A GSA BASED INPUT CURRENT HARMONIC OPTIMIZATION FOR 
EFFICIENCY IMPROVEMENT OF BLDC MOTOR DRIVE SYSTEM 
SUITABLE FOR ELECTRICAL VEHICULAR APPLICATION

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Abstract: Brushless DC motors are extensively used for electric vehicular applications owing to their excellent torque versus speed characteristics, good efficiency and high power density. This paper describes efficiency improvement of a position sensorless brushless DC motor with improved pulse width modulation (PWM) scheme for the inverter drive compared to existing ones. The inverter switching is based on Selective Harmonic Elimination (SHE). Gravitational Search based Algorithm (GSA) is used to calculate the optimized switching angles for the purpose. The proposed method reduces Total Harmonic Distortion (THD) from the input current and armature flux thereby reducing the core losses. Also the power requirement with the proposed switching technique is much lesser than the existing switching schemes. The benefit of the proposed efficiency improvement can be of great importance in Electric Vehicular (EV) applications. The effectiveness of the proposed scheme is demonstrated through simulation and experimental results.

Key words: Brushless DC motor, Sensorless control, Selective Harmonic Elimination (SHE), Total Harmonic Distortion, Gravitational Search Optimization (GSA).

1. Introduction

Brushless DC (BLDC) motors have been widely used in various industries, automation and appliances due to their higher efficiency, improved ruggedness and power density. These motors are generally available in diverse power ratings. BLDC motors with higher efficiencies are helpful for Electric Vehicular (EV) applications especially over wide torque and speed ranges. Since the voltage requirement for a BLDC drive remains fixed, the current can be a factor to estimate the motor efficiency. BLDC motors with trapezoidal back electromotive force (EMF) characteristics require six discrete rotor position information sensors for the inverter, as they are used to trigger the switches. Brushless DC motors require lower maintenance due to the lack of mechanical commutator and they have high power density. For these reasons they are ideal for high torque to weight ratio applications [1]. The most important drawback of BLDC machine is high initial cost and relative higher complexity due to the

converter part. Reducing losses through the injection of proper direct axis current in the stator winding is also present [2]. The control algorithm determines the optimal direct axis current according to the operating speed and loading conditions. Efficiency improvement through using both axial and radial flux also has been studied [3]. Utilizing both radial and axial gaps can increase the effective area for torque generation and the fill-factor for the coil winding. By optimizing core and permanent magnet to minimize the electromagnetic loss while maintaining the same level of torque, the magnetic saturation of the core is also reduced. The iron loss can be reduced by the flux-weakening control [4-6]. To reduce the air gap flux by the demagnetizing effect due to the d-axis armature reaction, d-axis current is controlled. Optimal control method of armature current vector is proposed in order to minimize the controllable losses [7]. Switching techniques for BLDC motor torque ripple reduction is proposed previously using microcontroller [8]. Field programmable gate array based PWM is also presented in [9]. Also Fuzzy logic based speed control is discussed with some possible efficiency improvement [10]. In the recent past, sensorless position control strategies for BLDC motor control have been adopted of which Hybrid sliding mode control [11], I-f starting sequence and real time flux estimation [12], offline finite element method assisted position and speed estimation [13], hysteresis comparator [14] are of great relevance. However, current switching based loss minimization is not available in literatures with proposed optimization. A PWM and PAM based sensorless high speed BLDC drive have been proposed recently [15].

In the proposed control, the phase current waveform is switched effectively to eliminate some lower order harmonics which will reduce the harmonics generated by the stator flux. A GSA [16] based selective harmonic elimination PWM technique for the inverter switching angle optimization is used. The GSA algorithm effectively
calculates switching angles offline for proposed SHE based PWM. The proposed switching will make certain that the BLDC will have minimal core losses. Also, this will cause minimal input power consumption for the BLDC motor.

Fig. 1 shows the equivalent circuit of a BLDC motor.

![Equivalent Circuit of BLDC Motor](image)

2. BLDC Motor Model

The three phase voltage equations from the equivalent circuit of Fig. 1 for the BLDC motor can be written as [17],

\[
\begin{bmatrix}
v_a \\
v_b \\
v_c
\end{bmatrix} =
\begin{bmatrix}
R_a & 0 & 0 \\
0 & R_b & 0 \\
0 & 0 & R_c
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix} +
\begin{bmatrix}
L_a & 0 & 0 \\
0 & L_b & 0 \\
0 & 0 & L_c
\end{bmatrix}
\begin{bmatrix}
di_a \\
di_b \\
di_c
\end{bmatrix} +
\begin{bmatrix}
e_a \\
e_b \\
e_c
\end{bmatrix}
\tag{1}
\]

Where, \( v \) is the stator voltage, \( R \) is the stator resistance, \( i \) is the stator current, \( L \) is the stator inductance and \( e \) is the back emf. The above quantities are defined for three phases \( a-b-c \). The mechanical dynamic equation for the motor can be given as,

\[
T_{em}(t) = \omega(t) + J \frac{d\omega}{dt} + T_L(t)
\tag{2}
\]

Where, \( T_{em} \) is developed electromagnetic torque, \( \omega(t) \) is rotor angular velocity, \( B \) is viscous friction constant, \( J \) is rotor moment of inertia and \( T_L \) is load torque.

The back emf (\( a \) phase) can be described as,

\[
e_a = k_e \omega(t)
\tag{3}
\]

Where, \( k_e \) is per-phase back emf. The voltage equation in Laplace domain can be obtained from (1) and (2) for phase \( a \) and can be specified as,

\[
V_{am}(s) = R_a i_a(s) + L_a s i_a(s) + k_e \omega(s)
\tag{4}
\]

From (4), the phase current can be given as,

\[
I_a(s) = \frac{V_{am}(s) - k_e \omega(s)}{R_a + s L_a}
\tag{5}
\]

The electromagnetic torque \( T_{em} \) can be expressed as,

\[
T_{em} = \frac{(e_a i_a + e_b i_b + e_c i_c)}{\omega}
\tag{6}
\]

3. Proposed SHE PWM based loss minimization

For a trapezoidal emf machine considered, the back emf and phase current waveforms for phase a considering 120° switching mode are given in Fig. 2(a) for existing switching and Fig. 2(b) for SHE PWM switching required for speed control applications.

![Phase Current Waveform](image)

Fig. 2. Phase current waveform for 120° conduction mode (a) existing switching (b) SHE PWM switching

The expression for the current for 120° back emf and phase current waveforms for phase \( a \) is given as,

\[
I_a = \left( \frac{4f}{\pi} \right) \left( \frac{1}{3} \cos \alpha \sin \alpha + \frac{1}{3} \cos 3 \alpha \sin 3 \alpha + \frac{1}{7} \cos 7 \alpha \sin 7 \alpha \ldots \right)
\tag{7}
\]

Putting \( \alpha = 30° \) in equation (7), the harmonic spectrum can be given in Fig. 3 as,
Using the proposed switching with removal of 5th and 7th harmonics as shown in the proposed Selective Harmonic Elimination (SHE) based PWM switching of Fig. 2(b), the harmonic spectrum can be shown in Fig. 4 as,

\[
W_e = K_v B_{\text{max}}^2 f^2 \text{ Watts} \tag{10}
\]

Again, taking harmonic components in account (10) can be modified as,

\[
W_e = K_v B_{\text{max}}^2 f^2 + K_v B_{\text{max}}^2 f^2 + K_v B_{\text{max}}^2 f^2 + \ldots \text{term} \tag{11}
\]

As evident from equation (9) and (11), both the losses contain harmonic terms. Removal of the 5th and 7th order harmonics from the phase current will ensure reduced harmonic content in the induced flux linkages. Removal of these harmonics from the phase current will contribute to an induced flux waveform with minimal core losses. Consequently, the input power requirement for the BLDC motor will reduce. With the removal of lower order harmonics in phase current waveform, the torque pulsation also gets reduced.
4. Harmonic elimination using Gravitational Search Optimization (GSA)

The Gravitational Search Algorithm proposed by [16] is based on the well known Newton’s law of gravity and laws of motion. It affirms that every particle attracts every other particle and the force acting between them is directly proportional to product of their masses and inversely proportional to square of their effective separation distance. In GSA, the optimizing quantity can be referred to as agents with specified mass which will represent a possible solution by virtue of the position they take which is decided by the attractive force. The optimization agents are initialized in the search space defined by some constraints. The algorithm considers each agent or solution will be having four specifications: its position, its inertial mass, its active gravitational mass and passive gravitational mass. As mentioned, the position the agent takes in the search space is the potential solution. The gravitational and inertial masses of the agent are specified by a fitness function. The fitness function is the function which gets optimized using the solved position of the agent. The fitness function judges the solution by the direct modification of the agent’s mass.

The agents are attracted to each other in the search space by the gravitational force. The force causes global movement of the agents towards the agents with heavier masses. Agents having heavier masses will have higher fitness values compared to agents with lighter masses as gravitational attractive force is directly proportional to the mass product. These agents with heavier masses will be considered as better solutions and the agents with lighter masses will be discarded. The convergence of the algorithm is marked by dominance of agents with heavier masses. The agent with the heaviest mass will represent an optimum solution in the chosen search space. The algorithm of the GSA can be given as,

(a) Initialization of a solution within the chosen search space abide by constraints: In the beginning, a random solution is chosen within the chosen workspace. In a system with \( N \) agents, let the position of the \( i^{th} \) agent is chosen as a solution given as \( \mathbf{a}_i = (a_{i1}^d, a_{i2}^d, ..., a_{id}^d) \) for \( i = 1, 2, ..., N \). For an \( n^{th} \) dimension problem \( \mathbf{a}_i^d \) represents the position of \( i^{th} \) agent in \( d^{th} \) dimension. Gravitational constant \( G_0 \) is also initialized in this stage.

(b) Evaluation of fitness of the solution using the fitness function of the problem: In this problem, the fitness function chosen is the current Total Harmonic Distortion. This can be represented as \( f(\mathbf{a}) \).

(c) Updating Gravitational constant and masses, best and worst solutions: The Gravitational constant \( G \) is dependent on the initial chosen value \( G_0 \) in the first step. The iteration count is taken as \( t \). \( \delta \) is a constant and \( T \) is the total number of iterations for a particular problem. The expression for \( G \) as a function of iteration is,

\[ G(t) = G_0 e^{-\delta(t/T)} \]  

Since the present problem is a minimization problem, the best and worst solution can be specified as,

\[ \text{best}(t) = \min_{j \in \{1, ..., N\}} \text{fit}_j(t) \]  
\[ \text{worst}(t) = \max_{j \in \{1, ..., N\}} \text{fit}_j(t) \]  

(13)

\( \text{fit}(t) \) is the value of the fitness function for each \( j \). The gravitational and inertial masses taken as \( M_i(t) \) can be iteratively given as,

\[ M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \]  

Where, \( m_i(t) = \text{fit}_i(t) - \text{worst}(t) \) for iteration \( t \) \( \text{fit}_i(t) \) is the value of the fitness function of agent \( i \).

(d) Calculation of force and total force in different directions with calculation of acceleration: Gravitational force from agent \( i \) to \( j \) can be given as,

\[ F_{ij}^d(t) = G(t) M_{pi}(t) M_{wj}(t) \frac{(\mathbf{a}_j^d(t) - \mathbf{a}_i^d(t))}{R_{ij}(t) + \varepsilon} \]  

(15)

Where, \( M_{pi} \) and \( M_{wj} \) are the passive and active gravitational masses of agent \( i \) and \( j \) respectively. A small error constant \( \varepsilon \) is used to adjust for divide by zero error removal. \( R_{ij} \) is the Euclidean coordinate distance between the agents \( i \) and \( j \). The total force can be calculated as,

\[ F_i^d(t) = \sum_{j=1, |j| \neq i}^{N} \text{rand}_j F_{ij}^d(t) \]  

(16)

As observed, it is the random weighted sum of the force exerted on all other agents. The can have a tendency to get trapped at local optimum and this can be a problem. This is avoided by choosing exploration at the beginning of the iteration and then going for exploitation as the iteration reaches final stages with reduced number of agents. This approach
can be implemented by introducing using a variable ‘\(k_{\text{best}}\)’ which is a function of iteration \(t\). It has an initial value of \(k_0\) which continually decreases as a final solution is achieved. \(k_{\text{best}}\) linearly decreases up till a final single optimized solution. Modifying (16), the expression thus becomes,

\[
F_{ij}^d(t) = \sum_{j \in k_{\text{best}}, j \neq i}^N \text{rand} F_{ij}^d(t) \tag{17}
\]

The force thus calculated is used to calculate the acceleration using well recognized Newton’s second law of motion.

\[
a_i^d(t) = \frac{F_i^d(t)}{m_i} \tag{18}
\]

(e) Calculation of velocity and position to reach final solution: The velocity as calculated from the values of current velocity and acceleration can be expressed as,

\[
v_i^d(t + 1) = \text{rand} \cdot v_i^d(t) + a_i^d(t) \tag{19}
\]

To randomize the search with the given search space, a variable \(\text{rand}\) is chosen within 0 and 1. The potential solution can be calculated from the position as,

\[
\alpha_i^d(t + 1) = \alpha_i^d(t) + v_i^d(t + 1) \tag{20}
\]

(f) Updating the position: At the end of each iteration, the new positions are updated. Steps (b) to (f) are repeated until stop criteria is reached. In the chosen problem, the 5\(^{th}\) and 7\(^{th}\) harmonics are eliminated from the input current waveform by taking three switchings per quarter cycle. The fundamental value of the current waveform \(a_1\) is adjusted to a desired value \(M\). Thus, the problem for minimization becomes,

\[
\text{Minimize the fitness function,} \quad f(\alpha) = \frac{1}{\sum_{n=3,5}^7} (a_n)^2 \tag{21}
\]

Subject to, \(0 < \alpha_1 < \alpha_2 < \alpha_3 < \frac{\pi}{2}\),

\[
a_i = M \quad \text{and} \quad k_5(a_5-\varepsilon)^2 + k_7(a_7-\varepsilon)^2 \leq \varepsilon
\]

Where, \(\varepsilon\) is the error limit for acceptable minimizations for harmonic and \(k_5, k_7\) are the constants representing the burden for minimizing the 5\(^{th}\) and 7\(^{th}\) harmonic respectively.

In the present problem, \(k_5, k_7\) are assigned values of 20 and 40 respectively. Initially, a solution for the switching angles is chosen within the range 0 to \(\pi/2\). Initial value of \(G_0\) is taken as 80. Number of iterations (also the stopping criterion) is taken as 400. Fig.6 shows the variation of the switching angles with the change in modulation index for inverter output. GSA is chosen for the proposed optimization as it has an advantage of minimal adjustment of controlling parameters over other techniques.

![Fig. 6. Variation of switching angles with modulation index](image)

For comparison, similar harmonic optimization is solved using Particle Swarm Optimization (PSO) [18] and Genetic Algorithm (GA) [19] as shown in Fig.7.

![Fig. 7. Convergence of the fitness function vs. number of iterations](image)

As seen from Fig.7, convergence is achieved with lesser iterations using GSA as compared to the other two techniques. Thus it takes lesser computational time to use GSA to compute offline for the present problem. The switching angles are calculated offline and then stored in the microcontroller memory as a function of modulation index in the form of mixed model equations [20] for online usage. Thus the proposed technique also minimizes the computational burden of the processor for direct online calculation of the switching angles.
5. Simulation and Experimental results

A simulation study for the proposed scheme is carried out using MATLAB/Simulink R2013b. To validate the simulation, an experimental study is also conducted using an experimental BLDC motor (specification of the motor is provided in Table-1). The proposed scheme adopts a sensorless control strategy with the help of a PIC microcontroller. Initially the PWM registers of the microcontroller are initialized, and pulses are generated with a default frequency of 0.5 Hz. The reference speed is set at initial value, the proposed switching is applied with frequency A/D conversion is initiated. After the conversion calculation of the instance of zero crossing is done along with the calculation for frequency and speed. Repeatedly the reference speed is checked against a set speed at an interval of 500 msec. If the reference speed is not equal to set speed it will be incremented and again back EMF is sensed through A/D new speed is calculated and PWM pulses are generated accordingly. For any speed/torque change, PI controllers are used to stabilize the motor within around 100 msec. The required 48V DC for the BLDC motor is obtained through a DC voltage source.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>BLDC Motor Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor type</td>
<td>Surface Permanent Magnet</td>
</tr>
<tr>
<td>Rated Power</td>
<td>350W</td>
</tr>
<tr>
<td>Rated Voltage</td>
<td>48V</td>
</tr>
<tr>
<td>Rated Speed</td>
<td>450 r/min</td>
</tr>
<tr>
<td>No. of poles</td>
<td>12</td>
</tr>
<tr>
<td>Winding</td>
<td>3-phase star connected</td>
</tr>
<tr>
<td>Resistance</td>
<td>2.5 Ω</td>
</tr>
<tr>
<td>Inductance</td>
<td>11.2 mH</td>
</tr>
</tbody>
</table>

The PIC microcontroller is used for generating SHE based PWM for the BLDC driver. The experimental setup is shown in Fig.8. The simulated waveform for the phase current for normal phase current is shown in Fig. 9(a). With the proposed switching the simulated waveform is shown in Fig. 9(b).

Fig. 8. Block diagram of the experimental setup

Fig. 9. Simulated waveform for phase current for speed reference of 450 rpm for 120° conduction mode with (a) normal switching (b) proposed SHE PWM switching

The experimental waveforms were stored using a Tektronix make digital storage oscilloscope. The normal phase current is shown in Fig. 10(a) and with proposed switching is shown in Fig. 10(b).
The torque ripple profile for the machine using the proposed control for 120° conduction mode is shown in Fig. 11. The Fig. 11(a) shows the simulation and Fig. 11(b) the experimental torque ripple at full load and half rated load at 120° conduction mode.

The input power versus motor speed at different loads is plotted to justify the lower power requirement for the proposed SHE-PWM control.

As observed from the plot of Fig. 12, with the proposed control, the power requirement decreases than existing switching control scheme.

Fig. 13 shows the variation of core loss $P_c$ and copper loss $P_{cu}$ with change in loads at fixed speed of 400 t/min. With the proposed control, both the losses are minimized than the losses that occur when the BLDC operates without any current switching.
5. Conclusions
A simple efficiency optimization scheme for BLDC drive system suitable for EV applications is proposed. The efficiency optimization is realized by eliminating some unwanted lower order harmonics using GSA from the motor current. The core losses are also reduced in the proposed switching of the BLDC drive system. This also accounts for lower power consumption of the motor than existing switching techniques. The simulation and experimental results sum up the suitability of the proposed scheme.

References

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