Robust Nonlinear Control for Direct Torque Control of Induction Motor Drive Using Space Vector Modulation

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Abstract: To solve the problems of torque ripple and inconstant switch frequency of inverter in the conventional direct torque control (DTC), a novel DTC method using space vector modulation (SVM) is proposed based on input–output feedback linearization technique, where hysteresis controller is substituted by input–output feedback linearization and switch table is substituted by SVM. In order to preserve the system robustness with respect to rotor resistances variations and uncertainties. IM drive simulation model with novel SVM-DTC is created and studied using MATLAB. Simulation results demonstrate the feasibility and validity of the proposed DTC system by effectively accelerating system response, reducing torque and flux ripple and a very satisfactory performance has been achieved.

Key words: input–output feedback linearization, induction motor, SVM, direct torque control, key parameter variation.

1. Introduction

The direct torque control (DTC) scheme has been increased due to several factors such as quick torque response and robustness against the motor parameter variations [1,2]. The conventional DTC algorithm using the hysteresis based voltage switching method has relative merits of simple structure and easy implementation. The performance of such a scheme depends on the error band set between the desired and measured torque and stator flux values. In addition, in this control scheme, the inverter switching frequency is changed according to the hysteresis bandwidth of flux and torque controllers and the variation of speed and motor parameters. Superior motor performance is achieved by narrower hysteresis bands especially in the high speed region. As a result, this approach will not be suitable for high power drives such as those used in tractions, as they require good torque control performance at considerably lower frequency.

To overcome the drawbacks problems, some researchers have suggested, the DTC scheme using the space vector modulation (SVM) techniques [3-5]. The control scheme in [6] is sensitive to parameters uncertainty, especially to the stator resistance variations and the stability will be affected by parameter variation. To solve this problem, feedback linearization techniques have been applied to the control of nonlinear plants such as robot manipulators, induction motors, PM synchronous motors and synchronous reluctance motor [7-15]. The main objective is to force the speed and torque of an induction motor to follow their reference trajectories. The basic idea is to first transform a nonlinear system into a linear one by a nonlinear feedback, and then use the well-known linear design techniques to complete the controller design. These techniques, however, require the full knowledge of the system parameters and load conditions with the sufficient accuracy. Recently, an adaptive input–output linearization technique, adaptive backstepping, and adaptive sliding mode have been applied to the induction motor drives [16-23]. Although good performance can be obtained.

The contribution of this paper is to describe a robust DTC-SVM method for a torque and flux control of induction motor drive based on input–output feedback linearization technique. The results show that a satisfactory control performance is obtained.

2. The IM model

With the simplifying assumptions relation to the IM, the model of the IM expressed in the stationary "αβ" axes reference frame can be expressed by:

\[
\frac{di_{a}}{dt} = \left( \frac{R_s}{\sigma L_s} + \frac{R}{\sigma L_r} \right) i_{a} + \omega \Phi_{b} - \omega \Phi_{a} + \frac{V_{sa}}{\sigma L_s} \\
\frac{di_{b}}{dt} = \left( \frac{R_s}{\sigma L_s} + \frac{R}{\sigma L_r} \right) i_{b} + \omega \Phi_{a} - \omega \Phi_{b} + \frac{V_{sb}}{\sigma L_s} \\
\frac{d\Phi_{a}}{dt} = V_{sa} - R_{i} i_{a} \\
\frac{d\Phi_{b}}{dt} = V_{sb} - R_{i} i_{b}
\]

(1)

where \( i_{a}, \Phi_{a}, V_{s}, R \) and \( L \) denote stator currents,
The generated torque of the induction motor can be expressed in terms of stator currents and stator flux linkage as

\[ T_e = \frac{3p}{2}(\Phi_{s\alpha}i_{\beta} - \Phi_{s\beta}i_{\alpha}) \]

(2)

where \( p \) is the number of pole pairs.

The mechanical dynamic equation is given by

\[ \frac{d\omega_m}{dt} - \frac{3p}{2}(\Phi_{s\alpha}i_{\beta} - \Phi_{s\beta}i_{\alpha}) - \frac{T_i}{J} \]

(3)

where \( J \) and \( T_i \) denote the moment of inertia of the motor, the load torque and \( \omega_m \) is the rotor speed.

The generated torque of the induction motor can be rewritten in a compact form as

\[ \dot{x} = f(x) + g_1(x)\dot{V}_{ss} + g_2(x)V_{ss} \]

\[ y = h(x) \]

(4)

where \( x \) is defined as:

\[ x = \left[ i_{s\alpha}^2, i_{s\beta}^2, \Phi_{s\alpha}^2, \Phi_{s\beta}^2 \right]^T \]

\[ g_1(x) = \left[ \frac{1}{\sigma L_s} 0 1 0 \right]^T \]

\[ g_2(x) = \left[ 0 \frac{1}{\sigma L_s} 0 1 \right]^T \]

(5)

At this stage, the generated torque \( T_e \) and the squared modules of the stator flux linkage \( \Phi_{s\alpha}^2 + \Phi_{s\beta}^2 \) are assumed to be the system outputs. Therefore, by considering

\[ h_1(x) = T_e = \frac{3p}{2}(\Phi_{s\alpha}i_{\beta} - \Phi_{s\beta}i_{\alpha}) \]

\[ h_2(x) = \Phi_{s\alpha}^2 + \Phi_{s\beta}^2 \]

(7)

Define the controller objectives \( y_1 \) and \( y_2 \) as

\[ y_1 = h_1(x) \]

\[ y_2 = h_2(x) \]

(8)

3. Input-output feedback linearization

To linearize the nonlinear model in (4), the controlled variable is differentiated with respect to time until the input appears. This can be easily done by introducing the Lie derivative.

3.1 Lie Derivatives

Consider system (4). Differentiating the output \( y \) with respect to time yields:

\[ \dot{y} = \frac{\partial h}{\partial x} \dot{x} + \frac{\partial h}{\partial \dot{x}} \ddot{x} + \frac{\partial h}{\partial x} f(x) + \frac{\partial h}{\partial \dot{x}} g(x) \]

(9)

where \( L_i h(x) = \frac{\partial h}{\partial x} f(x) + L_i h(x) - \frac{\partial h}{\partial \dot{x}} g(x) \)

The function \( L_i h(x) \) is called the Lie Derivative of \( h(x) \) with respect to \( f(x) \), and corresponds to the derivative of \( h \) along the trajectories of the system \( x = f(x) \). Similarly, \( L_i h(x) \) is called the Lie Derivative of \( h \) with respect to \( g \), and corresponds to the derivative of function \( h(x) \) along the trajectories of the system \( \dot{x} = g(x) \).

3.2 Relative degree of a nonlinear system

For nonlinear systems, the relative degree \( r \) of system (4) corresponds to the number of times the output \( y = h(x) \) has to be differentiated with respect to time before the input \( u \) appears explicitly in the resulting equations.

System (4) is said to have a relative degree \( r, 1 \leq r \leq n \) in \( R^n \) if \( \forall x \in R^n \):

\[ L_i h(x) = 0 \quad i = 1, 2, \ldots, r-1 \]

\[ L_i h(x) \neq 0 \]

(10)

where

\[ L_i h(x) = \sum_{i=1}^{n} L_i h(x) \]

Using the above notation, we can obtain that

3.2.1 Relative degree of the torque

\[ \dot{y}_1 = L_f h_1(x) + L_{g_1} h_1(x) \dot{V}_{ss} + L_{g_2} h_2(x) V_{ss} \]

\[ = \frac{\partial h}{\partial x} f(x) + \frac{\partial h}{\partial \dot{x}} g_1(x) \dot{V}_{ss} + \frac{\partial h}{\partial \dot{x}} g_2(x) V_{ss} \]

(11)

with
\[
\begin{align*}
L_i h_1 &= -\frac{3p}{2} \Phi_{\beta} = \left( -\frac{R_s}{\sigma L_s} + \frac{R_i}{\sigma L_i} \right) i_{sa} - \omega_i i_{\phi} + \frac{\omega_i}{\sigma L_s} \Phi_{\beta} \\
&\quad + \frac{3p}{2} \Phi_{\alpha} = \left( -\frac{R_s}{\sigma L_s} + \frac{R_i}{\sigma L_i} \right) i_{\phi} + \omega_j i_{sa} - \frac{\omega_j}{\sigma L_s} \Phi_{\alpha} \\
L_j h_1 &= \frac{3p}{2} \left( i_{\phi} - \frac{1}{L_s} \Phi_{\alpha} \right) \\
L_g h_1 &= \frac{3p}{2} \left( i_{sa} - \frac{1}{L_s} \Phi_{\alpha} \right) \\
\end{align*}
\]

The relative degree of \( y_1(x) \) is \( r_1 = 1 \).

### 3.2.2 Relative degree of the flux

\[
\begin{align*}
\dot{y}_2 &= L_j f_2(x) + L_j g_2(x) V_{\phi} + L_s g_2(x) V_{sa} \\
&= \frac{\partial}{\partial x} f_2(x) + \frac{\partial}{\partial x} g_2(x) V_{\phi} + \frac{\partial}{\partial x} g_2(x) V_{sa} \\
&= (12)
\end{align*}
\]

with

\[
\begin{align*}
L_i h_2 &= -2R_s \left( \Phi_{\alpha} \cdot - \Phi_{\beta} \right) \\
L_j h_2 &= 2\Phi_{\alpha} \\
L g h_2 &= 2\Phi_{\beta} \\
\end{align*}
\]

The relative degree of \( y_2(x) \) is \( r_2 = 1 \).

### 3.2.3 Relative degree of the system

The total degree of the system is equal to order \( N(r_1 + r_2 = N = 2) \). The system is exactly linearizable.

### 4. Decoupling matrix

The matrix defining the relation between the physical input \( u \) and the output derivative \( y(x) \) is given by the expression \((13)\).

\[
\begin{align*}
\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} &= A(x) + E(x) \begin{bmatrix} V_{sa} \\ V_{\phi} \end{bmatrix} \\
\end{align*}
\]

with

\[
A(x) = \begin{bmatrix} L_i h_1 \\ L_j h_2 \end{bmatrix} \\
E(x) = \begin{bmatrix} L_g h_1 \\ L_g h_2 \end{bmatrix} \\
\begin{bmatrix} 3p \left( i_{\phi} - \frac{1}{L_s} \Phi_{\alpha} \right) \Phi_{\beta} \\ 2\Phi_{\alpha} \end{bmatrix} = \begin{bmatrix} 3p \left( i_{\phi} - \frac{1}{L_s} \Phi_{\alpha} \right) \Phi_{\beta} \\ 2\Phi_{\alpha} \end{bmatrix} \\
\begin{bmatrix} \Phi_{\beta} \\ \Phi_{\alpha} \end{bmatrix} \end{align*}
\]

\[
\det(E) = 3p \left[ \frac{1}{L_s} \Phi_{\beta} + \Phi_{\alpha}^2 \right] + i_{\phi} \Phi_{\beta} + i_{sa} \Phi_{\alpha}
\]

using the induction motor model of \((1)\)

\[
\begin{align*}
i_{sa} &= \frac{1}{\sigma L_s} \Phi_{sa} - \frac{M}{\sigma L_i L_r} \Phi_{in} \\
i_{\phi} &= \frac{1}{\sigma L_s} \Phi_{\phi} - \frac{M}{\sigma L_i L_r} \Phi_{in}
\end{align*}
\]

Linking \((15)\) and \((16)\)

\[
\det(E) = -3p \left[ \frac{M}{\sigma L_i L_r} \Phi_{\phi} + \Phi_{sa} \Phi_{\alpha} \right]
\]

It is clear that the matrix \( E(x) \) is always reversible, the product of stator flux and rotor flux can not be equal to zero the following input-output feedback linearization is introduced for the system shown in \((4)\)

\[
\begin{align*}
V_{sa} &= E^{-1}(x) [-A(x) + \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}] \\
\end{align*}
\]

where

\[
V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}
\]

are the new inputs.

substituting \((18)\) in \((13)\), the system dynamic are

\[
\begin{align*}
V_i &= h_i(x) \\
V_2 &= h_2(x)
\end{align*}
\]

To ensure a perfect regulation and track the desired signals of the flux and torque towards their reference, \( V_1, V_2 \) are chosen as follows:

\[
\begin{align*}
V_i &= \Phi_{i\text{ref}} + k_1 \Phi_{i\text{ref}}^2 + k_2 (T_{i\text{ref}} - T_i) \\
V_2 &= \Phi_{i\text{ref}} + k_1 \Phi_{i\text{ref}}^2 + k_2 (T_{i\text{ref}} - T_i)
\end{align*}
\]

where subscript ‘ref’ denotes the reference value. \((k_1, k_2)\) are constant design parameters to be determined in order to make the decoupled system \((20)\) stable. The behavior of the linearized model is imposed by the poles placement methods. Theses coefficients are selected such as the equation \(s + k_1, s + k_2\) are the polynomials of Hurwitz.

### 5. Voltage space vector modulation

The voltage vectors, produced by a 3-phase PWM inverter, divide the space vector plane into six sectors as shown in Fig. 1.
In every sector, each voltage vector is synthesized by basic space voltage vector of the two side of sector and one zero vector. For example, in the first sector, \( \bar{V}_{s_{\text{ref}}} \) is a synthesized voltage space vector and expressed by:

\[
S_{\text{ref}} V_{T} = V_{0} T + V_{1} T + V_{2} T
\]

(21)

where, \( T_{0} \), \( T_{1} \), and \( T_{2} \) is the work time of basic space voltage vectors \( V_{0} \), \( V_{1} \), and \( V_{2} \) respectively.

The rest of the period spent in applying the null-vector. For every sector, commutation duration is calculated. The amount of times of vector application can all be related to the following variables:

\[
X = \frac{T}{E} \sqrt{2} V_{\text{ref}}
\]

(25)

\[
Y = \frac{T}{E} \left( \frac{\sqrt{2}}{2} V_{\text{ref}} + \frac{\sqrt{6}}{2} V_{\text{sat}} \right)
\]

(26)

\[
Z = \frac{T}{E} \left( \frac{\sqrt{6}}{2} V_{\text{ref}} + \frac{\sqrt{2}}{2} V_{\text{sat}} \right)
\]

(27)

The application durations of the sector boundary vectors are tabulated as follows:

<table>
<thead>
<tr>
<th>Sector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{1} )</td>
<td>Z</td>
<td>Y</td>
<td>-Z</td>
<td>-X</td>
<td>X</td>
<td>-Y</td>
</tr>
<tr>
<td>( T_{2} )</td>
<td>Y</td>
<td>-X</td>
<td>X</td>
<td>Z</td>
<td>-Y</td>
<td>-Z</td>
</tr>
</tbody>
</table>

The third step is to compute the three necessary duty cycles as:

\[
T_{\text{on}} = \frac{T_{1} - T_{2}}{2}
\]

(28)

\[
T_{\text{bon}} = T_{\text{on}} + T_{1}
\]

(29)

\[
T_{\text{con}} = T_{\text{bon}} + T_{2}
\]

(30)

The last step is to assign the right duty cycle \( T_{\text{con}} \) to the right motor phase according to the sector.

<table>
<thead>
<tr>
<th>Sector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{a} )</td>
<td>( T_{\text{bon}} )</td>
<td>( T_{\text{con}} )</td>
<td>( T_{\text{on}} )</td>
<td>( T_{\text{bon}} )</td>
<td>( T_{\text{con}} )</td>
<td>( T_{\text{bon}} )</td>
</tr>
<tr>
<td>( S_{b} )</td>
<td>( T_{\text{on}} )</td>
<td>( T_{\text{con}} )</td>
<td>( T_{\text{bon}} )</td>
<td>( T_{\text{bon}} )</td>
<td>( T_{\text{con}} )</td>
<td>( T_{\text{bon}} )</td>
</tr>
<tr>
<td>( S_{c} )</td>
<td>( T_{\text{con}} )</td>
<td>( T_{\text{bon}} )</td>
<td>( T_{\text{on}} )</td>
<td>( T_{\text{bon}} )</td>
<td>( T_{\text{con}} )</td>
<td>( T_{\text{bon}} )</td>
</tr>
</tbody>
</table>

6. Sensitivity study and simulation results

In this section, the effectiveness of the proposed algorithm for torque and flux control of an induction motor is verified by computer simulations. The specifications for the used induction motor are listed in table (3). The block scheme of the investigated direct torque control with space vector modulation (DTC-SVM) for a voltage source inverter fed IM is presented in (Fig. 3).

A series of tests were conducted to check the performance of the proposed DTC-SVM. In all sketched figures, the time axis is scaled in seconds.
6.1 Speed reversal from 100 rad/sec to -100 rad/sec

The motor reference speed is changed from 100 rad/s to -100 rad/sec at 0.5s and then again, speed is set to 100 rad/s at 1 s, without any change in parameters during the operating time. The performance of the proposed controller for such kind of speed reference is shown in Fig. 4. Plots the reference speed and, actual motor speed with respect to time. It is observed that the actual motor speed follow the reference with good accuracy.

6.2 Variation of the load torques.

Fig. 5 depicts the simulation results after the introduction of load torque of 10 Nm between 0.5 s and 1s after a leadless starting. We can see the insensitivity of the control algorithm to load torque variation and the stator flux responses are not affected by this perturbation.
6.3 Variation in the rotor resistance

These tests investigate the influence of the electrical parameters change on the drive performance. Fig. 6 depicts the drive performance for brusque changes in the rotor resistance. It can be seen that the impact of the electrical parameters change on the drive performance is more important. However, those results shown also that the drive robustness and rejection of the perturbations is significantly enhanced.

Fig. 5. Drive response under load torque change. 
(a) reference and actual rotor speed, (b) electromagnetic torque, (c) phase current, (d) stator flux magnitude.

Fig. 6. Drive response under rotor resistance change: 
(a) reference, (b) reference and actual rotor speed, (c), stator flux magnitude.
6.4 Drive response under flux-weakening

This test concerns the drive dynamic under flux-weakening operating. As depicted in Fig. 7, the flux and speed are not affected by the reference flux reduction. Phase current ripple has also a notable reduction.

Fig. 7. Drive response under flux-weakening. (a) Reference and actual rotor speed (b) stator flux magnitude, (a) phase current

6.5 Variation in the inertia coefficient.

Fig. 8 shows the drive dynamic under different values of inertia with constant speed reference. It is clear that the speed tracking is little affected by those changes.

7. Conclusion

In this paper, we present a robust direct torque control method for voltage inverter – fed induction motor based on a space vector modulation (SVM) scheme combined with input–output feedback linearization technique. The overall speed and flux control system was verified to be robust to the variations of motor mechanical and electrical parameters variations. Simulation studies were used to demonstrate the characteristics of the proposed method. It is shown that the proposed controller has better tracking performance and robustness against parameters variations as compared with the conventional direct torque control.
## Appendix

### Table 3

<table>
<thead>
<tr>
<th>Induction motor parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>4 KW</td>
</tr>
<tr>
<td>Pole pair</td>
<td>P=2</td>
</tr>
<tr>
<td>Nominal speed</td>
<td>1440 rpm</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>0.1545 H</td>
</tr>
<tr>
<td>Rotor inductance</td>
<td>0.1568 H</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>0.15 H</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>1.2 Ω</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>1.8 Ω</td>
</tr>
<tr>
<td>Machine inertia</td>
<td>0.07 kg.m²</td>
</tr>
<tr>
<td>Viscous coefficient</td>
<td>0.00031 kg.m²/s</td>
</tr>
<tr>
<td>Rated frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Reference flux</td>
<td>$\Phi_{ref} = -1.1$ Wb</td>
</tr>
</tbody>
</table>

## References