Space Vector Modulation For Three Phase Induction Dielectric Heating

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I. INTRODUCTION

The overall performance and the cost of the heating system will be one of the important issues to be considered during the design process for the next generation of Induction Dielectric Heating (IDH) applications. The power conversion circuit (Three phase Pulse Width Modulation (PWM) inverter) of IDH applications must achieve high efficiency, low harmonic distortion, high reliability and low electromagnetic interference (EMI) noise. Three phase PWM inverter are becoming more and more popular in present day induction heating system.[17], [19], [21].

Sinusoidal Pulse Width Modulation (SPWM) has been used to control the three phase inverter output voltage. To maintain a good performance of the drive the operation has been restricted between 0 to 78 % of the value that would be reached by square wave operation. [3],[7],[17].

The various modulation strategies have been developed [3],[4],[5],[9],[10],[12],[11],[13],[17],[20],[19],[22], and analyzed. The space vector modulation (SVM) [2],[18] has been offered significant flexibility to optimize switching waveforms and it has been well suited for digital implementation.

For the IDH application, full utilization of the DC bus voltage is extremely important to achieve the maximum temperature under all conditions. The current ripple in three phase pulse width modulation inverter under steady state operation can be minimized using SVM compared to any other PWM methods for voltage control mode.

A symmetrical space vector modulation pattern has been proposed, to reduce Total Harmonic Distortion (THD) without increasing the switching losses. The design and implementations a 3-phase PWM inverter for 3-phase IDH to control temperature using space vector modulation(SVM) has been described.

II. PRINCIPLE OF SPACE VECTOR PWM

The circuit model of a typical three-phase voltage source PWM inverter is shown in Figure 1. $S_1$ to $S_6$ are the six power switches that shape the output voltage, which are controlled by the switching variables $a, a', b, b'$ and $c, c'$. When an upper MOSFET is switched on, i.e., when $a, b$ or $c$ is 1, the corresponding lower MOSFET is switched off, i.e., the corresponding $a', b'$ or $c'$ is 0. Therefore, the on and off states of the upper MOSFET $S_1, S_2$ and $S_3$ can be used to determine the output voltage [15].

The relationship between the switching variable vector $[a, b, c]^T$ and the line-to-line voltage vector $[V_{ab}, V_{bc}, V_{ca}]^T$ is given by eq. 1 in the following:

$$
\begin{align*}
V_{ab} &= V_{DC} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \\
V_{bc} &= V_{DC} \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \\
V_{ca} &= V_{DC} \begin{bmatrix} -1 & 0 & -1 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}
\end{align*}
$$

Also, the relationship between the switching variable vector $[a, b, c]^T$ and the phase voltage vector $[V_{an}, V_{bn}, V_{cn}]^T$ can be expressed below.

$$
\begin{align*}
V_{an} &= \frac{V_{DC}}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \\
V_{bn} &= \frac{V_{DC}}{3} \begin{bmatrix} -1 & 2 & -1 \\ 2 & -1 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \\
V_{cn} &= \frac{V_{DC}}{3} \begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}
\end{align*}
$$

As illustrated in Figure 1, there are eight possible combinations of on and off patterns for the three upper power switches. The on and off states of the lower power devices are opposite to the upper one and so are easily determined once the states of the upper power MOSFET’s are determined. According to eq. 1 and 2, the eight switching vectors, output line to neutral voltage (phase voltage), and output line-to-line voltages in terms of DC-link Vdc, are given in Table I and
Table I
SWITCHING VECTORS, PHASE VOLTAGES AND OUTPUT LINE TO LINE VOLTAGE

<table>
<thead>
<tr>
<th>Voltage Vectors</th>
<th>Switching Vector</th>
<th>Line to line voltage</th>
<th>Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>V0(000)</td>
<td>OFF OFF OFF ON ON ON</td>
<td>0 0 0</td>
<td>Zero</td>
</tr>
<tr>
<td>V1(010)</td>
<td>ON OFF OFF OFF ON ON</td>
<td>1 0 0</td>
<td>VDC Active</td>
</tr>
<tr>
<td>V2(101)</td>
<td>ON OFF OFF OFF OFF ON</td>
<td>-VDC VDC 0</td>
<td>VDC Active</td>
</tr>
<tr>
<td>V3(011)</td>
<td>OFF ON OFF OFF ON OFF</td>
<td>VDC 0</td>
<td>0 Active</td>
</tr>
<tr>
<td>V4(001)</td>
<td>OFF OFF ON ON OFF OFF</td>
<td>-VDC 0</td>
<td>VDC Active</td>
</tr>
<tr>
<td>V5(111)</td>
<td>ON ON OFF OFF OFF OFF</td>
<td>VDC VDC 0</td>
<td>VDC Active</td>
</tr>
</tbody>
</table>

Figure 2 shows the eight inverter voltage vectors [9] (V0 to V7).

Space Vector Modulation (SVM) refers to a special switching sequence of the upper three power MOSFETs of a three-phase inverter. The source voltage has been utilized most efficiently by the space vector modulation (SVM) compared to sinusoidal pulse width modulation [9] as shown in Figure 3.

In the vector space, according to the equivalence principle, the following operating rules are obtained:

\[ V_1 = -V_4, V_2 = -V_5, V_3 = -V_6 \]
\[ V_0 = V_7 = 0, V_1 + V_3 + V_5 = 0 \]  

(3)

In one sampling interval, the output voltage vector \( V_t \) can be written as

\[ V_t = \frac{t_0}{T_s} V_0 + \frac{t_1}{T_s} V_1 + \ldots + \frac{t_7}{T_s} V_7 \]  

(4)

where \( t_0, t_1, \ldots, t_7 \) are the turn-on time of the vectors \( V_0, V_1, \ldots, V_7; t_0, t_1, \ldots, t_7 > 0 \).
\[ \sum_{i=0}^{7} t_i = T_s; T_s \text{ is the sampling time.} \]

According to eq. 3 and 4, there are infinite ways of decomposition of \( V \) into \( V_1, V_2, \ldots, V_7 \). However, in order to reduce the number of switching actions and make full use of active turn-on time for space vectors, the vector \( V \) is commonly split into the two nearest adjacent voltage vectors and zero vectors \( V_0 \) and \( V_7 \) in an arbitrary sector. For example, in sector I, in one sampling interval, vector \( V \) can be expressed as

\[ V = \frac{T_1}{T_s} V_1 + \frac{T_2}{T_s} V_2 + \frac{T_0}{T_s} V_0 + \frac{T_7}{T_s} V_7 \]  

(5)

where \( T_s - T_1 - T_2 = T_0 + T_7 \geq 0, T_0 \geq 0 \) and \( T_7 \geq 0 \).

Let the length of \( V \) be \( mV_{DC} \), where \( m \) is modulation index, then

\[ \frac{m}{\sin \frac{2\pi}{3}} = \frac{T_1}{T_s} \frac{1}{\sin \left( \frac{\pi}{3} - \alpha \right)} = \frac{T_2}{T_s} \frac{1}{\sin \alpha} \]  

(6)

where \( \alpha \) is the phase shift.
The switching sequence is From eq. 6 to 8;

Thus,

\[
\begin{align*}
T_1 &= \frac{2}{\sqrt{3}} T_r m \cos(\omega t + \frac{\pi}{6}) \\
T_2 &= \frac{2}{\sqrt{3}} T_r m \cos(\omega t + \frac{\pi}{2}) \\
T_0 + T_7 &= T_r - T_1 - T_2
\end{align*}
\]

(7)

where \(2n\pi \leq \omega t \leq 2m\pi + \pi/3\).

The length and angle have been determined by active and zero (space) vectors. Where \(V_1, V_2, ..., V_6\) are called as active vectors, and \(V_0, V_7\) are called zero (space) vectors. The decomposition of voltage \(V\) in different sectors has been presented in Table II. Equations 5 and 6 have been commonly used for formulation of the space vector modulation. It has been shown that the turn on times \(T_i (i = 1, ..., 6)\) for active vectors are identical in different space vector modulation[17],[20],[19],[22]. The different distribution of \(T_0\) and \(T_7\) for zero vectors yields different space vector modulation.

There are not separate modulation signals in each of the three space vector modulation technique [6]. Instead, a voltage vector is processed as a whole [1]. For space vector modulation, the boundary condition for sector I is:

\[
T_s = T_1 + T_2, T_0 = T_7 = 0
\]

From eq. 6 to 8;

\[
m = \frac{\sin \frac{\pi}{3}}{\sin(\frac{2\pi}{3} - \alpha)}
\]

The boundary of the linear modulation range is the hexagon [6],[8] as shown in Figure 3. The linear modulation range is located within the hexagon. If the voltage vector \(V\) exceeds the hexagon, as calculated from eq. 7, then \(T_1 + T_2 > T_s\) and it is unrealizable. Thus, for the over modulation region space vector modulation is outside the hexagon. In six step mode, the switching sequence is \(V_1 - V_2 - V_3 - V_4 - V_5 - V_6\) [6], Furthermore, it should be point out that the trajectory of voltage vector \(V\) should be circular while maintaining sinusoidal output line-to-line voltages. From Figure 4, it has been seen that for linear modulation range, the length of vector \(m V_{DC}\) should be \(V = (\sqrt{3}/2)V_{DC}\), the trajectory of \(V\) becomes the inscribed circle of the hexagon and the maximum amplitude of sinusoidal line-to-line voltages is the source voltage \(V_{DC}\).

Moreover, for space vector modulation, there is a degree of freedom in the choice of zero vectors in one switching cycle, i.e., whether \(V_0\) and \(V_7\) or both.

For continuous space vector schemes, in the linear modulation range, both \(V_0\) and \(V_7\) are used in one cycle, that is, \(T_7 \geq 0\) and \(T_0 \geq 0\).

For discontinuous space vector schemes, in the linear modulation range, only \(V_0\) or only \(V_7\) is used in one cycle, that is \(T_7 = 0\) and \(T_0 = 0\).

III. DESIGNING STEP FOR SVM GENERATION

To implement the space vector modulation, the voltage equations in the abc reference frame can be transformed into the stationary dq reference frame [9] as shown in Figure 6.

\[
f_{d(0)} = K_s f_{abc}
\]

(10)

where, \(K_s = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\), \(f_{d(0)} = [f_d, f_q, f_0]^T\), \(f_{abc} = [f_a, f_b, f_c]^T\), and \(f\) denotes either a voltage or a current variable.

As described in Figure 6, this transformation is equivalent to an orthogonal projection of \([a, b, c]^T\) onto the two-dimensional perpendicular to the vector \([1, 1, 1]^T\) (the equivalent \(d - g\) plane) in a three-dimensional coordinate system. As a result, six non-zero vectors and two zero vectors are possible. Six nonzero vectors \((V_1 - V_6)\) shape the axis of a hexagonal as shown in Figure 4, and feed electric power to the load. The angle between any adjacent two non-zero vectors is 60 degrees. Meanwhile, two zero vectors \((V_0\) and \(V_7\) are at the origin and apply zero voltage to the load. The eight vectors are called the basic space vectors and are denoted by \(V_0, V_1, V_2, V_3, V_4, V_5\) and \(V_7\). The same transformation can be applied to the desired output voltage

| TABLE II |
|——|——|
| Sector I | Sector II |
| \((0 \leq \omega t \leq \frac{\pi}{6})\) | \((\frac{\pi}{6} \leq \omega t \leq \frac{\pi}{3})\) |
| \(T_1 = \frac{2}{\sqrt{3}} T_r m \cos(\omega t + \frac{\pi}{6})\) | \(T_2 = \frac{2}{\sqrt{3}} T_r m \cos(\omega t + \frac{\pi}{3})\) |
| \(T_0 + T_7 = T_r - T_1 - T_2\) | \(T_0 + T_7 = T_r - T_1 - T_2\) |
| Sector III | Sector IV |
| \((\frac{\pi}{3} \leq \omega t \leq \frac{\pi}{2})\) | \((\frac{\pi}{2} \leq \omega t \leq \pi)\) |
| \(T_5 = \frac{2}{\sqrt{3}} T_r m \cos(\omega t + \frac{\pi}{3})\) | \(T_4 = \frac{2}{\sqrt{3}} T_r m \cos(\omega t + \frac{\pi}{6})\) |
| \(T_0 + T_7 = T_r - T_1 - T_2\) | \(T_0 + T_7 = T_r - T_1 - T_2\) |
| Sector V | Sector VI |
| \((\frac{\pi}{2} \leq \omega t \leq \frac{3\pi}{4})\) | \((\frac{3\pi}{4} \leq \omega t \leq \frac{5\pi}{6})\) |
| \(T_5 = \frac{2}{\sqrt{3}} T_r m \cos(\omega t + \frac{\pi}{3})\) | \(T_6 = \frac{2}{\sqrt{3}} T_r m \cos(\omega t + \frac{\pi}{6})\) |
| \(T_0 + T_7 = T_r - T_1 - T_2\) | \(T_0 + T_7 = T_r - T_1 - T_2\) |
to get the desired reference voltage vector \( V_{\text{ref}} \) in the \( d-q \) plane.

The objective of space vector modulation technique is to approximate the reference voltage vector \( V_{\text{ref}} \) using the eight switching patterns.

Therefore, space vector modulation can be implemented by the following steps:

**Step 1.** Determine \( V_d, V_q, V_{\text{ref}} \), and angle \( \alpha \).

**Step 2.** Determine time duration \( T_1, T_2, T_0 \).

**Step 3.** Determine the switching time of each MOSFET (\( S_1 \) to \( S_6 \)).

**A. Determine \( V_d, V_q, V_{\text{ref}} \), and angle \( \alpha \)**

From Figure 6, the \( V_d, V_q, V_{\text{ref}} \), and angle \( \alpha \) can be determine as follows:

\[
V_d = V_{an} - V_{bn} \cdot \cos 60 - V_{cn} \cdot \cos 60 = 0
\]

\[
V_q = 0 + V_{bn} \cdot \cos 30 - V_{cn} \cdot \cos 30 = \frac{\sqrt{3}}{2} V_{bn} - \frac{\sqrt{3}}{2} V_{cn}
\]

\[
|V_{\text{ref}}| = \sqrt{V_d^2 + V_q^2} = \sqrt{V_{an}^2 + V_{bn}^2 - V_{cn}^2}
\]

\[
\alpha = \tan^{-1} \left( \frac{V_d}{V_q} \right) = \omega t = 2\pi f t,
\]

where \( f \) is fundamental frequency.

**B. Determine time duration \( T_1, T_2, T_0 \)**

From Figure 7, the switching time duration can be calculated as follows:

- Switching time duration at sector 1

\[
T_s \cdot V_{\text{ref}} \cdot \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} = T_1 \cdot \frac{2}{3} \cdot V_{DC} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}
+ T_2 \cdot \frac{2}{3} \cdot V_{DC} \cdot \begin{bmatrix} \cos(\pi/3) \\ \sin(\pi/3) \end{bmatrix}
\]

where, \( 0 > \alpha > 60^\circ \)

\[
T_1 = T_s \cdot \frac{\sin(\pi/3 - \alpha)}{\sin(\pi/3)}
\]

\[
T_2 = T_s \cdot \frac{\sin(\alpha)}{\sin(\pi/3)}
\]

\[
T_0 = T_s - (T_1 + T_2)
\]

where, \( T_s = \frac{1}{f} \) and \( \alpha = \frac{|V_{\text{ref}}|}{V_{DC}} \cdot \frac{\pi}{3} \). Switching time duration at any sector

\[
T_1 = \frac{\sqrt{3}}{V_{DC}} \cdot |V_{\text{ref}}| \left( \sin \left( \frac{\pi}{3} - \alpha + n \cdot \frac{1}{3} \pi \right) \right)
\]

\[
T_2 = \frac{\sqrt{3}}{V_{DC}} \cdot |V_{\text{ref}}| \left( \sin \left( \alpha - \frac{n}{3} \pi \right) \right)
\]

\[
T_0 = T_s - T_1 - T_2
\]

where, \( n=1 \) through 6 (that is, Sector I to VI) \( n \cdot \frac{\pi}{3} - \pi \leq \alpha \leq \frac{\pi}{3} \).

**C. Determine the switching time of each MOSFET (\( S_1 \) to \( S_6 \))**

Based on Figure 8, the switching time at each sector has been summarized in Table III, and it will be built in Simulink model to implement SVM.

**IV. Simulation Results**

Simulation results were performed using simulink block as shown in Figure 9. The DC bus \( V_{DC} \) is equal to 325V.
TABLE III
SWITCHING TIME CALCULATION AT EACH SECTOR

<table>
<thead>
<tr>
<th>Sector</th>
<th>Upper Switches</th>
<th>Lower Switches</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$S_1 = T_1 + T_3 + T_7$</td>
<td>$S_4 = T_6$</td>
</tr>
<tr>
<td></td>
<td>$S_2 = T_2 + T_7$</td>
<td>$S_6 = T_1 + T_6$</td>
</tr>
<tr>
<td></td>
<td>$S_3 = T_3 + T_7$</td>
<td>$S_8 = T_2 + T_6$</td>
</tr>
<tr>
<td></td>
<td>$S_5 = T_4 + T_7$</td>
<td>$S_7 = T_3 + T_6$</td>
</tr>
<tr>
<td></td>
<td>$T_1 = T_n/2$</td>
<td>$T_0 = T_n/2$</td>
</tr>
</tbody>
</table>

Sample circuit parameters are given in Table IV. Simulation space vector generator has been shown in Figure 9. Three phase PWM inverter output line to line voltage, output current, 3 phase to 2 phase dq transformation voltages and 3 phase to 2 phase dq transformation currents are shown in Figure 10, 11, 12, 13 respectively. Simulation summaries and results are given in Table V, VI respectively.

A spectral analysis of all waveforms is performed and all harmonics are presented in Table VII. These results shows that acceptable performances can be obtained at all testing frequencies since the total harmonic distortion (THD) did never reach 10 %. At high switching frequency the PWM converter generate a voltage having an amplitude close to the desired value.

V. CONCLUSIONS

Space vector modulation only requires on reference space vector to generate three phase sine waves. The amplitude and frequency of load voltage can be varied by controlling the reference space vector. Furthermore, this algorithm is flexible and suitable for advanced vector control. The strategy of the switching minimizes the distortion of load current as well as loss due to minimize number of commutations in the inverter.

Simulation of Space Vector Modulation (SVM) technique has been done using MATLAB. This aim on the one hand to prove the effectiveness of the SVM in the contribution in the switching power losses reduction. SVM is among one of the best solution to achieve good voltage transfer and reduce harmonic distortion in the output of three phase inverter for IDH. It also provide excellent output performance optimized.

TABLE IV
CIRCUIT PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>220V/50Hz</td>
</tr>
<tr>
<td>$V_{DC}$</td>
<td>325 volt</td>
</tr>
<tr>
<td>$L_m$</td>
<td>0.93mH</td>
</tr>
<tr>
<td>$f_{sw}$</td>
<td>4kHz</td>
</tr>
</tbody>
</table>

TABLE V
SIMULATION RESULTS

<table>
<thead>
<tr>
<th>Switching Freq.</th>
<th>Set Temp. in °C</th>
<th>Final Temp. in °C</th>
<th>$V_{DC}$ in Volt</th>
<th>Frequency in Hz</th>
<th>Load current in Amp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>120</td>
<td>1167</td>
<td>139.21</td>
<td>41.91</td>
<td>9.036</td>
</tr>
<tr>
<td>2000</td>
<td>120</td>
<td>1170</td>
<td>153.49</td>
<td>41.74</td>
<td>6.107</td>
</tr>
<tr>
<td>200000</td>
<td>120</td>
<td>1168</td>
<td>151.33</td>
<td>41.74</td>
<td>5.186</td>
</tr>
<tr>
<td>2000000</td>
<td>120</td>
<td>11990</td>
<td>175.93</td>
<td>41.74</td>
<td>4.763</td>
</tr>
</tbody>
</table>
efficiency and high reliability compared to similar three phase inverter with conventional pulse width modulations.

REFERENCES


