ANALYSIS AND DESIGN OF MAGNETIC LOSS SATURATION IN
INDUCTION MOTOR

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Abstract – In this paper the non-linear system control can be solved by using theory of adaptive control with observer theory application. In this paper is analyzed IM sensor less vector control system based on Model Reference Adaptive System. Estimation of rotor flux, rotor speed and stator resistance identification is based on known stator voltages and currents. In this paper has been analyzed saturation of mutual inductance in the low speed region and impact of changes of stator and rotor resistance. Mutual inductance \( L_m \) is chosen by method described in reference.

Index Terms: Alternative energy, Induction motor control, core loss, Magnetic saturation.

1. Introduction

With the development of microelectronic circuits (DSP, FPGA, dSPACE, etc.) high-speed performances are now realized using vector control of IM drives. In these systems is intensively researched IM rotor speed estimation using measurement stator currents and voltages. This problem based on IM parameter changes due to saturation in iron and thermal changes of stator and rotor resistance.

II. Rotor flux and speed observer

The analyzed sensorless induction motor vector control is shown in Fig. 1 [1]. The control system is established in synchronously rotating \( d,q \) reference frame. The angular frequency of this frame \( \omega^* \) is at the same time reference value of first harmonic angular frequency of supply voltage vector. The observed vector control system is based on two-axis control; \( d \)-component of stator current vector \( i_{sd}^* \) that is reference value of rotor flux, and \( q \)-component of stator current vector \( i_{sq}^* \) that is reference value of IM electromagnetic torque. The reference value of angular frequency \( \omega^* \) and reference values of the stator voltage vector components \( u_{sd}^* \) and \( u_{sq}^* \) are formed by using set reference value of rotor speed \( \omega^* \) and reference value of rotor flux \( i_{sd}^* \). The modern microelectronics and power electronics circuits enable "copying" of first harmonic supply voltage \( \bar{u}_{s}^* \) from electronics circuits to IM stator, and actual stator voltage vector and reference voltage vector are equal (\( \bar{u}_{s} = \bar{u}_{s}^* \)). Also, their angular frequencies are equal (\( \omega_{s} = \omega_{s}^* \)).

![Figure1. Sensorless vector control system](image-url)
The feedback signals in the control system are measured components of stator current vector $i_{sd}$ and $i_{sq}$ and estimated rotor speed $\hat{\omega}$. The components of the stator current vector are given by transformation of measured phase currents in original $a,b,c$ reference frame.

For rotor speed estimation $\hat{\omega}$ and forming angular velocity $\omega^*_s$ has been used the method based on rotor flux adaptive control with observer theory application.

For rotor flux estimation can be used space-vector voltage equation of stator or voltage equation of rotor. The well-known space-vector voltage equation of stator is expressed as:

$$u_s = R_s i_s + \frac{d\Psi}{dt} + j\omega_c \Psi,$$  \hspace{1cm} (1)

and estimated rotor flux is:

$$\Psi_r = \frac{L_m}{L_s} (\Psi_s - \sigma L_s i_s).$$  \hspace{1cm} (2)

On the other hand, rotor flux vector can be estimated using space-vector voltage equation of rotor winding which being expressed by stator current vector has the form:

$$0 = -\frac{L_m}{L_s} \frac{d}{dt} \frac{1}{T_r} \left[ \frac{1}{T_r} + j(\omega^*_r - \omega) \right] \Psi_r + \frac{d\Psi}{dt}.$$  \hspace{1cm} (3)

where $\sigma = 1 - \frac{L^2_m}{L_s L_r}$ and $T_r = \frac{L_s}{R_s}.$

The equations (1) and (2) describe the voltage model, and equation (3) describes the current model [1].

The input values for rotor flux estimation using (1), (2) and (3) are stator supply vector $u_s$, angular frequency $\omega$, and measurement stator current vector $i_s$. The stator and rotor resistances $R_s$ and $R_r$ are thermal dependent. It is supposed that relative changes of stator and rotor resistance are equal. The stator and rotor leakage inductance $L_{s\sigma}$ and $L_{r\alpha}$ have constant (unsaturated) values, because in observed vector control system stator and rotor currents don’t reach especially large values ($I_s < 2I_{sn}$). Mutual inductance $L_{m}$ varies rapid during saturation in iron. This effect is especially expressed in low speed region and cannot be neglected. Thus, rotor flux position in reference $d,q$ frame has been described with three vectors (Fig. 2): rotor flux vector is estimated by using voltage model $\Psi^r$, current model $\Psi^c_r$ and actual rotor flux $\Psi_r$.

So, it is necessary to find such method of adaptive control that rotor flux vectors given from the current model and from the voltage model are just equal to actual rotor flux vector.

In this paper has been proposed rotor-flux-based estimator with the current model as a reference model. The $d$-axis of synchronously rotating $d,q$ frame is aligned with the flux vector $\Psi^r$. The rotor flux vector $\Psi^r$, being estimated using the voltage model is output value of adaptive model. By solving the difference in rotor flux vectors given from the voltage and current model $\Psi^r - \Psi^c_r$, is possible to estimate rotor speed $\hat{\omega}$. Also, verification of tuned stator and rotor resistance and mutual inductance is possible.

Rotor flux vector described using current model is reference vector controlled by the components of reference stator current vector $i^*_{sd}$ and $i^*_{sq}$, in condition $|\Psi^r_r| = \text{const.}$ ($d\Psi^r_r / dt = 0$).

By introducing this type of control, equation (3) can be expressed as:

$$0 = -\frac{L_m}{T_r} i^*_{sd} + \left[ \frac{1}{T_r} + j(\omega^*_r - \omega) \right] \Psi^r_s.$$  \hspace{1cm} (4)

Figure 2. Position of rotor flux vector $\Psi^r_s$ in stationary reference frame $\alpha, \beta$ and synchronously rotating reference frame $d,q$. 
If equation (4) is described in scalar mode then d-component of rotor flux vector \( \psi'_{rd} \) \( (\psi'_{rq} = 0) \) and angular velocity of \( d,q \) reference frame \( \omega_s \) become:

\[
\psi'_{rd} = L_{rd} i'_{id},
\]

(5)

\[
\frac{d\psi'_{rd}}{dt} = \omega_s = \omega + \frac{i'_{iq}}{T_{r} i'_{id}}.
\]

(6)

Because of the errors of estimated rotor speed \( \omega \) and motor parameters \( L_m \) and \( T_r \) the actual angle \( \gamma^* \) is not the flux angle. This is a common problem of all vector control systems. The used signs indicate: * – reference value, v – vector estimated from the voltage model, c – vector estimated from the current model, ^ – estimated value.

To calculate the fluxes from the voltage model of (1) and (2) a numerical integrating process is needed. However, it is difficult to maintain the vector control system stability during numerical integration process due to IM parameter changes. In order to overcome this problem, the current model using observer theory modifies the voltage model as follows [1]:

\[
\frac{d\hat{\omega}}{dt} = \frac{K_m}{T_o} \psi_{rq},
\]

(9)

that, essentially, present proportional integrator (PI) controller with rotor flux component \( \psi_{rq}^v \) as an input value and it must converge to zero. The output of PI speed controller is estimated rotor speed \( \hat{\omega} \). The constant value \( K_m \) is the proportional gain and, \( T_o \) is the integral gain of rotor speed PI controller. The rotor speed estimator is shown in Fig. 3.

![Figure 3. The rotor speed estimator](image-url)

For the adequate rotor speed estimation is used the following equation [5]:

\[
\frac{d\hat{\omega}}{dt} = \frac{K_m}{T_o} \psi_{rq}^v,
\]

(9)

where \( 1/T_c \) is the gain of the flux observer.

The quality of rotor flux and speed estimation is strongly dependent on gain \( 1/T_c \). By the simulation program it is shown that \( T_c \) must be equal to small positive value. In this paper \( T_c = 0.048 \) [ms].

Stator and rotor fluxes are estimated by (7) and (8) of the adaptive model. If all IM parameters in the simulation are properly tuned then vectors \( \psi_r \) and \( \psi_c \) are absolutely equal in dynamic state as well as in the stationary state. When there is an estimation error of motor parameters (parameters in observer, especially mutual inductance \( L_m \)), then vectors \( \psi_r \) and \( \psi_c \) are different just in dynamic state, but in stationary state are equal.

Therefore, taking into account that there is rotor flux component \( \psi_{rq}^v \) in negative feedback control loop of the PI controller of rotor speed (Fig. 1) it will really converge to zero. Fig. 3 and 4 depict the analyzed rotor flux and speed observer.

The stator resistance depends indirect on rotor speed estimation during estimation of vector rotor flux components. Their actual value varies with inner temperature of IM. Thus, it is necessary to know exactly the value of stator resistance for stability and speed control accuracy. For rotor resistance estimation is used equation as follows [1]:

\[
\frac{d\hat{R}_s}{dt} = \text{sign}(\omega) i_{iq} \mu (\psi_{rd} - \psi_{rd}^e),
\]

(10)

where \( \mu \) is the identification gain and must be set equal to a small positive value. The gain \( \mu \) defines convergence speed of rotor resistance from initial value to actual value. It was obtained by trial and error procedure (\( \mu = 2 \)). The described procedure of rotor flux estimation, rotor resistance identification is...
represented in Fig. 4.

Figure 4. Stator flux and rotor resistance estimator

It is proposed that relative changes of rotor resistance are equal to relative changes of stator resistance. Process of stator resistance identification and rotor speed estimation is simultaneously.

The measurements of hysteresis loops and associated loss were carried out in conformance with the requirements of IEC 60404-3 standard, i.e. for sine wave of flux density. The extended type B uncertainty of loss measurement related to errors introduced by the measurement system was lower than 1.5 %. The steel grade under examination for the purpose of this paper was ET 122-30 (0.3 mm thick). The grades ET 120-27 and ET 130-35 are examined. The frequency and the amplitude of flux density were equal to 5 Hz and 1.8 T, respectively. It was assumed, that for this frequency and gauge of examined steel the dynamic effects from eddy currents could be neglected.

The estimated set of model parameters is given in Table 1. The algorithm has reached the final fitness value $3.86 \times 10^{-6}$ determined as the sum of squared errors in 33 reference points because of exceeding the assumed number of iterations. The number of data points used was redundant in comparison to the problem dimensions in order to diminish the possible effect of measurement errors.

The simulated system is designed to be resonant at forth harmonic. Both even and odd harmonic components are produced during the transformer energizing. Among them, the second harmonic is dominant harmonic. The magnitudes of the harmonics during energizing vary with time. No abrupt changes are found in time varying harmonic magnitudes. An exponential function thus is able to approximate these time varying harmonics.

Table 1. Estimated set of model parameters for ET 122-30 steel sample

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$1.26 \times 10^{-6}$</td>
</tr>
<tr>
<td>$a$</td>
<td>17.3</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2546.5</td>
</tr>
<tr>
<td>$k$</td>
<td>17.3</td>
</tr>
<tr>
<td>$M_s$</td>
<td>$1.82 \times 10^{6}$</td>
</tr>
</tbody>
</table>

3. Simulation results

The input parameters of simulation program are IM parameters (appendix) and command variables $\omega^*$ and $i_{sd}^*$. The run-up of unloaded IM has been simulated to rotor speed of 6.28 [s$^{-1}$] (stator frequency is then $f_s = 1$ [Hz]). If desired condition is constant value of rotor speed ($\omega = 6.28$ [s$^{-1}$]) at applied step load, vector control systems must increase stator frequency $f_s$. Fig. 5 shows static characteristic of electromagnetic torque in mode $\psi_r = 1.2 \psi_{rm}$ at frequencies of stator voltage supply at $f_s = 1$ [Hz] and $f_s = 4.06$ [Hz].

It is shown in Fig. 5 that for rotor speed at 6.28 [s$^{-1}$] and stator voltage frequency at 4.06 [Hz] corresponds electromagnetic torque of 12.32 [Nm] ($1.2 M_{in}$). Thus, it is expected stator frequency increasing in dynamic state. To illustrate this effect motor was loaded in time $t = 159$ [ms] with load torque 20 % higher than nominal

![Figure 5. IM torque-speed characteristic by condition $\psi_r = 1.2 \psi_{rm}$ and $f_s = 4.06$ Hz](image-url)
torque value ($M_t = 1.2 M_{tn}$). The actual speed and estimated rotor speed are shown in Fig. 6. This result has been reached with saturated mutual inductance ($L_m = 0.23 \text{ H}$).

![Figure 6. Actual and estimated rotor speed by step load torque at 1,2 M_{tn}](image)

Frequency of stator voltage has been reached at 4.06 [Hz]. This is the same value defined by torque-speed characteristic in Fig. 5. In Fig. 6 is correct at the active load torque only.

The following figures show the harmonic supervision results of test system using the transform based harmonic surveying analysis. Fig.7 and Fig.8 show the fundamental and 4th harmonic voltage waveform derived from the magnitudes from FFT. Because there is no decaying or growing in the fundamental component, the magnitude obtained from FFT agrees with the magnitude of the fundamental voltage waveform.

![Figure 7. Source Currents](image)

![Figure 8. Load Currents (Phase A)](image)

The 4th harmonic, the magnitude from FFT is much smaller than that from system the 4th harmonic decays fast. Fig.9 and Fig.10 show the fundamental and 2nd harmonic current from the analysis and the magnitudes from FFT. Due to the slow decaying and growing speeds, the magnitudes are agreed well with the magnitudes from FFT.

![Figure 9. Source Currents (Phase A)](image)

The system runs at steady state with only ideal diode rectifier load connected. After the active filter takes action at 0.06 second, the induction motor is switched into the test system at 0.12 second. The motor starting transient lasts for several cycles and dies down approximately at 0.2 second. The test system finally
settles down at steady state with both motor and diode rectifier load connected.

Figure 10. Source Currents Filter (Phase A)

4. Conclusion

In this paper the sensorless vector control system is based on the current model as the reference model and voltage model as the adaptive model. Proposed rotor flux and rotor speed estimation algorithm is simpler than conventional MRAS algorithm that inversely defines adaptive and reference model.

The analyzed vector control system is based on rotor speed estimation by solving an error in estimated $q$-component of rotor flux and by stator resistance estimation by solving an error in estimated $d$-component of rotor flux. The rotor flux and rotor speed estimation strongly depend on the observer gain $1/T_c$.

The researchers, in this paper, have been shown that mutual inductance $L_m$ saturation strongly depends on quality of rotor speed estimation. Unsaturated value of mutual inductance $L_m$ in analyzed observer gives bad rotor speed estimation, and the vector control system becomes unstable. By determining mutual inductance in the way described in reference [3] actual speed is practically in conformity with the estimated rotor speed.

References


