follow:

\[
\begin{align*}
x_3 &= L_i h_3 + \bar{r}_p b \\
y_3 &= h_3 = i_{pb}, \quad y_4 = i_{pb,ref}
\end{align*}
\]  

Where:

\[
x_3 = i_{pb}, \quad L_i h_3(x) = \frac{1}{L_p} (v_u - R_i i_{pb}), \quad \bar{r}_p b = \frac{1}{L_p} v_{pb}
\]  

**Subsystem 4:**

The fourth subsystem is characterized by one state \(x = i_{q}\) and one control input \(u = v_{ra}\).

\[
\frac{di_{q}}{dt} = \frac{1}{L_p} (v_{ra} - v_u - R_i i_{ra})
\]
The fourth subsystem (12) can also be written as follow:

\[
\begin{align*}
    x_4 &= L_p h_4 + \sigma_{p_0} \\
    y_4 &= h_4 = l_{p_0} y_{sd} = y_{p_0, ref}
\end{align*}
\]  

(13)

Where:

\[
x_4 = l_{p_0}, \quad L_p h_4(x) = \frac{1}{L_p}(-v_a - R_s i_{p_0}), \quad \sigma_{p_0} = \frac{1}{L_p} v_{p_0}
\]

2.5. DC voltage controller synthesis

The synthesis of the DC voltage controller is based on the first subsystem. The first tracking error is defined as:

\[
z_i = x_i - y_{id}
\]

(14)

Its derivative is:

\[
z'_i = L_i h_i + L_s h_s u - y'_{id}
\]

(15)

The Lyapunov function is chosen as:

\[
V_i = \frac{1}{2} z_i^2
\]

(16)

The derivative of (16) is given by:

\[
V'_i = z_i z'_i = z_i (L_i h_i + L_s h_s u - y'_id)
\]

(17)

The dc power \( p_{dc} \) represent the control law of the first subsystem. It is selected such as the Lyapunov function \( V_i \) should be definite negative [8], as follow:

\[
p_{dc} = \frac{-k_i z_i + y_{id}}{L_i h_i}
\]

(18)

Where: \( k_i \) is a positive constant.

2.6. Current controller synthesis

The errors \( z_2, z_3, \) and \( z_4 \) are defined as:

\[
z_i = x_i - y_{id}, \quad i = 2, 3, 4
\]

(19)

The Lyapunov functions are given by the following expression:

\[
V_i = \frac{1}{2} z_i^2, \quad i = 2, 3, 4
\]

(20)

And consequently, their derivatives are given by:

\[
V'_i = z_i z'_i = z_i (L_i h_i + \sigma_{p_0} - y'_{id})
\]

(21)

To make \( V'_i < 0 \), \( V'_2 < 0 \) and \( V'_3 < 0 \) [8] we must choose:

\[
\begin{align*}
    \sigma_{p_0} &= -k_i z_i - L_i h_i + y'_{id} \\
    \sigma_{p_0} &= -k_i z_i - L_i h_i + y'_{id} \\
    \sigma_{p_0} &= -k_i z_i - L_i h_i + y'_{id}
\end{align*}
\]

(22)

Where: \( k_2, k_3 \) and \( k_4 \) are positive constants.

We also have:

\[
\begin{align*}
    \sigma_{p_0} &= -k_i z_i - L_i h_i + y'_{id} \\
    \sigma_{p_0} &= -k_i z_i - L_i h_i + y'_{id} \\
    \sigma_{p_0} &= -k_i z_i - L_i h_i + y'_{id}
\end{align*}
\]

(23)

Where:

\[
\begin{align*}
    \sigma_{p_0} &= -k_i z_i - L_i h_i + y'_{id} \\
    \sigma_{p_0} &= -k_i z_i - L_i h_i + y'_{id} \\
    \sigma_{p_0} &= -k_i z_i - L_i h_i + y'_{id}
\end{align*}
\]

(24)

3. Simulation Results

Harmonic current filtering, reactive power compensation, load current balancing and neutral current elimination performance of the FL-SAPF with the proposed control have been examined under balanced and unbalanced nonlinear load. The parameters used are shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1 System Parameters</th>
</tr>
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<tbody>
<tr>
<td>RMS value of phase voltage</td>
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<tr>
<td>DC-link capacitor C</td>
</tr>
<tr>
<td>Source impedance R_s, L_s</td>
</tr>
<tr>
<td>Filter impedance R_f, L_f</td>
</tr>
<tr>
<td>Line impedance R_l, L_l</td>
</tr>
<tr>
<td>DC-link voltage reference ( v_{dc, ref} )</td>
</tr>
<tr>
<td>Diode rectifier load R_d, L_d</td>
</tr>
<tr>
<td>Switching frequency ( f_s )</td>
</tr>
<tr>
<td>Sampling frequency</td>
</tr>
<tr>
<td>( k_1, k_2, k_3, k_4 ) constants</td>
</tr>
</tbody>
</table>
Fig. 2. Simulation results of the proposed backstepping controller. (a) Source current, (b) Load current, (c) Source voltage and source current of a-phase, (d) Neutral current. (e) DC-link voltage $v_{dc}$.

Fig. 3. Simulation results with PI controller. (a) Source current, (b) Load current, (c) Source voltage and source current of a-phase, (d) Neutral current, (e) DC-link voltage $v_{dc}$.

Fig. 4. Harmonic spectrum. (a) With backstepping controller. (b) With PI controller, before compensation.
The harmonic spectrums of ac supply current before and after compensation are illustrated in Fig. 5. It results that the active filter decreases the total harmonic distortion (THD) in the supply currents from 16.94% to 2.18% with PI controller. However, with backstepping controller, the THD is increased to 1.29% which proves the effectiveness of the proposed nonlinear controller.

The dynamic behavior under a step change of the load in phase b at t = 0.3s (to make unbalanced of the nonlinear load) is presented in Figs. 3 and 4. It can be observed that the network current become perfectly sinusoidal after the control application, the neutral current is compensated, and unity power factor operation is successfully achieved, even in this transient state.

The absence of an overshoot in dc voltage response during load change, low rise time and low source current THD, demonstrates the superiority of the backstepping controller compared to its counterpart traditional PI controller.

4. CONCLUSION

This work presented a backstepping control technique applied to a three-phase four-leg shunt APF. The developed controller was aimed to compensate for harmonics and unbalance in the case of distorted nonlinear load currents, by making the source currents in phase with their corresponding phase voltages. Also, the controller was capable of eliminating the current flowing in the neutral line. Finally, the simulation results validated both the steady state and dynamic behavior of the proposed controller.

References