SPACE VECTOR MODULATION OF MULTILEVEL H-CASCADED CONVERTER

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ABSTRACT

In this paper, a new space vector modulation algorithm for multilevel inverter is advanced based on imaginary coordinate, by which the available vertexes calculation and time calculation of vectors will be more simple than traditional method. Moreover, the corresponding selection method of vertex and vector is also developed. The additional control rules such as voltage balance, high dv/dt prevent, sub-harmonic elimination, etc. could be implemented easily with this method, so that the inverter performance could be improved.

Key Words: Space Vector Modulation, H-Cascaded Converter, Modulation Strategy, Switches, Circle and Switching frequency
1 INTRODUCTION

Multilevel converters are increasingly studied widely recently thanks to their much approximately sinusoidal output waveform [1][2]. Multilevel converters can leave out the bulky, heavy transformers or reactors. This advantage is much of utility since the size and cost of the transformers and reactors is the main embarrassment for improving the capability of converters. Presently, there are four types of circuit topology in multilevel converters: diode-clamped multilevel converters, flying-capacitor multilevel converters, P2 multilevel converter and H-cascade multilevel converters. There are several modulation strategies as followed for cascade multilevel converter:

i) Stair Waveform PWM (Pulse Width Modulation) [2]. This technique is based on the use of the available voltage levels in a staircase to derive an approximately sinusoidal output waveform. Through precisely selecting the different duration time for each level, the lower order harmonics can be eliminated and restrained.

ii) Multilevel Space Vector Modulation (abbreviated as ML-SVM)[3][4][5]. This is a natural extension of the classical two-level SVM to n-level converter. The modulation principle of this technique is the same as the two-level SVM; i.e. each space vector is approximated by optimizing and allocating of the nearest fundamental space vectors.

iii) Carrier Phase-Shifted SPWM (abbreviated as CPS-PWM)[6]. The modulation principle is to compare a sinusoidal reference waveform with a group of phase-shifted triangle carriers. Thus, a group of SPWM waveforms is generated to control the power devices. Clearly, the equivalent switching frequency is much higher than that of each device. This technique can lead to a fast dynamic respond and a wide transmission band.

2 SPACE VECTOR MODULATION

The most important on line modulation method is today the space vector modulation. It has very near relationship with carrier based modulation.

In fig1 (figure) and table1 the quantities of a three phase converter are explained with the help of space vector explained. In a stationary state, a space vector shows a constant length and rotates in a complex $\alpha\beta$-surface with the circle frequency for symmetric and sinus voltages. For multilevel converters, this method is difficult to use due to a large number of switches. From DC-voltage $V_d$ the switches 1 to 6 can switch in the phase $u$, $v$ and $w$ between the potentials -$V_d$ and +$V_d$.

Space Vector Explanation:

$$v = \begin{bmatrix} v_u \\ v_v \\ v_w \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{bmatrix} \begin{bmatrix} v_{UM} \\ v_{VM} \\ v_{WM} \end{bmatrix}$$

(1)

<table>
<thead>
<tr>
<th>$v_0$</th>
<th>$v_1$</th>
<th>$v_2$</th>
<th>$v_3$</th>
<th>$v_4$</th>
<th>$v_5$</th>
<th>$v_6$</th>
<th>$v_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>v</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>w</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: The logical states of the standard vectors

Figure 1b shows the room allocation of the stand-steady coordinate system of three phase legs $u$, $v$ and $w$. The logical position of three transistor branches is defined with:

0 if the branch is on negative potential or
1 if the branch is on positive potential.

With three branches eight possible logical states or eight standard vectors $v_0$, $v_1$, …, $v_7$ arise and two vectors $v_0$ –alle branches in negative− and $v_7$ –alle branches in the positive potential– are the zero vectors. The vectors divide the whole vector space into six sectors $S_1$ …… $S_6$ or four quadrants $Q_1$ …… $Q_4$. The table 1 shows the logical states of the transistor branches.
3 PRINCIPLE OF MODULATION

In the following example (fig.2) it will be shown, how from eight standard vectors a standard voltage vector can be generated. The resultant vector $v_r$ (equation 2) is, e.g., in the sector $S_1$, to the area between the standard vectors $v_1$ and $v_2$. Vector $v_r$ arises from the addition of the both in the directions from $v_1$ and $v_2$ to disassembled edge vectors $v_r$ and $v_l$.

$$|v_s|_{\text{max}} = |v_1| = \ldots = |v_4| = \frac{2}{3} V_d \quad (2)$$

Following results are achieved:

Vector $v_r$ is the resultant vector of $v_1$ and $v_2$. Through the logical states of $v_1$ and $v_2$ within the time span become $v_i$ and $v_1$

$$T_s = T^*_\rho \frac{|v_1|}{|v_1|_{\text{max}}}; T_i = T^*_\rho \frac{|v_i|}{|v_i|_{\text{max}}} \quad (3)$$

where $T^*_\rho$ is reference time period.

Through the switching patterns $v_1$ and $v_2$ are known from the table 1. The times $T_r$ and $T_i$ are to be determined. The conclusion arises from the formula (3):

To determine $T_r$ and $T_i$, the amounts from $v_1$ and $v_2$ must be confessed. Condition is that the resultant standard voltage vector $v_r$ will be given according to amount and phases of current controller. Following two questions remain open.

1) How the rest of the pulse period behaves $T^*_\rho - (T_r + T_i)$?

2) In which order are the vectors $v_1$ and $v_2$ or $v_i$ and $v_l$ realised?

In the remaining time span $T^*_\rho - (T_r + T_i)$ one of the zero vectors $v_0$ or $v_7$ is given.
In the final effect the following equation is realised as:

\[ v_r = v_i + v_l + v_0 \quad (or \quad v_f) \]

\[ \frac{T_p}{T_p} v_r + \frac{T_p}{T_p} v_l + \frac{T_p}{T_p} (T_p + T_l) v_0 (or v_f) \quad (4) \]

Then the question is, in which order three vectors – two vectors and a zero vector – are given.

The table 2 shows necessary switching states in sector S. It is recognizable that that order is advantageous with which every branch within a pulse period must be switched only once. If last switching state is \( v_0 \), then the result can be realised as:

\[ v_0 \Rightarrow v_1 \Rightarrow v_2 \Rightarrow v_7 \quad , \quad \text{Inversely if it is} \quad v_i , \quad \text{then it should be:} \]

\[ v_i \Rightarrow v_1 \Rightarrow v_2 \Rightarrow v_0 \]

With these strategies the switching losses are minimum.

If one explains the switching states (fig. 3) of the two pulse periods following on each other, a known picture arises from the pulse width modulation.

This figure makes clear that the period \( T_p \), accepted up to now, amounts only to half of the real pulse period \( T_p \) for the realisation of a voltage vector. The real pulse period \( T_p \) contains the realisation of two – same or different, this depends on the concrete implementing of the modulation- vectors. Till yet the process of the realisation was explained in the sector S., regardless of the vector position within the sector.

Every reference vector can be generated by decomposition of the reference vectors in the edge components which are oriented in the directions of two neighbouring standard vectors in any position in the whole pointer level.

From the fact:

that the current controller delivers the reference value of a new voltage vector \( v_i \), the modulation after every carrier period \( T \),

that every (modulation- or) pulse period contains the realisation of two voltage vectors, gives a relationship between puls frequency \( f_p = 1 / T_p \) and carrier frequency \( 1 / T \).

Fig. 3 states theoretically that two carrier periods \( T \) correspond to a pulse period \( T_p \). This relationship becomes practical, nevertheless, seldom in use. In principle counts that the current controller realises the given vector \( v_i \) within at least one or several pulse periods. Thereby becomes possible to think a reasonable relation between pulse frequency and carrier frequency that a sufficient pulse frequency allows at the same time enough big carrier period.

3.1 Calculation and Function of Switching Time

After the introduction of principle of SVM, now the use of this principle will be given. The inverter will be given a signal, how and how long should its switches be switched, if the resultant vector will be given according to magnitude and phase.

3.1.1 Modulation of Two Level Converter

From equations (2-3) it is clear that the calculation of switching time \( T_r \) and \( T_l \) depends on the informations of magnitudes of vectors \( v_r \) and \( v_l \). (fig. 4):

1) Either through the uniform components \( v_{sd} , v_{sq} \) in dq coordinates. The total phase angle is given from the sum of the angles \( \theta \), and the phase position of \( v_i \) is within the diagram

\[ \theta = \theta + \arctan \left( \frac{v_{sq}}{v_{sd}} \right) \quad (5) \]

2) or through the sine form components \( v_{sa} , v_{sb} \) in αβ-coordinates. In this form is the information on phase angle not explicit, but consists on implicit in the components.

Two strategies exist in order to calculate components.

Strategie1:

First the angle \( \theta \) and then \( \gamma \) will be calculated with the help of equ. (5) (equation) and fig. 4, and then the components from the following equations, which give the total surface vector.

\[ \left| v_s \right| = \frac{2}{\sqrt{3}} |v_i| \sin(60^\circ - \gamma) ; \quad (6) \]

With the magnitude:

\[ \left| v_s \right| = \sqrt{v_{sd}^2 + v_{sq}^2} \quad (7) \]
Strategie 2:

After the transformation of coordinate components $V_{sd}$ and $V_{sq}$ are given from $V_{v}$, $V_{v}$.

The vectors $V_{r}$ and $V_{l}$ will be calculated from table 3 for every sector.

The introduced strategies can be used simultaneously in order to calculate the switching time period $T_{e}$ and $T_{l}$ (Table 4). The switching time depends on the hardware configuration of the controller. The use of 2nd strategy seems to be complicated (Table 3), but if we consider it exactly, it has only three terms (equ. 8).

\[
\begin{align*}
a &= |V_{v}| \cdot \frac{1}{\sqrt{3}} |V_{v}\beta| ; \\
b &= |V_{v}| \cdot \frac{1}{\sqrt{3}} |V_{v}\beta| ; \\
c &= \frac{2}{\sqrt{3}} |V_{v}\beta| \quad (8)
\end{align*}
\]

It will be here stressed that in table 4 $T_1 = T_r$ and $T_2 = T_l$ is.

The position of phase $V_{v}$ can be calculated with the help of following considerations. Through the symbol of $V_{v}$ it will be decided, in which quadrant is the voltage vector situated.

As the magnitudes of $V_{r}$, $V_{l}$ are always positive and term $b$ changes its symbol passing through one sector into the other. Through the symbol of $b$ it will be checked, in which quadrant it is.

The reference vector can be put in any of the six sectors of a hexagonal, that contains the changing states of vector. This problem can be easily solved through anti clockwise rotation of $V_{v}$ (reference vector) at the angle of $(S-1)\pi/3$, where $S$ is the number of sectors. This rotation displaces every reference vector at $60^\circ$. The changing states of the vectors for multilevel controlling will be determined through reverse rotation. The diagram of a hexagonal is explained in fig. 5. Besides this, the transformation is easy to explain [7], if a sector $S$ is fixed. Through addition of $V_{v}$ and $V_{v} \beta$, the numerical values of the switching period can be reduced, without using trigonometric function calcula-

<table>
<thead>
<tr>
<th></th>
<th>$V_0$</th>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$v$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$w$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
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</table>

Table 2: Switching states in Sector $S_1$
tions as with conventional algorithms. The space vectors of two and three level converter are depicted in fig. 6 and 8. Mathematically x and y dimensions (fig. 6 and equ. 9-12) can be calculated in radial and tangent direction and is also explained in detail in [9].

Two level centered space vector diagram:

\begin{align*}
x &= \frac{MV}{2} \sin \theta \sin(\theta + 120^\circ) \quad (9) \\
y &= \frac{MV}{16}(2 + \sqrt{3}M \sin(\theta - 120^\circ)) \quad (10)
\end{align*}

Two level discontinuous PWM:

\begin{align*}
x &= \frac{MV}{2} \sin \theta \sin(\theta + 120^\circ) \\
y &= \frac{MV}{8}(2 + \sqrt{3}M \sin(\theta - 120^\circ)) 
\end{align*}

In equ. (9-12) is \( M \), Modulation \( M = \frac{V_{ref}}{V_d} \). The switching period, \( 0 \leq \theta \leq 60^\circ \), where as \( V_d \) is dc-bus voltage and \( V_{ref} \) is reference Voltage.

Comparing (11) and (12) with (9) and (10) reveals that discontinuous modulation can be achieved by simply doubling the y dimension of hysteresis bounding box. Discontinuous PWM-strategy is being explained in detail in [10].

Fig. 3: Puls sample of voltage vectors in sector

![Diagram of voltage vectors and pulses](image-url)
<table>
<thead>
<tr>
<th>Sector 1</th>
<th>((0 \leq \omega t \leq \pi/3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_1 = \frac{\sqrt{3}}{2}MT_p \cos(\omega t + \pi/6)), (T_2 = \frac{\sqrt{3}}{2}MT_p \cos(\omega t + 3\pi/2))</td>
<td></td>
</tr>
<tr>
<td>(T_0 + T_7 = T_p - T_1 - T_2)</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector 2</th>
<th>((\pi/3 \leq \omega t \leq 2\pi/3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_1 = \frac{\sqrt{3}}{2}MT_p \cos(\omega t + \pi/2)), (T_4 = \frac{\sqrt{3}}{2}MT_p \cos(\omega t + 7\pi/6))</td>
<td></td>
</tr>
<tr>
<td>(T_0 + T_7 = T_p - T_2 - T_3)</td>
<td></td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Sector 3</th>
<th>((2\pi/3 \leq \omega t \leq \pi))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_1 = \frac{\sqrt{3}}{2}MT_p \cos(\omega t + 3\pi/2)), (T_4 = \frac{\sqrt{3}}{2}MT_p \cos(\omega t + \pi))</td>
<td></td>
</tr>
<tr>
<td>(T_0 + T_7 = T_p - T_3 - T_4)</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector 4</th>
<th>((\pi \leq \omega t \leq 4\pi/3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_1 = \frac{\sqrt{3}}{2}MT_p \cos(\omega t + 7\pi/6)), (T_4 = \frac{\sqrt{3}}{2}MT_p \cos(\omega t + \pi/2))</td>
<td></td>
</tr>
<tr>
<td>(T_0 + T_7 = T_p - T_4 - T_5)</td>
<td></td>
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<tr>
<th>Sector 5</th>
<th>((4\pi/3 \leq \omega t \leq 5\pi/3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_1 = \frac{\sqrt{3}}{2}MT_p \cos(\omega t + 2\pi/3)), (T_4 = \frac{\sqrt{3}}{2}MT_p \cos(\omega t + 5\pi/6))</td>
<td></td>
</tr>
<tr>
<td>(T_0 + T_7 = T_p - T_5 - T_6)</td>
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<thead>
<tr>
<th>Sector 6</th>
<th>((5\pi/3 \leq \omega t \leq 2\pi))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_1 = \frac{\sqrt{3}}{2}MT_p \cos(\omega t + \pi/3)), (T_4 = \frac{\sqrt{3}}{2}MT_p \cos(\omega t + 11\pi/6))</td>
<td></td>
</tr>
<tr>
<td>(T_0 + T_7 = T_p - T_1 - T_6)</td>
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</table>

Table 3: Edge components in the dependence of the position of voltage vector

Fig. 4: Possibilities for the default of \(v_s\)
Table 4: Calculation of the switching time period

| S1 | Q1 | $|v_{s_a}| - \frac{1}{\sqrt{3}} |v_{s_{\beta}}|$ | $\frac{2}{\sqrt{3}} |v_{s_{\beta}}|$ |
|----|----|----------------------------------|------------------|
| S2 | Q1 | $|v_{s_a}| + \frac{1}{\sqrt{3}} |v_{s_{\beta}}|$ | $- |v_{s_a}| + \frac{1}{\sqrt{3}} |v_{s_{\beta}}|$ |
|    | Q2 | $- |v_{s_a}| + \frac{1}{\sqrt{3}} |v_{s_{\beta}}|$ | $|v_{s_a}| + \frac{1}{\sqrt{3}} |v_{s_{\beta}}|$ |
| S3 | Q2 | $\frac{2}{\sqrt{3}} |v_{s_{\beta}}|$ | $|v_{s_a}| - \frac{1}{\sqrt{3}} |v_{s_{\beta}}|$ |
| S4 | Q1 | $|v_{s_a}| - \frac{1}{\sqrt{3}} |v_{s_{\beta}}|$ | $\frac{2}{\sqrt{3}} |v_{s_{\beta}}|$ |
| S5 | Q1 | $|v_{s_a}| + \frac{1}{\sqrt{3}} |v_{s_{\beta}}|$ | $- |v_{s_a}| + \frac{1}{\sqrt{3}} |v_{s_{\beta}}|$ |
|    | Q2 | $- |v_{s_a}| + \frac{1}{\sqrt{3}} |v_{s_{\beta}}|$ | $|v_{s_a}| + \frac{1}{\sqrt{3}} |v_{s_{\beta}}|$ |
| S6 | Q3 | $\frac{2}{\sqrt{3}} |v_{s_{\beta}}|$ | $|v_{s_a}| - \frac{1}{\sqrt{3}} |v_{s_{\beta}}|$ |

Fig. 5: Modulation of a two level inverter according to fig. 1
3.2.2 Modulation of a 3 level H-cascaded converter

The circuit and space vector diagram of a 3 level and 3 phase H-cascaded converter is explained in fig. 7. In fig. 8a, however a sector is divided in 4 small triangles. The number (no.) of sectors depends on the no. of levels of the converters. In order to get minimum harmonic distortion with multilevel modulation, only the three next space vectors of the converter can be used to find a reference vector [11-12] (none of them should be a zero vector). Fig. 8a-c show the space vector diagram in αβ-coordinate system of three level and phase u, v and w. The logical and easy position of branches of transistor are defined with:

1, if the branch on negative potential or with
2, if the branch on positive potential or with
0, if the branch on zero potential
of dc bus voltage is switched.

A three level H-cascaded converter (fig. 7) has $3^3 = 27$ switching states. Nineteen of them are active switching states and make 18 space vectors (fig. 8a). Three of them are zero vectors and lie in the origin. The reference vector can be chosen freely within a sector. There are a large no. of combination of vectors and too many solutions.

Space vectors can be divided into 4 groups (fig. 8a) as explained below:

(i) The “large vectors” (200, 220, 020, 022, 002 and 202) assign the output voltages of the converter to either the highest or lowest voltage levels. As they do not connect any output, they do not effect the voltage balance of the capacitors. In fact, these six vectors are equivalent to the active ones of the two level converter.

(ii) The “medium vectors” (210, 120, 021, 012, 102 and 201) connect each output to a different dc-link voltage level. Under balanced conditions, their tip end in the middle of segments that join two consecutive large vectors. The length of the medium vectors defines the maximum amplitude of the reference vector for linear modulation and steady state conditions, which is $\frac{\sqrt{3}}{2}$ the length of the large vectors.

(iii) The “small vectors” (100-211, 110-221, 010-121, 011-122, 001-112, and 101-212) connect the AC outputs to two consecutive dc-link voltage levels. Their length is half the length of large vectors. They are double vectors, which means that two states of the converter can generate the same voltage vector.
(iv) The “zero vectors” (000, 111 and 222) are in origin of the diagram. They connect all the outputs of the vector to the same dc-link voltage and therefore they do not produce any current in the dc side.

These Space vectors can be treated as a two level converters, whose origin can be changed out from {000; 111; 222} to {100; 211}. At the center of these six hexagonal lied vectors are known as “equivalente zero vectors” and are abundant. As a whole a three level converter has six space vectors with abundancy and therefore can build six hexognal with its own rogin (see fig. 8b). Similarly the origin can be changed from {000; 111; 222} to {110; 221}, {010; 121}, {011; 122},{001; 112} and {101; 212}. Some switched states are abundant and produce the same vectors. The difficult task of the space vectors was to choose the reference vector. This problem has been recently solved by Celanovic [8], by using linear coordinate transformation.

Fig. 9 shows modulation of a 3 level converter and the explanation of centered space vector diagrams. In figures 10-13 the simulation results with software “Simplorer” are explained. Fig. 10 gives the input phase and line to line voltages, where as fig. 11 shows the output phase and line to line voltages. Figure 12 gives common mode voltage and fig. 13 gives the output currents. All these results are of the circuit according to fig. 7. For circuit in figure 7 sine-triangle PWM method has been used. All abbreviations and parameters are explained at end of article before conclusion.

\[ \begin{align*}
\text{Parameters: } & V_s = 1.55kV, C = 2.46mF, f_1 = 50Hz, f_s = 3kHz, M = 1, L_L = 7.14mH, e = 3.81kV \\
\end{align*} \]

Fig. 7: Circuit of a 3 level and 3 phase H-cascaded converter
Fig. 8: (a-c) Modulation of a 3 level and three phase H-converter
Fig. 9: Modulation of a 3 level converter (a) Space vector diagram (A Sector $S_1$ from fig. 8) 
(b) Explanation of centered space vector diagram (c) 60°-Discontinuous PWM 
(d) 30°-Discontinuous PWM

Voltage/V                      Voltage/V
\[
\begin{array}{cccccc}
100 & 200 & 210 & 211 & 210 & 200 & 100 \\
\end{array}
\]

Fig. 10: Input (a) phase voltages (b) line to line voltages

Voltage/V
\[
\begin{array}{cccccc}
100 & 200 & 210 & 211 & 210 & 200 & 100 \\
\end{array}
\]

Fig. 11: Output (a) phase voltage (b) line to line voltage

Voltage/V                      Current/A
\[
\begin{array}{cccccc}
100 & 200 & 210 & 210 & 200 & 100 \\
\end{array}
\]

Fig. 12: Common mode voltage

Fig. 13: Output currents

Abbreviations:  
fig. = figure, equ. = equation, $V_s$ = Effective value of Voltage $M$ = Modulation, $C$ = capacity, $f_1$ = Basic frequency, $f_s$ = Switching frequency, $L_L$ = Last inductor, $e$ = electro motive force
4 CONCLUSION

A space vector modulation scheme is proposed for 2 and 3 level H-cascaded inverters. The main feature of the modulation scheme lies in its ability to eliminate even order harmonics in the inverter output voltages. The approach uses a rotating bounding box and a simple set of switching rules in synchronous d-q frame to select between the two nearest space vector regions. The algorithm is readily extended to incorporate variable dimensions for the bounding box, to achieve a near constant switching frequency. The output phase voltage is half of the output line to line voltage.

REFERENCE