Power Angle Control of Virtual Synchronous Generator

Gamal M. Sarhan 1, Amr M. Abdin 2, Mohamed M. Shaalan 3
1 Elect. Eng. Dept., College of Eng., Northern Border University, Arar, Kingdom Saudi Arabia
2 Elect. Power & Machine Dept., Faculty of Eng. Ain-Shams University, Cairo, Egypt
3 Elect. Eng. Dept., Benha Faculty of Engineering, Benha University, Qalubiya, Egypt
e-mail: Mohamed.shaalan@bhit.bu.edu.eg

Abstract— Inverter-based distributed generators (DGs) can be operated in parallel comprising an isolated microgrid or connected to the main power system. In the first case there must be an advanced control system to adjust frequency and power sharing between inverters constituting the microgrid. In grid mode the DGs inject electrical power to grid without any contribution to rotating inertia that give the power system its distinctive stability characteristics and conventional control liability. Improvement to DGs by the relatively new control methods based on virtual synchronous generator (VSG) and synchronverter (SV) have been suggested to control such DGs to emulate conventional synchronous generator. By these methods a controlled amount of virtual inertia can be injected in mains besides load sharing. This paper suggests a control method of such systems based on direct control of power angle between inverter voltage and grid voltage. In this method the grid phase angle is directly computed from the grid 3-phase voltages without phased locked loop (PLL) or other synchronization methods. A linearized small signal model of power angle is used to implement this control method. The model can also emulate rotating inertia, injecting a controllable amount of energy into grid during frequency transients to enhance transient stability. Damping effect of damper bars is taken into account in model and proves to be effective in damping out oscillations.

Index Terms— Distributed generation, Damping, Frequency control, inverter-dominated power system, load sharing, microgrid, parallel inverters, pulse width modulation (PWM) inverter, renewable energy, static synchronous generator (SG), synchronverter (SV), microgrids, power control, renewable energy sources (RES), virtual synchronous generator (VSG).

I. Introduction

Distributed generators (DGs) emerging as a natural consequence of advanced application of renewable energy sources (RES), such as photovoltaic and wind turbines. Renewable energy sources principally cover large geographical areas and economic considerations necessitate utilization of these resources in places where this energy available. It is different from this point of view from conventional power system where electrical power is generated in power stations and then utilized at load centers usually far from power station using transmission and distribution systems. In places far from national grid such renewable DGs can be operated together in form of small or micro grids (MG). MGs based on renewable energy sources and inverter have attracted too many research efforts and operated on principles completely different from those used in conventional power grid. A large number of DGs are operated connected to main grid and planned to share a remarkable proportion of total generated power. In this case the conventional power system well-settled operation and control methods need special algorithms to adopt these DGs in way similar to conventional SG.

The terminology Droop Control, Synchronverter (SV) and Virtual Synchronous Generators (VSG) refer to control strategies to operate inverter side of DG to satisfy the above requirement [1]-[11]. Usually a frequency droop algorithm emulate real governor control of active power sharing of DG and a voltage droop algorithm is used to control reactive power sharing [1]-[3]. The swing equation of SG rotor is emulated to give Virtual dynamic response equivalent to actual generator [8]-[18]. In these control methods, commonly a phase-locked loop (PLL) is used to synchronize the inverter to the grid [5].

Another way to increase inertia contribution of DG is to adopt it as a virtual inertia. This can be attained for any DG by adding a short-term-energy-storage (Batteries), combined with a suitable control mechanism for the power electronics converter of the DG. In this way DG behaves as a source of energy similar to freewheeling effect of real SG inertia during short time intervals, and contributes to stabilization of the grid frequency during fluctuations in the grid loads [8], [12]-[19].

In this paper a linearized small signal model of the swing equation [20] is used to adjust the power angle is directly to control active power share. The 3-phase voltage signals of grid are mathematically manipulated to obtain peak grid voltage, phase angle and frequency. A control algorithm is used to calculate the magnitude voltage and angle of the PWM referenced to grid voltage phase-angle. No need of PLL or similar synchronizing method since the inverter voltage is directly referenced to the grid voltage.

II. MODEL OF SYNCHRONOUS GENERATOR

In steady state condition the equivalent circuit of a cylindrical rotor synchronous generator connected to an infinite bus is at presented in Fig.1. The Kirchhoff voltage low in this case gives the following phasor equations [20].

\[ E = I_a (R_a + jX_s) \]

Fig. 1 Synchronous machine equivalent circuit.
\[ E = V + [R_a + jX_a]I_a \text{ or } E = V + Z_d I_a \]  
\[ Z_S = R_a + jX_a = [Z_d] \omega \]  

Where \( Z_S \) is the synchronous impedance, \( R_a \) is the armature winding resistance and \( X_a \) is the synchronous reactance.

The phasor diagram in this case is shown in Fig.2. The angle \( \delta \) between excitation emf \( E \) and terminal voltage \( V \), is related to developed torque and power as follows:

\[ I_a = \frac{|E - jV\angle\gamma|}{|Z_a|\angle\gamma} \]  

Thus, the real power \( P_{3a} \) and reactive power \( Q_{3a} \) are:

\[ P_{3a} = 3 \frac{|E||V|}{|Z_a|} \cos(\gamma - \delta) - 3 \frac{|V|^2}{|Z_a|} \cos \gamma \]  

\[ Q_{3a} = 3 \frac{|E||V|}{|Z_a|} \sin(\gamma - \delta) - 3 \frac{|V|^2}{|Z_a|} \sin \gamma \]

If \( R_a \) is neglected with respect to \( X_a \), then \( Z_a = jX_a \) and \( \gamma = 90^\circ \). Previous Equations reduce to:

\[ P_{3a} = 3 \frac{|E||V|}{X_a} \sin \delta \]  

\[ Q_{3a} = 3 \frac{|V|^2}{X_a} (|E| \cos \delta - |V|) \]

Eq. 4 shows that if \( |E| \) and \( |V| \) are held fixed and the power angle \( \delta \) is changed by varying the mechanical driving torque, the power transfer varies sinusoidal with the angle \( \delta \). From \((4a)\), the theoretical maximum power occurs when \( \delta = 90^\circ \).

\[ P_{max(3a)} = 3 \frac{|E||V|}{X_a} \]

In general, stability considerations dictate that a synchronous machine achieve steady-state operation at a power angle considerably less than \( 90^\circ \). The control of real power flow is controlled by the prime-mover governor droop characteristics.

Equation \((4b)\) shows that for small \( \delta \), \( \cos \delta \) is nearly unity and the reactive power can be approximated to

\[ Q_{3a} \approx 3 \frac{|V|^2}{X_a} \]

\[ A. \text{ The mechanical model of synchronous generator and Swing Equation} \]

Consider a synchronous generator that is subjected to disturbance results in an accelerating \( (T_m > T_o) \) or decelerating \( (T_m < T_o) \) torque \( T_a \) on the rotor.

\[ T_a = T_m - T_o - T_{\text{damping}} \]

Where \( T_{\text{damping}} \) is the torque produced due to relative motion between the rotor and the resultant rotating air gap field. Due to this motion there will be induction torque in damper bars opposing this relative motion. This is called the damping torque.

This is the final form of swing equation in per unit with \( \delta \) in electrical radian \[20].

\[ \frac{H}{\pi f_o} \frac{d^2 \delta}{dt^2} + D \frac{d \delta}{dt} + P_{max} \cos \delta \frac{d \delta}{dt} \Delta \delta = 0 \]  

\[ \text{Where } H \text{ is the per unit inertia constant and damping coefficient } D \text{ as:} \]

\[ H = \frac{\text{Kinetic energy at rated speed}}{\text{machine power rating}} \]

\[ D = \text{damping constant due to damper winding effect} \]

The swing equation is a nonlinear function of the power angle. However, for small disturbances, the swing equation may be linearized with little loss of accuracy as follows. Consider a small deviation \( \Delta \delta \) in power angle from the initial operating point \( \delta_0 \) and \( \omega_n \) i.e.,

\[ \delta = \delta_0 + \Delta \delta \]

Substituting in \((7)\), the above equation reduces to linearized equation in terms of incremental changes in power angle, i.e.

\[ \frac{H}{\pi f_o} \frac{d^2 \Delta \delta}{dt^2} + D \frac{d \Delta \delta}{dt} + P_{max} \cos \delta_0 \frac{d \Delta \delta}{dt} \Delta \delta = 0 \]  

\[ \text{Where: } \omega_n = 2\pi f_o = \omega_s \text{ equals the grid frequency and } P_s = P_{max} \cos \delta_0 \text{ is called the synchronizing power.} \]

In terms of the standard second-order differential equation, we have

\[ \frac{d^2 \Delta \delta}{dt^2} + 2\zeta \omega_n \frac{d \Delta \delta}{dt} + \omega_n^2 \Delta \delta = 0 \]

Where \( \omega_n \), the natural frequency of oscillation is given by:

\[ \omega_n = \sqrt{\frac{H}{\pi f_o} \frac{P_s}{P_s}} \]

\[ \zeta \text{ is the damping ratio, given by:} \]

\[ \zeta = \frac{D}{2 \sqrt{H f_o}} \]

And \( \omega_d \) is the damped frequency of oscillation given by:

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]

The characteristic equation is

\[ S^2 + 2\zeta \omega_n S + \omega_n^2 = 0 \]  

\[ \text{Fig. 2 Power-angle curve.} \]
For normal operating conditions.

For $\zeta < 1$ the roots of the characteristic equation are complex

$$S_1, S_2 = -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2} = -\zeta \omega_n \pm j \omega_d$$  \hspace{1cm} (15)

It is clear that for positive damping, roots of the characteristic equation have negative real part if synchronizing power coefficient $P_s$ is positive. The response is bounded and the system is stable.

Taking inverse Laplace transforms results in the zero-input response:

$$\Delta \delta = \frac{\alpha \Delta \delta}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t + \beta)$$  \hspace{1cm} (16)

$$\Delta \omega = \frac{\alpha \omega \Delta \delta}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin \omega_d t$$  \hspace{1cm} (17)

where $\beta = \cos^{-1} \zeta$

The motion of rotor relative to the synchronously revolving field is

$$\delta = \delta_0 + \frac{\alpha \Delta \delta}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t + \beta)$$  \hspace{1cm} (18)

and the rotor angular frequency is

$$\omega = \omega_0 - \frac{\alpha \omega_0 \Delta \delta}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin \omega_d t$$  \hspace{1cm} (19)

The response time constant is

$$\tau = \frac{1}{\zeta \omega_n} = \frac{2H}{\pi f_0 D}$$  \hspace{1cm} (20)

and the response settles in approximately four time constants, and the settling time is

$$t_s \approx 4 \tau$$

III. SUGGESTED POWER ANGLE CONTROL

Fig. 4 illustrates the main idea of the power angle control scheme used in this paper. Equations (4a) and (4b) clarify that provided $Y_2 \gg R_2$, the active power is dependent only on power angle $\delta$ while reactive power is dependent on the difference between magnitudes of inverter voltage and grid voltage. The nonlinear swing equation is linearized for small perturbation $\Delta \delta$ around an operating point $\delta_0$ as given in Eq. 7 – Eq. 9.

$$\sum \frac{d^2 \delta}{dt^2} = \frac{d \delta}{dt}$$

$$\sum \frac{d \delta}{dt} = \pi f_0 \Delta \delta$$

$$\frac{d \delta}{dt} = \Delta \delta$$

Fig. 3 The Suggested Power Angle Control Scheme

Fig. 4 The Small Signal Model of Power Angle of

The linearized swing equation model given by Eq. 9 is shown pictorially in Fig. 5. Here Eq. 4 – Eq. 6 are used to calculate required variables and constants dependent on measurement taken from 3-phase voltage signals of grid and inverter.

Eq. 10 shows that the swing equation for small signal perturbation takes the form of the standard second order system and its response is bounded with under-damped oscillations that finally vanished out. The main factors affecting response are the damping ratio $\zeta$ and the damped frequency $\omega_d$ related to damping constant $D$ and inertia constant $H$ of the synchronous generator model. Since $H$ is the virtual inertia to be emulated for benefit of grid, $D$ can be adjusted for the best response of the Synchronverter.

A. The Frequency, Phase and Voltage Acquisition

Fig. 6 shows the 3-phase acquisition circuit to get the phase angle, peak voltage and frequency of any 3-phase quantities. Direct measurement these signals is based on the following simple analysis. There is no need of phase locked loop normally used in other works [1],[2].

Consider any 3-phase signals:

$$v_a = V_{\text{max}} \sin(\omega t)$$  \hspace{1cm} (21)

$$v_b = V_{\text{max}} \sin(\omega t - \frac{2\pi}{3})$$  \hspace{1cm} (22)

$$v_c = V_{\text{max}} \sin(\omega t + \frac{2\pi}{3})$$  \hspace{1cm} (23)

The equivalent 2-phase signals are:

$$v_a = \frac{1}{2} (v_a - \frac{1}{3} v_b - \frac{1}{3} v_c) = V_{\text{max}} \sin \omega t$$  \hspace{1cm} (24)

$$v_b = \frac{1}{2} \sqrt{3} (v_b - \frac{1}{3} v_a - \frac{1}{3} v_c) = V_{\text{max}} \cos \omega t$$  \hspace{1cm} (25)

Differentiating Eq. 24, Eq. 25 with w.r.t. time gives:

$$\frac{dv_a}{dt} = \omega v_{\text{max}} \cos \omega t$$  \hspace{1cm} (26)

$$\frac{dv_b}{dt} = -\omega v_{\text{max}} \sin \omega t$$  \hspace{1cm} (27)

Then the required measurements are obtained directly as:

$$V_{\text{max}} = \sqrt{v_a^2 + v_b^2}$$  \hspace{1cm} (28)

$$\omega = \tan^{-1} \left(\frac{v_a}{v_b}\right)$$  \hspace{1cm} (29)

$$\omega = \frac{1}{v_{\text{max}}} \sqrt{\left(\frac{dv_a}{dt}\right)^2 + \left(\frac{dv_b}{dt}\right)^2}$$  \hspace{1cm} (30)
IV. SIMULATION RESULTS

The system described above is simulated using Matlab/Simulink package depending on parameters of a 16-kVA synchronous generator given in table 1. Response of the suggested signal-acquisition subsystem is presented first and then overall VSG system response is presented and discussed.

A. Response of the 3-phase Signal Acquisition Circuit (Peak Voltage, Phase Angle and Frequency)

In power system the electrical quantities cannot be changed abruptly. Moreover frequency changes are in a very narrow range around nominal system frequency. To test accuracy and response the suggested acquisition system, a 3-phase voltage with 400-V peak phase voltage and ramp changed frequency from 48 to 52-Hz is used.

Fig.6 presents simulation results of the three measurements; Fig.6a presents the frequency/time changes of the input test signal. Fig.6b shows the frequency measurement, where the output-signal tracks exactly the input signal with nearly zero error. Fig.6c shows the error in phase angle measurement. Due to input band-pass filter there is small error in phase angle that vanishes at center frequency (50 Hz) and increases to 0.0545 radian (3.12°) at both ends of frequency range. Fig.6d shows the error in peak voltage measurement, again the input band-pass filter introduces a small error (0.015% of measured voltage) at both ends of frequency range.

B. Results of Power Angle Control

The power part of model shown in Fig. 4 can be simulated by a sinusoidal PWM inverter with a variety of modulating techniques or by an ideal ac voltage source. In first case the high frequency switching of power converter affects all measurement and waveforms. Moreover the simulation time necessary to implement PWM technique represents a large overhead of the total sampling time. On the contrary, the second method is free of such defects. Really, with the present technological advances an inverter plays the role of a sinusoidal power amplifier with negligible errors.

Fig.7a presents the simulation results of the suggested scheme of VSG based on a space vector modulator inverter operating at 10-kHz and sampling time 0.1-μS. These results are intended mainly to prove feasibility and correctness of the whole model. There are two reference inputs, one for active power sharing and the other is for the reactive power sharing. The events are recorded during simulation period from 0.4 Sec to 1.8 Sec. As shown in Fig.7a the input mechanical power the active power reference signal $P_m$ simulates an increase of prime mover power and the electrical power share $P_e$ developed follows it. Power angle $\delta$ varies according to this power balance as shown in figure. The transient response is not clear for two reasons, first due to switching noises previously mentioned and secondly due to relatively large time scale. This will be examined in the coming results.

The reactive power share $Q$ follows exactly its reference $Q_{set}$ as shown in figure. In conjunction the VSG phase voltage follows variation of $Q$ as depicted by Equ.6. the phase voltage reaches nearly 380-V peak at reactive power share of 7-kVAR, of course deciding factor is the series reactance $X_s$ corresponding to the synchronous reactance of corresponding real SG and reactance of connecting feeder. In practical system $X_s$ can be very small relative to synchronous reactance. Optimizing value of $X_s$ is not a matter of this paper.

For the sake of comparison Fig.7b shows the same results presented Fig.7a where inverter is simulated by an ideal voltage source. There is no difference except of the ripples
superimposed on signals due to the switching effects of inverter

In rest of this paper inverter is simulated by an ideal voltage source to facilitate and speed up model simulation time and to give chance to examine the most important factors affecting model operation.

C. Effect of Damping Constant D and Inertia Constant H

In actual SG, the inertia constant \( H \) is dependent on rotating mass of rotor and prim-mover while the damping constant \( D \) is dependent of damper bars effect and partly on mechanical friction. In VSG model \( H \) and \( D \) are numeric parameters that can be varied for the benefit of the grid without material penalty. For a specific value of \( H \), \( D \) can be adjusted for the best dynamic performance. Fig. 8 show the effect of different values of \( D \) at \( H = 0.01S \), 0.02S and 0.04S; corresponding respectively to 0.1, 0.2 and 0.4 of the inertia constant of the SG (Table I).

The dynamic response of \( \delta \) for a step \( P_m \) from 0.5pu to 0.6pu shows an under-damped response. Due to nonlinearities in model, the approximations assumed to make it linear and the phase shift introduced by the input filter the response doesn’t agree closely with the second order system.

However the damping coefficient \( D \) can still be calculated referring to Eq.12 to give a critically damped response \( (\zeta = 1) \) can be computed as:

\[
D = 2 \frac{P_P}{\sqrt{\Delta}}
\]  

The Fig.8 shows that response at this value gives no critically damped one due to the previously mentioned reasons but reasonably good dynamic response comparing with other values around it. However this value of \( D \) doesn’t depend only on \( H \) but also on \( \Delta = P_m \cos \delta_0 \) and need to be calculated on line. To reduce cross-coupling between different controllers making use of the fact that \( \delta_0 \) is usually small, it is assumed that \( P_P = P_m \).

Figure 8 that the best response of the suggested method of control is obtained for \( H \) below 50% of that of the corresponding real SG. This means that the contribution to grid total inertia is lower than that of the corresponding SG.

Table 1: Parameters of Synchronous Generator

<table>
<thead>
<tr>
<th>Power</th>
<th>Voltage</th>
<th>Frequency</th>
<th>Poles</th>
<th>Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>16kVA</td>
<td>400V</td>
<td>50-Hz</td>
<td>4</td>
<td>Round</td>
</tr>
<tr>
<td>( X_0 )</td>
<td>( X_0 )</td>
<td>( H )</td>
<td>( D )</td>
<td></td>
</tr>
<tr>
<td>1.65 pu</td>
<td>0.0045pu</td>
<td>0.09854pu</td>
<td>0.02005pu</td>
<td></td>
</tr>
</tbody>
</table>
However the relatively fast response enable contributing inertia by another way as it is explained in the flowing.

**D. Operating VSG as virtual inertia**

VSG can contribute to system stability by injecting energy during frequency dips of the network due to load transients. The dc bus must incorporate batteries and the control algorithm sets power sharing to track the negative of derivative of grid-frequency variation [8], [9] (very similar to freewheeling effect). That is:

\[ P_{ref} = P_0 - K \frac{d\theta_{grid}}{dt} \]

Where \( P_0 \) is the steady state active power share of VSG. The second term a component resembles the inertial power share present only when grid frequency changes. The constant \( K \) can take any value; limitations of this term are only related to the inverter ratings and storage batteries capacity.

In Fig. 9 the grid frequency simulates a frequency dip by a single cycle of sine-wave with amplitude of 1/2 Hz imposed on the fixed 50 Hz grid frequency (i.e. the grid frequency change between 49.5-50Hz) as shown in Fig. 9a. This transition vanishes in one second. The VSG model uses small value of \( H = 0.005 \text{ s} \), and damping constant \( D = 0.002 \) to make response fast enough to track frequency dips. Fig. 9b shows the electrical power injected or extracted by the VSG.

**V. CONCLUSION**

In this paper, the principle of power angle control of VSG is suggested and verified via simulation. The system performance is satisfactory and the VSG can play the role of the real SG with active and reactive power sharing. The model can also emulate real inertia effect to aid effectively network stability.

**References**


[18] K. Visscher and s.w.h. de Haan "Virtual synchronous machines (VSG’s) for frequency stabilization in future grids with a significant share of decentralized generation" CIRED Seminar 2008.
