Abstract: This paper presents a solution of economic dispatch problem (EDP) with transmission losses using a Hopfield Neural Network (HNN) algorithm. A direct computation method has been developed and used for solving the ED problem, which employs a linear input-output model for the neurons. Formulations for solving the ED problem are explored. Through the application of these formulations, direct computation instead of iterations for solving the problem becomes possible. Unlike the usual Hopfield methods, which select the weighting factors of the energy function by trials, the proposed method determines the corresponding factors by calculations. The effectiveness of the developed method is identified through its application to the 15-unit system. Computational results manifest that the method has a lot of excellent performances.

Keywords: Economic Dispatch, Hopfield model, Energy function, Linear input-output model

1. Introduction

The economic dispatch problem (EDP) objective is to minimize production cost while satisfying demand and working area constraints for a given combination of active units. Aside from using the solutions of the EDP (combination of units with the least production cost) for its own merits in system operation, they are used to guide the solution method that solves the combinatorial part of the unit commitment problem. When the combinatorial part of the unit commitment problem is solved, solutions from the EDP are used to estimate the quality of different unit combinations.

In this chapter the construction and implementation of an exact method using the Hopfield Neural Network that solves both the economic dispatch problem is presented. The performance of such a method with respect to time and solution quality is a crucial part in the solution process of solving the unit commitment problem. The use of the Hopfield neural network methods to solve the EDP is therefore justifiable if the method produces optimal solutions and outperforms near-optimal solver with respect to computation time.

2. Problem Formulation

Economic dispatch (ED) is defined as the process of allocating generation levels to the thermal generating units in service within the power system, so that the system load is supplied entirely and most economically [1] and [2]. The objective of the ED problem is to calculate, for a single period of time, the output power of every generating unit so that all demands are satisfied at minimum cost, while satisfying different technical constraints of the network and the generators. The problem can be modeled by a system which consists of \( N \) generating units connected to a single bus-bar serving an electrical load \( D \). The input to each unit shown as \( F_i \) represents the generation cost of the unit. The output of each unit \( P_i \) is the electrical power generated by that particular unit. The total cost of the system is the sum of the costs of each of the individual units. The essential constraint on the operation is that the sum of the output powers must equal the load demand.

The standard ED problem can be described mathematically as an objective with two constraints as:

\[
\min F_T = \sum_{i=1}^{N} F_i(P_i) \quad (1)
\]

Subject to the following constraints:
\[ \sum_{i} P_i = D + L \] \hspace{1cm} (2)

\[ P_i^{\min} \leq P_i \leq P_i^{\max} \]

Where

- \( P_i \): Real power output of \( i \)-th generator (MW);
- \( F_i \): Total operating cost ($/h);
- \( F_i(P_i) \): Operating cost of unit \( i \) ($/h);
- \( D \): Total demand (MW);
- \( L \): Transmission losses (MW);
- \( P_i^{\min}, P_i^{\max} \): Operating power limits of unit \( i \) (MW);
- \( N \): total number of units in service.

The fuel cost function or input-output characteristic of the generator may be obtained from design calculations or from heat rate tests. Many different formats are used to represent this characteristic. The data obtained from heat rate tests or from the plant design engineers may be fitted by a polynomial curve. It is usual that, quadratic or cubic polynomial curve. It is usual that, quadratic or cubic polynomial curve.

The Willans line [5] described by:

\[ F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \] \hspace{1cm} (3)

Where, \( a_i, b_i, c_i \) are the cost coefficients (non-negative constants) of the \( i \)-th generating unit.

For some generators such as large steam turbine generators, however, the input-output characteristic is not always as smooth as (2.3). Large steam turbine generators will have a number of steam admission valves that are opened in sequence to obtain ever-increasing output of the unit [3], [4]. The fuel cost function in this case can be expressed as:

\[ F_i(P_i) = a_i + b_i P_i + c_i P_i^2 + \left[ e_i \sin (f_i (P_i^{\min} - P_i)) \right] \] \hspace{1cm} (4)

Where \( e_i \) and \( f_i \) are non-negative constants.

Alternatively, fuel cost functions may be represented by piecewise-linear cost functions, such as the Willans line [5] described by:

\[ F_i(P_i) = \text{inci}^k_i P_i + nl_i^k \] \hspace{1cm} k = 1,2,\ldots,3 \hspace{1cm} (5)

Where:

- \( \text{inci}^k_i \): incremental cost of segment \( k \) of unit \( i \) ($/MWh), \( k = 1, 2, \) and 3;
- \( nl_i^k \): no-load cost of segment \( k \) of unit \( i \) ($/h), \( k = 1, 2, \) and 3;

Eq(5) can be written in more detail as:

\[ F_i(P_i) = \text{inci}^1_i P_i + nl_i^1 \] \hspace{1cm} \( P_i^{\min} \leq P_i < e_i^1 \) \hspace{1cm} (6)

\[ F_i(P_i) = \text{inci}^2_i P_i + nl_i^2 \] \hspace{1cm} \( e_i^1 \leq P_i < e_i^2 \) \hspace{1cm} (7)

\[ F_i(P_i) = \text{inci}^3_i P_i + nl_i^3 \] \hspace{1cm} \( e_i^2 \leq P_i \leq P_i^{\max} \) \hspace{1cm} (8)

Where:

- \( P_i^{\min} \) and \( P_i^{\max} \) are the lower and upper generation limits of unit \( i \), respectively [MW];
- \( e_i^1 \) and \( e_i^2 \): are the first and second breaking points of the piece-wise linear cost function of unit \( i \), respectively [MW].

The total production cost expressed with piece-wise cost function is as follows:

\[ F_t = \sum_{i=1}^{N} F_i = \sum_{i=1}^{N} \left( \text{inci}^k_i P_i + nl_i^k \right) \] \hspace{1cm} (9)

\[ k = 1 \text{ if } P_i^{\min} \leq P_i < e_i^1 \]

\[ k = 2 \text{ if } e_i^1 \leq P_i < e_i^2 \]

\[ k = 3 \text{ if } e_i^2 \leq P_i \leq P_i^{\max} \] \hspace{1cm} (10)

3. Hopfield Neural Network Applied to Economic Dispatch

The EDP has been widely studied and reported by several authors in books and journals on power system analysis. Many techniques have been developed to solve this problem, e.g. the lambda-iterative method, gradient technique, Interior Point, Lagrange technique, linear programming, Quadratic Programming, Dynamic Programming, Simulated Annealing, Genetic algorithm (GA), Evolutionary Programming (EP), Neural Network and methods combining two or more of the above methods [6] and [7]. Most of these methods often suffer from the large amount of computational requirement or give just a good estimate (near optimal) of the solution to the EDP.

3.1. Hopfield Neural Network

The Hopfield model of Neural Networks was investigated by John Hopfield in the early 1980s. The Hopfield network has no special input or output neurons. All neurons are both input and output, and each neuron is connected to all other neurons in both directions (with equal weights in the two directions). Input is applied simultaneously to all neurons. The output of each neuron is then supplied to all other
neurons. The process continues until a stable state is reached, which represents the network output.

Hopfield Neural Network (HNN) is the widely used model for solving combinatorial optimization problems [8]. These networks have three major forms of parallel organization found in neural systems, namely, parallel input, parallel output channels, and a large amount of interconnectivity between the neural processing elements.

Two types of Hopfield Neural Network models are widely used namely the Binary (Discrete) Model and the Analog (Continuous) Model.

In the binary model, each neuron or processing element has two states, \( V_i^0 \) or \( V_i^1 \), which may be taken as 0 or 1. The inputs to the neuron come from two sources, one from the external inputs \( I_i \) and the other from the other neurons \( V_j \).

\[
U_i = \sum_{j=1}^{N} T_{ij} V_j + I_i \tag{11}
\]

Where:
- \( U_i \): The total input to neuron \( i \),
- \( T_{ij} \): The interconnection conductance from the output of neuron \( j \) to the input of neuron \( i \),
- \( I_i \): The external input to neuron \( i \),
- \( V_j \): The output of neuron \( j \).

Each neuron \( i \) samples its input randomly according to a threshold \( \theta_i \) rule given as

\[
V_i = \begin{cases} 0, & \text{if } U_i < \theta_i \\ 1, & \text{if } U_i > \theta_i \end{cases} \tag{12}
\]

The continuous or deterministic model of the Hopfield Neural Network is based on continuous variables. The output variable of neuron \( i \) has the range \( V_i^0 < V_i < V_i^1 \) and the input-output function is a continuous and monotonically increasing function of the input \( U_i \) to neuron \( i \). The model is a mutual coupling neural network and of non-hierarchical structure. The dynamic characteristic of each neuron can be described by the following differential equation [9] and [10].

\[
\frac{dU_i}{dt} = \sum_{j=1}^{N} T_{ij} V_j + I_i \tag{13}
\]

Where:
- \( T_{ii} \): The self-connection conductance of neuron \( i \).

The output of neuron is given by

\[
V_i = f_i(U_i) \tag{14}
\]

Where \( f_i(U_i) \) is the input-output function of the neuron \( i \).

The energy function of the continuous Hopfield model can be defined as:

\[
E = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} T_{ij} V_i V_j - \sum_{i=1}^{N} I_i V_i \tag{15}
\]

The time derivative of the energy function can be proved to be negative [11]. Therefore, in the computation process the model state always moves in such a way that the energy function gradually reduces and converges to a minimum.

This model is suitable for economic dispatch, while the discrete model is suitable for unit commitment.

3.2. Mapping Economic Dispatch to Hopfield Neural Network

The Hopfield model of neural networks [12] has been employed to solve the ED problem for units having continuous or piece wise quadratic fuel cost function [9] and [13], and even for units having prohibited zones constraint [14] and [15]. The conventional Hopfield model belongs to the kind of continuous and deterministic model, and the input-output relationship for its neurons is described by a modified sigmoidal function. Due to the use of sigmoidal function in the conventional Hopfield model, in solving the ED problems, a method involving numerical iterations is inevitably applied; this numerical iteration method often suffers from large amount of computational requirements. Adopting a modified sigmoidal function causes two other problems. The first, it incurs unreasonable or incorrect generation dispatch, which is attributable to the serious saturation phenomena existing in the input-output relationship represented by the sigmoidal function. The second; it is troublesome to select shape constant of the sigmoidal function.

A fast Hopfield Neural Network method to solve the ED problem is presented. The method employs a linear input-output model for the neurons. Formulations for solving the ED problem are explored. Through the application of these formulations, direct computation instead of iterations for solving the problem becomes possible. Not like the usual Hopfield methods, which select the weighting factors of the energy function by trials, this method determines the corresponding factors by calculation.
The adoption of a linear model describing the input-output relationship of the neuron has resulted in the avoidance of the aforementioned problems.

To solve the ED problem using the Hopfield method, energy function including both power mismatch, \( P_m \) and total fuel cost \( F \) is defined as follows:

\[
E = \frac{A}{2} \left( (D + L) - \sum_{i=1}^{N} P_i \right)^2 + \ldots
\]

\[
= \frac{A}{2} \sum_{i=1}^{N} \left( a_i + b_i P_i + c_i P_i^2 \right) + \frac{B}{2} \sum_{i=1}^{N} P_i^2
\]

\[
= \frac{A}{2} P_m^2 + \frac{B}{2} F_i
\]

\( A \) and \( B \) introduce the relative importance of their respective associated terms.

Comparing eq(15) with eq(16), we get:

\[
T_{ii} = -A - B \cdot c_i
\]

\( T_{ij} = -A \) \hspace{1cm} (17)

\[
i_i = A (D + L) - B (b_i / 2)
\]

\( (18) \)

At this stage the transmission losses \( L \) can be neglected and reconsidered later in section 4.

Substituting eq(17), eq(18) and eq(19) into (13), the dynamic equation becomes,

\[
\frac{dU_i}{dt} = A P_m - \frac{B}{2} \left( \frac{dF_i}{dP_i} \right)
\]

(20)

Application of the conventional Hopfield method to the ED problem, the power output value can be represented by the output \( V_i \) of neuron \( i \) using a modified sigmoidal function, described as follows [13] and [14]:

\[
P_i = f_i(U_i)
\]

\[
= P_i^{\min} + \frac{1}{2} \left( P_i^{\max} - P_i^{\min} \right) \left( 1 + \tanh \left( \frac{U_i}{u_0} \right) \right)
\]

(21)

Where:

\( u_0 \): the shape constant of the sigmoidal function.

To avoid the problems resulting from curve saturation, a linear model shown in figure 1 is used to describe the input-output relationship for the neuron instead of the sigmoidal function. Linear transfer function of the \( i \)th neuron is defined as follows:

\[
P_i = f_i(U_i)
\]

\[
= \begin{cases} 
    U_i - U_{i\text{min}} (P_i^{\max} - P_i^{\min}) + P_i^{\min} & \text{if } U_{i\text{min}} \leq U_i \leq U_{i\text{max}} \\
    P_i^{\max} & \text{if } U_i \geq U_{i\text{max}} \\
    P_i^{\min} & \text{if } U_i \leq U_{i\text{min}} 
\end{cases}
\]

(22)

Fig 1. The proposed linear input-output function

Substituting eq(22) in eq(20) the dynamic equation becomes:

\[
\frac{dU_i}{dt} = A P_m - \frac{B}{2} \left( \frac{dF_i}{dP_i} \right)
\]

(23)

With

\[
K_i = \frac{P_i^{\max} - P_i^{\min}}{U_{i\text{max}} - U_{i\text{min}}}
\]

(24)

Solving (23) the neuron’s input function, \( U_i(t') \) is obtained as:

\[
U_i(t') = \left( U_i(0) + \frac{K_{3i}}{K_{3i}} e^{K_{3i} t'} - \frac{K_{4i}}{K_{3i}} \right)
\]

(25)

With

\[
K_{3i} = \frac{B}{2} c_i K_{2i}
\]

(26)

From eq(22), the neuron’s output function, \( P_i(t') \), is obtained as:

\[
P_i(t') = \frac{2 K_{4i} P_m - b_i}{2c_i} + \ldots
\]

(27)

\[
\left( K_{3i} U_i(0) + K_{2i} - \frac{2 K_{4i} P_m - b_i}{2c_i} \right) e^{K_{3i} t'}
\]

Where \( K_{4i} = \frac{A}{B} \)
The second term in eq(27) decays exponentially, finally becomes vanishingly small and eventually setting \( t' = \infty \), eq(27) gives:

\[ P_i(x) = \frac{2K_{ab}P_m - b_i}{c_i} \quad (28) \]

Here \( P_i(x) \) represents the optimal generation level of unit \( i \), which is the required solution.

Back substituting of eq(28) in eq (27), a more simple formula for the generation function is given as:

\[ P_i(t) = P_i(x) + (P_i(0) - P_i(x)) e^{|K_{x}| t} \quad (29) \]

Where \( P_i(0) \) is obtained from eq(27) by letting \( t' = 0 \), to give:

\[ P_i(0) = K_{x} + K_{x}U_i(0) \quad (30) \]

It should be noted here that \( t' \) is not representing real time, it is a dimensionless variable.

Using the power mismatch definition and eq(28) we obtain:

\[ P_i = D + \frac{1}{2} \sum_{i=1}^{N} b_i \left[ 1 + K_{ab} \sum_{i=1}^{N} \left( \frac{1}{c_i} \right) \right] \quad (31) \]

Equations (28) through eq(31) constitute the Hopfield model for the economic dispatch problem. A non iterative direct computation process is, therefore, possible.

4. Inclusion of transmission losses in a hybrid algorithm

The transmission losses \( L \) can be either given from a load flow study or approximated by traditional representation using \( B \) coefficients:

\[ L = \sum_{i=1}^{N} \sum_{j=1}^{N} P_{ij} B_{ij} P_j + \sum_{i=1}^{N} B_{ii} P_i + B_{oo} \quad (32) \]

\[ L = PB^T + B_0 P + B_{oo} \quad (Matrix form) \quad (33) \]

Where

\( P: \) vector of generator loading \((P_1, P_2, \ldots, P_N)\),

\( B_i: \) loss-coefficient matrix,

\( B_0: \) loss-coefficient vector,

\( B_{oo}: \) loss constant.

A Bisection solution method for solving the economic dispatch including transmission losses combined to the Hopfield Neural Network is presented in the following steps:

Step1: initialization of the interval search \([D_3, D_1]\).

\( \varepsilon: \) a pre-specified tolerance.

Initialize the iteration counter \( k = 1 \).

\( D_3^k = D \); 

\( D_1^k = D_3^k + 0.1 * D_3^k \); 

\( D_k^k = D_j^k + (D_j^k - D_j^k) / 2 \); 

Step2: Determine the optimal generators’ power outputs \( P_i, \ i = 1, \ldots, N \) using the Hopfield Neural Network algorithm, by neglecting losses and setting the power demand as \( D_k = D_k^k \).

Step3: Calculate the transmission losses \( L_k \) for the current iteration \( k \) using eq(32); 

Step 4: if \( D_1^k - D_3^k < \varepsilon \), stop otherwise go to step 5; 

Step5: if \( D_k^k - L_k < D \), update \( D_j \) and \( D_2 \) for the next iteration as follows:

\( D_j^{k+1} = D_j^k \);

\( D_2^{k+1} = D_2^k - (D_2^k - D_2^k) / 2 \);

Replace \( k \) by \( k+1 \) and go to step 2;

Step 6: if \( D_k^k - L_k > D \), update \( D_1 \) and \( D_2 \) for the next iteration as follows:

\( D_{j+1} = D_j^k \);

\( D_{2+1} = D_2^k - (D_2^k - D_2^k) / 2 \);

Replace \( k \) by \( k+1 \) and go to step 2.

5. Results and Discussion

To demonstrate the performance of the Hopfield based ED solver, A 15-unit test system [16] is used, where the convergence criteria considered here is the unit generation constraints must be not violated. The system consists of 15-units where data is given in table1. For comparison the case of a load demand of 2650 MW is considered as in [16].

The total operating cost of the system is represented by the following polynomial,

\[ F_i = \sum_{i=1}^{N} F_i(P_i) = \sum_{i=1}^{N} \left( a_i + b_i \cdot P_i + c_i \cdot P_i^2 \right) \quad (34) \]

Where the polynomial coefficients are listed in table 1 along with generator minimum and maximum operating limits. 
The loss coefficients matrix $B_i/10^2$, vector $B_0$ and constant $B_{00}$ are shown in the following:

$$
\begin{align*}
0.0014 & \quad 0.0012 & \quad 0.0007 & \quad -0.0001 & \quad -0.0003 & \quad -0.0001 & \quad -0.0001 & \quad -0.0003 & \quad -0.0005 & \quad -0.0003 & \quad -0.0002 & \quad 0.0004 & \quad 0.0003 & \quad -0.0001 \\
0.0012 & \quad 0.0015 & \quad 0.0013 & \quad 0.0000 & \quad -0.0005 & \quad -0.0002 & \quad 0.0000 & \quad 0.0001 & \quad -0.0002 & \quad -0.0004 & \quad -0.0001 & \quad -0.0000 & \quad 0.0004 & \quad 0.0001 & \quad -0.0002 \\
0.0007 & \quad 0.0013 & \quad 0.0076 & \quad -0.0001 & \quad -0.0013 & \quad -0.0009 & \quad -0.0001 & \quad 0.0000 & \quad -0.0008 & \quad -0.0012 & \quad -0.0017 & \quad -0.0000 & \quad -0.0026 & \quad 0.0111 & \quad -0.0028 \\
-0.0001 & \quad 0.0000 & \quad -0.0001 & \quad 0.0034 & \quad -0.0007 & \quad -0.0004 & \quad 0.0011 & \quad 0.0050 & \quad 0.0002 & \quad 0.0032 & \quad -0.0011 & \quad -0.0000 & \quad 0.0001 & \quad 0.0011 & \quad -0.0026 \\
-0.0003 & \quad -0.0005 & \quad -0.0013 & \quad -0.0007 & \quad 0.0090 & \quad 0.0147 & \quad -0.0003 & \quad 0.0012 & \quad 0.0010 & \quad 0.0013 & \quad 0.0007 & \quad 0.0000 & \quad 0.0006 & \quad 0.0024 & \quad -0.0003 \\
-0.0001 & \quad -0.0002 & \quad -0.0009 & \quad 0.0004 & \quad 0.0148 & \quad 0.0016 & \quad 0.0000 & \quad 0.0000 & \quad 0.0005 & \quad 0.0018 & \quad 0.0111 & \quad -0.0000 & \quad 0.0117 & \quad 0.0017 & \quad 0.0003 \\
-0.0001 & \quad 0.0000 & \quad -0.0000 & \quad 0.0011 & \quad -0.0003 & \quad 0.0000 & \quad 0.0015 & \quad 0.0017 & \quad 0.0015 & \quad 0.0009 & \quad 0.0005 & \quad 0.0007 & \quad 0.0000 & \quad 0.0120 & \quad 0.0008 \\
-0.0001 & \quad 0.0001 & \quad 0.0000 & \quad 0.0050 & \quad -0.0012 & \quad -0.0006 & \quad 0.0017 & \quad 0.0168 & \quad 0.0082 & \quad 0.0079 & \quad -0.0023 & \quad -0.0363 & \quad 0.0006 & \quad 0.0078 & \quad 0.0005 \\
-0.0003 & \quad -0.0002 & \quad 0.0008 & \quad 0.0029 & \quad -0.0010 & \quad -0.0005 & \quad 0.0015 & \quad 0.0082 & \quad 0.0129 & \quad 0.0116 & \quad 0.0017 & \quad -0.0025 & \quad 0.0057 & \quad 0.0012 & \quad -0.0017 \\
-0.0005 & \quad -0.0004 & \quad -0.0012 & \quad 0.0032 & \quad -0.0013 & \quad -0.0008 & \quad 0.0009 & \quad 0.0079 & \quad 0.0116 & \quad 0.0200 & \quad 0.0027 & \quad 0.0034 & \quad 0.0009 & \quad 0.0011 & \quad -0.0088 \\
-0.0003 & \quad -0.0001 & \quad 0.0007 & \quad 0.0001 & \quad -0.0005 & \quad -0.0005 & \quad 0.0005 & \quad -0.0005 & \quad 0.0140 & \quad 0.0001 & \quad 0.0004 & \quad 0.0000 & \quad 0.0038 & \quad 0.0016 \\
-0.0002 & \quad 0.0000 & \quad -0.0000 & \quad 0.0000 & \quad -0.0002 & \quad -0.0001 & \quad 0.0007 & \quad 0.0036 & \quad 0.0025 & \quad 0.0034 & \quad 0.0001 & \quad 0.0054 & \quad 0.0000 & \quad 0.0000 & \quad 0.0028 \\
0.0004 & \quad 0.0004 & \quad -0.0026 & \quad 0.0001 & \quad -0.0002 & \quad -0.0000 & \quad 0.0000 & \quad 0.0000 & \quad 0.0000 & \quad 0.0001 & \quad 0.0000 & \quad 0.0000 & \quad 0.0004 & \quad 0.0000 & \quad 0.0013 \\
0.0003 & \quad 0.0010 & \quad 0.0111 & \quad 0.0001 & \quad -0.0024 & \quad -0.0017 & \quad 0.0005 & \quad -0.0012 & \quad -0.0011 & \quad -0.0008 & \quad -0.0004 & \quad -0.0010 & \quad 0.0578 & \quad -0.0094 \\
-0.0001 & \quad -0.0002 & \quad -0.0028 & \quad -0.0002 & \quad -0.0003 & \quad 0.0003 & \quad -0.0008 & \quad -0.0078 & \quad -0.0072 & \quad -0.0088 & \quad 0.0168 & \quad 0.0028 & \quad 0.0002 & \quad -0.0094 & \quad 0.1283 \\
\end{align*}
$$

$$
B_{00} = 0.0055.

The seventh column of table 1 shows the optimal generators’ power outputs when the transmission losses is neglected. Total production cost is $ 32542.30. The problem was carried out on Pentium M 1.73 MHz using the presented Hopfield method with $U_{\min} = -0.5$, $U_{\max} = 0.5$ and $P_{in} = 0.001$. The computation time was about 0.14 s. The same test system was solved in [16], the total production cost is obtained as $ 32549.8$. It can be seen that the presented Hopfield approach could provide a better solution within a much shorter time. The last column of table 1 shows the optimal generators’ power outputs when the transmission losses is taken into account. The pre-specified tolerance was taken as 0.001. Total production cost is $ 32880.42, and the transmission losses equal to 32.1138 MW. The computation time was about 0.51 s for 21 iteration. The iterative search results using the HNN method for iterations (1 to 11 and last iteration 21) are given in table 2.
### Table 2. Iterative search results using HNN method for the 15-unit system.

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<th>$P_i$ ($k=1$) (MW)</th>
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### 6. Conclusion

A Hopfield neural network method combined to bisection has been developed for ED problems solution with transmission losses. This fast-computation solver, overcomes the drawbacks of the conventional segmoidal function by adopting a linear input/output transfer function, resulting in a superior Hopfield neural network as one calculation process is required (i.e. No iterations). This led to a very short computing time and suitability for on-line usage. The proposed method is relatively simple, straightforward, efficient, easy to apply and requires no training. Its connective conductances and external input can be determined directly by employing system data.
References


