ON THE COGGING TORQUE REDUCTION CAPABILITY OF FRACTIONAL-SLOT PM MACHINES

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Abstract: The paper is devoted to an investigation of the cogging torque exhibited by two brushless surface mounted PM machines which share the same rotor topology and differ by the slot per pole and per phase: (i) a unity one and (ii) a fractional one equal to $2/5$ which has been found a viable candidate to achieve many interesting features especially a high torque production capability with a reduced cogging torque. In a first step, an analytical formulation of the cogging torque is derived, then applied to predict the cogging torques of both machines. It has been found that the machine with a fractional slot per pole and per phase produces a lower cogging torque. Finally, a 2D FEA validation of the analytical results is provided combined with the computation of the electromagnetic torques developed by both machines.

Key words: Fractional-slot permanent magnet machines, cogging torque, analytical formulation, finite element analysis, comparison.

1. Introduction

Most if not all topologies of permanent magnet machines suffer more or less from the torque oscillation due to the interaction between the PMs located in one side of the air gap and the magnetic circuit teeth located in the other side, yielding the so-called “cogging torque”.

Many works have been reported in the literature regarding the assessment of cogging torque. Some authors proposed analytical models based on the virtual work method [1], [2], [3]. Others presented models based on the Maxwell stress method [4], [5]. The use of combined analytical-FEA methods have also been extensively treated [6], [7].

The established models are commonly developed for the sake of the cogging torque reduction. This latter represents a state of the art topic. The world-wide reported cogging torque reduction technique is the skewing, [4], [9], [8], [10], with a one-slot pitch of either the PMs or the teeth in the case of integer-slot PM machines. Nevertheless, the skewing remains a penalty for the machine manufacturability. This drawback could be by-passed considering fractional-slot permanent magnet machines where the skewing turns to be useless if suitable slot/pole combinations are selected.

The growing interest in fractional-slot permanent magnet machines is also due to [15], [14], [13], [11], [12]:

- their short end-winding leading to reduced copper losses and improved compactness, especially when double-layer slots are considered,
- their high slot fill factor especially when coupled to segmented stator structures,
- their interesting fault-tolerance capabilities thanks to their low mutual inductances, especially when single-layer slots are considered.

Recently, Zhu et al. [16], proposed an analytical approach based on a subdomain model for the computation of the open-circuit magnetic field in surface-mounted permanent-magnet machines with any pole and slot combinations, including fractional slot machines.

The present work develops the Zhu approach to investigate the cogging torque exhibited by two brushless surface mounted PM machines which share the same rotor topology and differ by the slot per pole and per phase.
2. Cogging Torque Formulation

Let us apply the approach developed by Zhu et al. [16] to a 12 slots/10 poles surface mounted PM machine whose cross-section is illustrated in figure 1.

Fig. 1: Layout of a 12 slots/10 poles surface mounted PM machine highlighting three types of subdomains

The magnets are surface mounted and radially magnetized. Prior to deal with the analytical model, the following assumptions are accounted for in order to simplify the electromagnetic phenomena [16]:

- simplified stator slot shape,
- stator and rotor iron infinite permeability,
- neglected end effect,
- non conductive magnet material,
- magnet linear demagnetization.

The exact subdomain model consists in dividing the whole study domain of the field problem into three types of subdomains [16], such that: magnet (Region 1), air-gap (Region 2), and stator slots (Region 3) \((i = 1, 2, \ldots N_s)\). Then, by applying the boundary conditions into the scalar potential distributions in polar coordinates for each sub-region, taking into consideration the influence of interaction between slot openings, one can obtain the air gap flux density expression.

Let’s remind the general formulation of the scalar potential distributions expressed in terms of the polar coordinates \((r, \alpha)\), in the case of Region 1 and for multipolar machines [16]:

\[
\phi_1(k \neq 1) = \sum_k \Gamma_1(k, r) \cos(ka) + \Lambda_1(k, r) \sin(ka) \tag{1}
\]

where:

\[
\begin{align*}
\Gamma_1(k, r) &= A_1(k) r^k + B_1(k) r^{-k} + \frac{M_{sk} r}{\mu_r (1 - k^2)} \\
\Lambda_1(k, r) &= C_1(k) r^k + D_1(k) r^{-k} + \frac{M_{sk} r}{\mu_r (1 - k^2)}
\end{align*}
\tag{2}
\]

and in the case of Region 2 [16]:

\[
\phi_2(k \neq 1) = \sum_k \Gamma_2(k, r) \cos(ka) + \Lambda_2(k, r) \sin(ka) \tag{3}
\]

where:

\[
\begin{align*}
\Gamma_2(k, r) &= A_2(k) r^k + B_2(k) r^{-k} \\
\Lambda_2(k, r) &= C_2(k) r^k + D_2(k) r^{-k}
\end{align*}
\tag{4}
\]

and in the case of the \(i^{th}\) slot (Region 3) [16]:

\[
\phi_3i = \sum_m C_{3i}(m) \left[ \left( \frac{r}{R_{sb}} \right)^{F_m} - \left( \frac{r}{R_{sb}} \right)^{-F_m} \right] \times \sin \left[ F_m \left( \alpha + \frac{b_{oa}}{2} - \alpha_i \right) \right] \tag{5}
\]

where \(\mu_r\) is the relative permeability of magnets, \(k\) and \(m\) are harmonic orders, \(R_{sb}\) is the radius of the slot bottom, \(A_1(k)\) to \(D_1(k)\), \(A_2(k)\) to \(D_2(k)\), and \(C_{3i}(m)\) are coefficients to be determined by the boundary conditions, \(\alpha_i\) is the position of the \(i^{th}\) slot, \(b_{oa}\) is the slot opening angle corresponding to the slot opening width \(b_o\), and:

\[
F_m = \frac{m \pi b_{oa}}{b_o} \quad \& \quad b_{oa} = \frac{b_o}{R_s} \tag{6}
\]

In what follows, let us focus on the rotor which is illustrated in figure 2.

Fig. 2: Layout of the rotor of a 12 slots/10 poles surface mounted PM machine
The magnetization of magnets can be expressed as follows [16]:

\[ M = M_r \cdot r + M_\alpha \cdot \alpha \]  

(7)

In the case of radial magnetization \( M_\alpha = 0 \), which yields:

\[ M_r = \sum_{k=5,15,25,...} M_{rck} \cos(k\alpha) + M_{rsk} \sin(k\alpha) \]  

(8)

where:

\[ \left\{ \begin{array}{ll}
M_{rck} &= M_r \cos(k\omega_r t) \\
M_{rsk} &= M_r \sin(k\omega_r t)
\end{array} \right. \]

with:

\[ M_{rck} = \frac{4p_r B_r}{k\pi \mu_0} \sin\left(\frac{k\pi \alpha_p}{2p_r}\right) \quad k = 1, 3, 5... \]  

(10)

where \( \mu_0 \) is the relative permeability of air, \( B_r \) is the magnet residual flux density and \( \alpha_p \) is the pole arc to pole pitch ratio, \( \omega_r \) is the rotor speed, and \( p_r \) is the rotor pole pair.

According to equation (3), the scalar potential on the surface of the stator bore \( (r = R_s) \) is:

\[ \phi_2 = \sum_k \Gamma_2(k, R_s) \cos(k\alpha) + \Delta_2(k, R_s) \sin(k\alpha) \]  

(11)

Moreover, according to equation (5), the scalar potential on the surface of the stator bore can also be expressed as:

\[ \phi_s = \left\{ \begin{array}{ll}
\phi_{3i}(R_s) & \text{along the slot opening} \\
0 & \text{otherwise}
\end{array} \right. \]  

(12)

where \( \phi_{3i}(R_s) \) could be expressed as follows:

\[ \phi_{3i}(R_s) = \sum_m C_{3i}(m) \chi_m \sin[F_m(\alpha + \eta_0 - \alpha_i)] \]  

(13)

with:

\[ \chi_m = \frac{R_s}{R_{sb}} F_m - \frac{R_s}{R_{rb}} F_m \]  

(14)

According to continuity condition, one can write:

\[ \phi_2(R_s) = \phi_{3i}(R_s) \]  

(15)

In order to solve equation (15), \( \phi_{3i} \) can be expanded into Fourier series as:

\[ \phi_s = \sum_k \left[A_s(k) \cos(k\alpha) + B_s(k) \sin(k\alpha)\right] \]  

(16)

where:

\[ \left\{ \begin{array}{l}
A_s(k) = \sum_m C_{3i}(m) \chi_m \eta_{si}(m,k) \\
B_s(k) = \sum_m C_{3i}(m) \chi_m \xi_{si}(m,k)
\end{array} \right. \]  

(17)

\[ \eta_{si} \text{ and } \xi_{si} \text{ are expressed as follows:} \]

\[ \left\{ \begin{array}{l}
\eta_{si} = \eta_s \cos(\xi_{si} k) - \xi_s \sin(\xi_{si} k) \\
\xi_{si} = \xi_s \sin(\xi_{si} k) + \xi_s \cos(\xi_{si} k)
\end{array} \right. \]  

(18)

where:

\[ \eta_s = \frac{2mb_{oa} \sin^2(m\pi/2)}{m^2\pi^2 - k^2b_{oa}^2} \quad m\pi \neq k\eta_0 \]  

(19)

and:

\[ \xi_s = \frac{-2mb_{oa} \sin^2(m\pi/2)}{m^2\pi^2 - k^2b_{oa}^2} \quad m\pi = k\eta_0 \]  

(20)

Accounting for the equation set (11)-(17), one can establish the following:

\[ \left\{ \begin{array}{l}
A_s = A_2 R_s^k + B_2 R_s^{-k} = \sum_{m} C_{3i}(m) \chi_m \eta_{si}(m,k) \\
B_s = C_2 R_s^k + D_2 R_s^{-k} = \sum_{m} C_{3i}(m) \chi_m \xi_{si}(m,k)
\end{array} \right. \]  

(21)

Employing the boundary conditions considering an infinite permeability of the rotor yoke, continuous scalar potential, and radial flux density at \( (r = R_m) \) leads to the following harmonic coefficients of \( \phi_2 \):

\[ \left\{ \begin{array}{l}
K_{Brc} = A_{4k} \gamma(k) M_{rck} \\
K_{Brs} = A_{4k} \gamma(k) M_{rsk}
\end{array} \right. \]  

(23)

with in the case of an internal rotor machine:

\[ \left\{ \begin{array}{l}
A_{4k} = -\frac{1}{2} \mu_0 k \\
\gamma = \frac{2}{k^2 - 1} \left[ (1-k) - 2 \left( \frac{R_s}{R_m} \right)^{k+1} + (k+1) \left( \frac{R_s}{R_m} \right)^{2k} \right] \\
\rho = (\mu_{r+1}) \left[ 1 - \left( \frac{R_s}{R_c} \right)^{2k} - (\mu_{r-1}) \left( \frac{R_s}{R_c} \right)^{2k} \right] \\
B_{5k} = R_s^{-k} \\
B_{6k} = R_s^{k} \\
A_{4k} = \frac{1}{p} \left( \mu_{r+1} + (\mu_{r+1}) \left( \frac{R_s}{R_m} \right)^{2k} \right) \\
A_{6k} = \frac{1}{p} \left( \mu_{r-1} + (\mu_{r+1}) \left( \frac{R_s}{R_m} \right)^{2k} \right)
\end{array} \right. \]
The remaining harmonic coefficient $C_{3l}(m)$ of $\phi_{3l}$ could be calculated using the last boundary condition which considers a continuous radial component flux density at stator slot opening [16]. According to (22), the following equation set can be derived:

\[
\begin{aligned}
A_k R_s^{k-1} - B_k R_s^{k-1} &= G_1 - \lambda A_k(k) \\
C_k R_s^{k-1} - D_k R_s^{k-1} &= G_2 - \lambda B_k(k)
\end{aligned}
\]

(24)

with the exception of a stator rotor equation:

\[
\begin{aligned}
G_1 &= \left( \frac{R_m}{R_s} \right)^{k+1} \gamma(k) M_{rck} \\
G_2 &= \left( \frac{R_m}{R_s} \right)^{k+1} \gamma(k) M_{rsk} \\
\lambda &= -\frac{1}{R_s} \left[ A_{sk} + \left( \frac{R_m}{R_s} \right)^{2k} A_{sk} \right]
\end{aligned}
\]

The harmonic coefficient $C_{3l}(m)$ can be obtained by solving [16]:

\[
\delta_m C_{3l}(m) = \sum_k k \xi_k C_{3l}(m) + \gamma G_2 \xi_{sl}(m, k) \sum_k k \xi_k \xi_{sl}(m, k) \\
\sum_k k \xi_k \xi_{sl}(m, k) \sum_k k \xi_k \xi_{sl}(m, k) \\
\sum_k k \xi_k \xi_{sl}(m, k) \sum_k k \xi_k \xi_{sl}(m, k)
\]

(26)

where $j = 1, 2, 3, ..., M$, $l = 1, 2, 3, ..., N_s$, $M$ is the highest harmonic order of the scalar potential distribution along the slot openings, and

\[
\delta_m C_{3l}(m) = \frac{F_m}{R_s} \left[ \left( \frac{R_s}{R_m} \right)^{F_m - 1} + \left( \frac{R_s}{R_m} \right)^{-F_m - 1} \right]
\]

Equation (26) can be rewritten as follows:

\[
\delta_m C_{3l} = Y_{sl} - P_{sl} \sum_{l=1}^{N_s} \eta_{sl} \chi C_{3l} - Q_{sl} \sum_{l=1}^{N_s} \tau_{sl} \chi C_{3l}
\]

(28)

where:

\[
\begin{aligned}
\delta &= diag(\delta, \delta, ..., \delta, M) \\
C_{3l} &= [C_{3l}(1), C_{3l}(2), ..., C_{3l}(M)]^T \\
\eta_{sl} &= \left( \sum_{m=1}^{N_s} C_{3l}(m) \eta_{sl}(m, k) \right)_{M \times N} \\
\tau_{sl} &= \left( \sum_{m=1}^{N_s} C_{3l}(m) \tau_{sl}(m, k) \right)_{M \times N} \\
Y_{sl} &= \sum_{m=1}^{N_s} C_{3l}(m) \eta_{sl}(m, k) G_1 \\
\sigma_{sl} &= \sum_{m=1}^{N_s} C_{3l}(m) \tau_{sl}(m, k) G_2 \\
P_{sl} &= \sum_{m=1}^{N_s} C_{3l}(m) \xi_{sl}(m, k) G_1 \\
Q_{sl} &= \sum_{m=1}^{N_s} C_{3l}(m) \xi_{sl}(m, k) G_2 \\
\lambda &= diag(\lambda(1), \lambda(2), ..., \lambda(N)) \\
\chi &= diag(\chi(1), \chi(2), ..., \chi(M))
\end{aligned}
\]

Equation (28) shows that the scalar potential contributions of all slots are coupled together. These have to be solved simultaneously to completely account for such a slot coupling. To do so, equation (28) is rewritten as:

\[
\delta_m C_{3l} = Y_{sl} - P_{sl} \eta_{sl}^T \chi C_{3l} - Q_{sl} \tau_{sl}^T \chi C_{3l}
\]

(30)

where:

\[
\begin{aligned}
\delta_t &= diag(\delta, \delta, ..., \delta)_{N_s \times N_s} \\
C_{3l} &= \left[ C_{3l}^T, C_{3l}^2, ..., C_{3l}^{N_s} \right]^T \\
Y_{sl} &= \left[ Y_{sl}^T, Y_{sl}^2, ..., Y_{sl}^{N_s} \right]^T \\
\sigma_{sl} &= \left[ \sigma_{sl}^T, \sigma_{sl}^2, ..., \sigma_{sl}^{N_s} \right]^T \\
\tau_{sl} &= \left[ \tau_{sl}^T, \tau_{sl}^2, ..., \tau_{sl}^{N_s} \right]^T \\
\eta_{sl} &= \left[ \eta_{sl}^T, \eta_{sl}^2, ..., \eta_{sl}^{N_s} \right]^T
\end{aligned}
\]

(31)

The coefficient $C_{3l}(m)$ can be obtained considering a variable change as:

\[
C_{3l} = \chi C_{3l}
\]

Equation (28) shows that the scalar potential contributions of all slots are coupled together. These have to be solved simultaneously to completely account for such a slot coupling. To do so, equation (28) is rewritten as:

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\]

(30)

where:

\[
\begin{aligned}
\delta_t &= diag(\delta, \delta, ..., \delta)_{N_s \times N_s} \\
C_{3l} &= \left[ C_{3l}^T, C_{3l}^2, ..., C_{3l}^{N_s} \right]^T \\
Y_{sl} &= \left[ Y_{sl}^T, Y_{sl}^2, ..., Y_{sl}^{N_s} \right]^T \\
\sigma_{sl} &= \left[ \sigma_{sl}^T, \sigma_{sl}^2, ..., \sigma_{sl}^{N_s} \right]^T \\
\tau_{sl} &= \left[ \tau_{sl}^T, \tau_{sl}^2, ..., \tau_{sl}^{N_s} \right]^T \\
\eta_{sl} &= \left[ \eta_{sl}^T, \eta_{sl}^2, ..., \eta_{sl}^{N_s} \right]^T
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Y_{sl} &= \left[ Y_{sl}^T, Y_{sl}^2, ..., Y_{sl}^{N_s} \right]^T \\
\sigma_{sl} &= \left[ \sigma_{sl}^T, \sigma_{sl}^2, ..., \sigma_{sl}^{N_s} \right]^T \\
\tau_{sl} &= \left[ \tau_{sl}^T, \tau_{sl}^2, ..., \tau_{sl}^{N_s} \right]^T \\
\eta_{sl} &= \left[ \eta_{sl}^T, \eta_{sl}^2, ..., \eta_{sl}^{N_s} \right]^T
\end{aligned}
\]

(31)

The harmonic of scalar potential distribution along the slot openings 17 can be rewritten as:

\[
\begin{aligned}
\delta_m C_{3l} &= \chi C_{3l} \\
W_{sl} &= \delta_m \chi^{-1}
\end{aligned}
\]

(34)

The air-gap flux density can be expressed by [16]:

\[
\begin{aligned}
B_{ry} &= \sum_k B_{rck} \cos(k\alpha) + \sum_k B_{rsk} \sin(k\alpha) \\
B_{ry} &= \sum_k B_{rck} \cos(k\alpha) + \sum_k B_{rsk} \sin(k\alpha) \\
B_{ry} &= \sum_k B_{rck} \cos(k\alpha) + \sum_k B_{rsk} \sin(k\alpha)
\end{aligned}
\]

(36)

where:

\[
\begin{aligned}
B_{rck} &= K_{Br} r_{Br} - g_{Br} A_s \\
B_{rsk} &= K_{Br} r_{Br} - g_{Br} B_s \\
B_{rck} &= K_{Br} r_{Br} - g_{Br} A_s \\
B_{rsk} &= K_{Br} r_{Br} - g_{Br} B_s
\end{aligned}
\]

(37)
with in the case of an internal rotor machine:

\[
\begin{align*}
    f_{B_r} &= \left( \frac{r}{R_s} \right)^{k-1} \left( \frac{R_m}{R_s} \right)^{k+1} + \left( \frac{R_m}{r} \right)^{k+1} \\
    f_{B_\alpha} &= -\left( \frac{r}{R_s} \right)^{k-1} \left( \frac{R_m}{R_s} \right)^{k+1} + \left( \frac{R_m}{r} \right)^{k+1}
\end{align*}
\]

(38)

and:

\[
\begin{align*}
    g_{B_r} &= \Delta \left[ \left( \frac{r}{R_s} \right)^{k-1} + \left( \frac{R_m}{R_s} \right)^{k-1} \left( \frac{R_m}{r} \right)^{k+1} \right] \\
    g_{B_\alpha} &= \Delta \left[ \left( \frac{r}{R_s} \right)^{k-1} - \left( \frac{R_m}{R_s} \right)^{k-1} \left( \frac{R_m}{r} \right)^{k+1} \right]
\end{align*}
\]

(39)

with \( \Delta = \frac{2 \mu_0 \mu_r k}{R_s p} \).

Following the determination of the scalar potential distribution harmonics along the stator bore, the air-gap flux density distribution can be predicted using equations (36)–(37). Then, the cogging torque can be computed applying the Maxwell stress tensor, such that:

\[
T_{\text{cog}} = \frac{\pi l_{af}^2}{\rho_0} \sum_k (B_{rck} B_{ack} + B_{rsk} B_{ask})
\]

(40)

3. Case Study

This paragraph deals with the comparison of the cogging torques exhibited by two brushless surface mounted PM machines which share the same rotor topology and differ by the slot per pole and per phase: (i) a unity one, and (ii) a fractional one equal to 2/5. The selection of the slot/pole combination characterized by an \( S_{pp} = 2/5 \) is related to several reasons including, particularly:

- a high winding factor of (0.966),
- a low magnetic force,
- a low cogging torque due to its high \( LCM(N_s, 2p_r) \). It should be noted that the number of cogging torque periods per revolution is equal to the \( LCM(N_s, 2p_r) \). The higher is the cogging torque frequency the lower is its amplitude.

The cross-sections of the studied machines are shown in figures 3 and 4, respectively. Table 1 gives the geometrical parameters shared by both machines. It is worth noting that these parameters are not selected by any sizing approach especially regarding cogging torque reduction.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{exts} )</td>
<td>117 mm</td>
</tr>
<tr>
<td>( R_{ints} )</td>
<td>70.5 mm</td>
</tr>
<tr>
<td>( R_{extr} )</td>
<td>66 mm</td>
</tr>
<tr>
<td>( R_{intr} )</td>
<td>40 mm</td>
</tr>
<tr>
<td>( l_a )</td>
<td>70 mm</td>
</tr>
</tbody>
</table>

Table 1: Geometrical SMPMM parameters

3.1 Analytical Model Based Assessment of the Cogging Torque

Figure 5 shows the analytically-predicted cogging torques exhibited by both machines under study, with the continuous line and the mark-line illustrate \( T_{\text{cog}} \) of the unity \( S_{pp} \) machine, and the fractional \( S_{pp} \) one, respectively.

One can notice that the peak-to-peak cogging torque is higher in the case of the integer-slot PM machine. It represents about the triple of the one of the fractional-slot PM machine. This could be explained by the fact that the frequency of the cogging torque of the integer-slot PM machine is lower and hence has higher magnitude. In other words, this is directly linked to the value of the \( LCM(30, 10) = 30 \), which is lower, compared to the one of the fractional-slot machine \( LCM(12, 10) = 60 \).

The results shown in figure 5 highlight the advantage of using fractional-slot concentrated windings in so far that they exhibit intrinsically low cogging torque.
Fig. 5: Analytical cogging torque computed for both configurations: integer-slot distribution in continuous line and the fractional-slot distribution with mark-line distribution

3.2 Finite Element Analysis Based Assessment of the Cogging Torque

A. Case of Integer-Slot Machine

Figure 6 shows the no-load flux lines in a cross section of the magnetic circuit of the 30 slot/10 pole machine.

Figure 7 illustrates the no-load flux density distribution in a cross section of the magnetic circuit of the 30 slot/10 pole machine.

Figure 8 shows, in continuous line, five periods of the cogging torque waveform computed by FEA. For the sake of comparison, the cogging torque waveform analytically-predicted, as shown in figure 5, is recalled in mark-line. Comparing both cogging torque waveforms reveals a good agreement, which confirms the effectiveness of the accuracy of the analytical model applied for the cogging torque prediction.

Fig. 6: No-load flux lines in a cross section of the magnetic circuit of the 30 slot/10 pole machine

Fig. 7: No-load flux density distribution through a cross section of the magnetic circuit of the 30 slot/10 pole machine ($S_{pp} = 1$)

Fig. 8: Cogging torque waveform of the 30 slot/10 pole machine. Legend: (continuous line) FEA results, (mark-line) analytical results.

B. Case of fractional-Slot Machine

Figures 9 and 10 show the no-load flux lines and flux density map, respectively, in a cross section of the magnetic circuit of the 12 slot/10 pole machine.

Figure 11 illustrates, in continuous line, the cogging torque waveform computed by FEA, of the 12 slot/10 pole surface mounted PM machine and, in dashed-line, the one predicted by the analytical model.

One can notice that the cogging torques waveforms computed by FEA and predicted by the analytical model are in good agreement. The peak-to-peak amplitude is almost equal to 3Nm, while it is approximately equal to 12Nm in the case of 30 slot/10 pole machine.
C. Extension of the Comparison to the Torque Production Capability

It has been clearly shown that the cogging torque amplitude exhibited by the fraction-slot machine is almost equal to the quarter of the integer-slot machine one. In order to extend the comparison to the torque production capability of both machines, the torque-angle characteristics have been computed by FEA and are illustrated in figure 12. One can notice that the fractional slot machine offers higher mean torque with lower ripple.

4. Conclusion

The paper was devoted to an investigation of the cogging torques exhibited by two brushless surface mounted PM machines which share the same rotor topology and differ by their slot per pole and per phase, such that: (i) a unity one and (ii) a fractional one equal to 2/5.

An analytical model aimed at the prediction of the cogging torque was firstly developed. Then, it was applied to both machines. It has been found that the cogging torque amplitude exhibited by the fraction-slot machine is almost equal to the quarter of the integer-slot machine one. Finally, a 2D FEA validation of the analytical results was provided combined with the electromagnetic torque computation. This has revealed that the fractional-slot machine has a better torque production capability.
References


