Abstract—The work reported in this paper presents the design and performance evaluation of a new Hybrid adaptive algorithm deployed in fast Distance protection of a series compensated transmission system. The proposed algorithm is a hybrid scheme which incorporates multi resolution analysis (MRA) of wavelet transform along with adaptive window Discrete Fourier Transform (DFT). The phasor estimation technique possesses the advantage of recursive computing and adaptive compensation method for removal of decaying dc offset in the fault transient signal. The fault detection index is embedded along with variable data window such that the proposed scheme provides fast tripping decision with high accuracy. A dynamic model of a transmission system equipped with series compensation capacitor along with MOV (Metal Oxide Varister) is simulated using Power System Block Set of MATLAB/Simulink Software. Extensive simulation considering different percentage of series compensation has been carried out to obtain large set of fault transients signal samples. The evaluation results based on the criteria of speed and accuracy of line fault detection is presented, which demonstrates the efficacy of the proposed scheme.

Key words—Adaptive algorithm, Digital distance relay, Discrete Fourier Transform, Discrete Wavelet Transform, Series compensation.

1. Introduction
Series compensation plays the vital role in modern heavily loaded grid connected transmission lines. The series capacitor deployment makes sense because it is simple and could be installed for 15 to 30% of the cost of installing a new line. Series compensation in modern power systems influences the power flow in particular network segment, reduces active power losses, prevents sub synchronous oscillations, and connects more robustly different subsystems to stronger integrated network [1, 2]. The introduction of series compensation in existing networks requires not only extensive studies into the expected performance of the new system, but also into the influence of its introduction on the operation of existing protection control and monitoring systems. The protection of series compensated lines is considered to be one of the most difficult tasks because of some main reasons like apparent change in the impedance depending on whether the air-gap has flashed or not, Inversions of voltage and current signals, sub transients due to air-gaps and nonlinearity of Metal Oxide Varistors (MOV’s).

The protective distance relays, which make use of impedance measurements in order to determine the presence and location of faults, are "fooled" by installed series capacitance on the line when the presence or absence of the capacitor in the fault circuit is not known a priori. The fault voltage and current signals produced on the transmission line contain different frequency components. These components include decaying harmonics, as well as decaying DC components occurring due to resonance between the system inductance and series capacitor [2]. The constituent of transient fault current signal include sub-synchronous frequencies having frequency components varying around half the fundamental frequency value, odd harmonics due to MOV conduction during faults, high-frequency components caused by resonance between line capacitance and line inductance, and the fundamental components of the steady-state fault current. Thus the protective relaying approach for series compensated transmission lines differs from that of uncompensated transmission lines. One of the directions the progress in relay design is concerned with the improvement of conventional relay algorithms based on phasor concepts [1].

During the last decades a lot of fault locating and relaying algorithms have been developed to protect the transmission lines, including the one-end and two-end algorithm. We can find a number of advanced techniques proposed for the phasor estimation in literature to meet the high speed and accuracy in distance protection [3]-[12]. Among these, distance relays employing digital filters with band pass structures quickly extract the fundamental frequency component from the input signals and accurately estimate the fault location under most of the usual transmission system configuration. Even though the Discrete Fourier Transform (DFT) algorithm is most popular and standard in industry, its performance results in erroneous estimates due to the adverse effect of decaying dc component in the fault
transients [3]-[5]. The distance relay mal-operates (overreach / under reach) due to the decaying dc offset. Digital mimic filter is proposed to eliminate the dc offset in current waveforms [3]. But the method gives the best performance when the time constant of both the mimic filter and dc offset is same. DFT based dc offset removal algorithm proposed in [4] and [5] cannot achieve high speed because of long window length of the filters. An adaptive distance relay which can overcome the speed and accuracy problems with that of the fixed data window algorithm is proposed in [6] and a new adaptive window technique embedded with fault detector and an adaptive DC offset removal scheme is proposed in [7]. However the speed can be further enhanced by pre-processing the fault signals. A wavelet-based signal processing technique is considered to be an effective tool for this purpose [9]-[13]. One of its major applications in power system protection is in fault detection. With MRA technique of wavelet transforms, pyramid algorithms are proposed [9]-[12] for fault detection and phasor estimation. Since the phasors are estimated using approximate coefficients at higher levels, the speed and accuracy will be affected by larger window lengths and also steps should be taken for dc component elimination. Further, combined techniques such as wavelet transforms and Artificial neural network is proposed in [12] for fault detection and classification. An algorithm based on travelling waves for the protection of series compensated lines is proposed in [14]. An algorithm based on high-frequency signals have been proposed in [15], in which a specialized measurement unit consisting of stack tuner and line trap is used to capture the high-frequency components of the fault signal.

In light of above development efforts, considering scope for further improvement in this direction, this study presents a hybrid scheme embedded with wavelet transform –MRA and an adaptive window DFT algorithm to enhance the speed of line fault detection and phasor estimation without compromising accuracy. Theoretical basis and design of the scheme are described in forthcoming sections. The proposed technique proves to be computationally efficient and no statistical information concerning the signals is required. Extensive simulation tests are conducted on a realistic series compensated transmission system simulated in MATLAB/SIMULINK and comparative studies of the scheme with the resent method proposed in [7] are explored and sample results are presented.

2. Proposed hybrid algorithm fault detection and phasor detection

Figure 1 shows the sequential flow of proposed hybrid scheme. In the proposed hybrid scheme, the simulated voltage and current signals are decomposed at level one of wavelet transform-MRA and the coefficients at level one are considered by the relaying algorithm, for impedance extraction. The Relaying Algorithm is the Adaptive data window DFT with recursive computing, along with the fault detection index and variable data window technique. The decaying dc offset component is removed from fault signals using an adaptive compensation method [7].

The approximate coefficients at level one, A1, between 0 and Fs/2 (Fs → Sampling frequency), with the fundamental component details, is used by the phasor estimation algorithm for the fundamental phasor estimation to calculate the impedance, meeting the requirement in distance relays.

![Sequence of operation of the proposed hybrid scheme](image)

2.1 Wavelet Transform: Discrete Wavelet Transform (DWT)

In recent times, WT is a quite useful technique for characterizing the transient signals occurring in a power system. DWT is a time frequency analysis technique that is most suited for non-stationary signals.

DWT analyses the signal at different frequency bands by decomposing the signal into detail information and coarse approximations called wavelet function and scaling function.

The decomposition of the signal into different frequency bands is simply obtained by successive high pass and low pass filtering of time domain signal, which
is achieved by Wavelet Filter Banks (WFB) [9] [10]. The DWT technique is indicated in Fig. 2.

The original sampled signal \(x(n)\) is passed through a high pass filter \(h(n)\) and low pass filter \(l(n)\). Filtering a signal corresponds to the mathematical operation of convolution of the signal with the impulse response of the filter.

After the filtering, half of the samples are eliminated according to Nyquist’s rule. The signal, therefore, is decimated by 2, simply by discarding every other sample. This constitutes the level one decomposition of original signal. \(D_1\) and \(A_1\) are the detail coefficients and the approximation coefficients at level one respectively. They are defined as

\[
D_1[n] = \sum_{k} h[k - 2n] x[k] \quad (1-1)
\]

\[
A_1[n] = \sum_{k} l[k - 2n] x[k] \quad (1-2)
\]

This procedure called sub-coding is repeated for further decomposition. At every level, the filtering and sub-sampling results in half the time resolution and double the frequency resolution. Each level of decomposition analyses the signal at different frequency ranges at different resolutions, hence the name Multi Resolution Analysis (MRA).

As shown in Fig. 2, if the original Sampling frequency is \(F_s\), in level one of the decomposition structure, the signal information captured by \(D_1\) (output of high pass filter), is between \(F_s/4\) and \(F_s/2\) of the frequency band. \(A_1\), output of low pass filter, has the signal information between 0 to \(F_s/2\). Similarly, in level two, \(D_2\) captures information between \(F_s/8\) and \(F_s/4\) and \(A_2\) between 0 and \(F_s/4\) of the frequency band. The decomposition process can be repeated up to the required level. Thus, this operation is recursive as a tree or pyramid algorithm, yielding a group of signals that divides the spectrum of the original signal into octave bands with successively coarser measurements in time as the width of each spectral band narrows and decreases in frequency. Hence, the approximate or the expansion coefficients represent the approximation of the original signal with a resolution of one point per every decimation of the original signal by 2 and the detail coefficients represent details of the original signal at different levels of resolution respectively. These coefficients completely and uniquely describe the original signal and can be used in a way similar to the Fourier transform.

In the present proposed scheme the decomposed fault signals structure at level 1 is adopted for the fault detection and phasor estimation by the developed adaptive window algorithm to achieve high speed in distance relays taking the accuracy into account.

### 2.2 Fault Detection Index:

The technique used for the fault detection Index is as mentioned in [7], originally proposed by J.A. Jiang et al [8] based on a fault detection/location index for a single phase. An adaptive PMU-based fault-detection/location technique is developed using two-terminal synchronized phasors and distributed line model [1][2]. The fault-detection indices are solved as

\[
N = \frac{1}{2} [V_r \cdot Z_c I_r] - \frac{1}{2} \exp(\gamma L)[V_s \cdot Z_c I_s] \quad (2)
\]

\[
M = \frac{1}{2} [V_s + Z_c I_s] \exp(-\gamma L) - \frac{1}{2} [V_r - Z_c I_r] \quad (3)
\]

\[
D = \frac{abs(N)}{abs(M)} = 2 \gamma L \quad (4)
\]

Where \(D\) is the suggested fault location/detection index and \(L\) is the total length of the transmission line. \(\gamma\) is the propagation constant and \(Z_c\) is the surge impedance of the transmission line. \(V_r\) and \(I_r\) are the sending and receiving end voltages and currents respectively.

Under healthy conditions, the absolute values of \(N\) and \(M\) will be ideally held at zero and thus the index \(D\) would be very high. But, even a couple of fault samples are enough to ensure that \(D\) changes drastically enough so that by means of a threshold, the fault can be detected very fast. In the simulation studies, since only single end data is obtained, the above equations are modified to exclude the receiver end terms like \(V_r\) and \(I_r\). This will introduce a bias even under healthy conditions. However, the change in the value of \(D\) when a fault occurs remains unaffected. To remove the bias, the index is slightly modified as

\[
D_m = D - D_{pre} \quad (5)
\]

Where \(D\) is the value of the index for the current sample and \(D_{pre}\) is the same for the previous sample. \(D_m\) is zero under healthy conditions and increases very fast with even a couple of samples of fault data. A threshold (\(Th_D\)) is employed for fault detection.

### 2.3 Adaptive compensation for decaying dc offset [7]

The major component of the fault current is expressed as

\[
x(t) = X \cos(\omega t + \phi) + X_d \exp(-\alpha t) \quad (6)
\]

\(X\) Amplitude of the current signal

\(\omega = 2\pi * 60\) Fundamental angular frequency of the current Signal
\[ \phi \] phase angle of current signal

\[ X_\omega \exp(-\alpha t) \] Decaying dc offset

\[ 1/\alpha = \tau \] time constant of decaying d.c. in the signal

The sample set is obtained by sampling the signal for \( N \) samples per cycle as

\[ x_n = X \cos(\omega n/50N + \phi) + X_d \exp(-\alpha n/50N) \quad n = 1, 2, 3, ..., N-1. \] (7)

\[ \hat{x}_r = \frac{2}{N} \sum_{n=0}^{N-1} x_{n+r} \exp(-j\theta) \] (8)

Where \( N = \text{Samples per cycle of data} \)

\( r = \text{Index of the moving window} \)

\( \theta = \frac{2\pi}{N} \)

Considering (7) and (8) the fundamental current phasor is expressed as

\[ \hat{x}_r = A_x + B_x \] (9)

Where,

\[ A_x = Xe^{j(\phi_0 + \alpha \theta)} \] (10)

\[ B_x = \frac{2}{N} \sum_{n=0}^{N-1} X e^{-j\omega n/50} \sum_{n=0}^{N-1} e^{-j\omega n/50} \exp(-j\theta) \] (11)

The fault current signal when fault incepts at voltage zero predominantly consists of decaying dc components. But a full cycle DFT will not eliminate the decaying dc component if present. To get exact solution \( B_r \) is taken for consideration, by defining,

\[ a = e^{-j\omega n/50N} \]

(12)

Considering 3 moving window DFT estimates, after some algebraic solutions [7], we obtain

\[ d = \frac{\hat{x}_r + 2 \cdot a \hat{x}_r + 1}{a - d} \]

(13)

\[ \hat{x}_r + 1 = a \hat{x}_r \]

\[ A_x = \frac{\hat{x}_r + 2 - d \hat{x}_r}{a - d} \]

(14)

Where \( d \) is the decaying dc component

Then, the accurate current fundamental phasor can be estimated by,

\[ X = \text{abs}(A_x) \] (15)

\[ \phi = \angle(A_x e^{-j2\pi(r-1)/N}) \] (16)

Specifically (12) (15) can be applied to the case of an arbitrary window length \( K \) with modifications so that it satisfies the least square error criterion [7].

The computation for the phasor estimation for an arbitrary length \( K \) is

\[ \begin{bmatrix} X_c \cr X_s \cr M_1 \cr M_2 \cr M_3 \end{bmatrix} = \begin{bmatrix} M_1 & M_2 & \sum_{n=0}^{K-1} x_n \cos(n\theta) \
M_2 & M_3 & \sum_{n=0}^{K-1} x_n \sin(n\theta) \end{bmatrix} \]

(17)

where \( \theta = \frac{2\pi}{N} \)

\[ M_1 = \frac{1}{\Delta} \sum_{n=0}^{K-1} \sin^2(n\theta) \]

(18)

\[ M_2 = \frac{1}{\Delta} \sum_{n=0}^{K-1} \cos^2(n\theta) \]

(19)

\[ M_3 = \frac{1}{\Delta} \sum_{n=0}^{K-1} \cos(n\theta) \sin(n\theta) \]

(20)

\[ \Delta = \sum_{n=0}^{K-1} \cos^2(n\theta) \sum_{n=0}^{K-1} \sin^2(n\theta) - \sum_{n=0}^{K-1} \sin(n\theta) \cos(n\theta) \]

(21)

Further, the DFT computations for a moving window scheme [1] and recursive estimation procedure for the implementation of Fourier filtering [2] [7], to estimate the phasor, is adopted so that the numbers of computations are significantly reduced.

From (17), real and imaginary part of the complex phasor is given by

\[ \begin{bmatrix} X_{c,r} \
X_{s,r} \end{bmatrix} = \begin{bmatrix} M_1 & M_2 & \sum_{n=0}^{K-1} x_n \cos(n\theta) \
M_2 & M_3 & \sum_{n=0}^{K-1} x_n \sin(n\theta) \end{bmatrix} \]

(22)

\[ Y_{c,r} \cdot jY_{s,r} = \left[ \begin{bmatrix} Y_{c,r} \cdot jY_{s,r} \end{bmatrix} + x \times e^{jK\theta} \cdot x_{c,r-K} \right] e^{j\theta} \]

(23)

Where \( r \) denotes the index for the moving data window

Thus the complex phasor can be expressed as

\[ \hat{x}_r = X_{c,r} - jX_{s,r} \]

(24)
2.4. Adaptive Window Fourier Technique

The technique developed consists of three stages as proposed by Chen et al in [7]. The technique will be initiated by the fault detector.

**Stage 1)**

a) Once the fault is detected, DFT filter starts with initial data window \((\text{init}\_\text{win})\), calculates one estimate using (22)-(24).

b) After three consecutive estimates with the same data window, (13)-(16) are utilized to estimate the accurate phasor.

c) After passing \(\text{init}\_\text{win}\) samples, prefault samples are completely removed from the data window.

**Stage 2)**

d) The phasor is now computed using a variable data window technique.

e) After each estimation process, the filter progressively increases its data window length to estimate the phasor.

f) The dc decaying offset is adaptively removed from the fault signal and the noise immunity is adaptively varied with the window length.

**Stage 3)**

g) If the data window reaches full cycle length \((\text{full}\_\text{win})\), the window length will be fixed. It will not change.

h) Then the moving data window approach will be adapted by the filter for the phasor estimation.

3. Performance of the Proposed Algorithm to Distance protection in series compensated lines

Figure 3 is the single line diagram of series compensated three-phase transmission system model considered for the simulation study [16]. The diagram represents a three-phase, 50 Hz, 735 kV power system, transmitting power from a power plant consisting of six 350 MVA generators to an equivalent system through a 600 km transmission line. The transmission line is split into two 300 km lines connected between buses B1, B2, and B3. Each line is series compensated by capacitors. Both lines are also shunt compensated by a 330 Mvar shunt reactance. At the B2 substation a 300 MVA-735/230 kV transformer feeds a 230 kV-250 MW load.

![Fig.3 Series compensated 3-phase transmission line model](image)

Each series compensation bank is protected by metal-oxide varistors (MOV1 and MOV2). CB1 and CB2 are the two circuit breakers of line 1. A Three-Phase Transformer (Two Windings) block and a Three-Phase Transformer (Three Windings) block are used to model the two transformers. Saturation is implemented on the transformer connected at bus B2. The current-flux characteristic is set as per the Module specifications. The MOV consists of 60 columns and that its protection level (specified at a reference current of 500 A/column or 30 kA total) is set at 298.7 kV. This voltage corresponds to 2.5 times the nominal capacitor voltage obtained at a nominal current of 2 kA RMS. A gap connected in parallel with the MOV block is fired when the energy absorbed by the surge arrester exceeds a critical value of 30 MJ. To limit the rate of rise of capacitor current when the gap is fired, a damping RL circuit is connected in series.

The distance relay placed at B1 gives the trip decision based on the fault impedance estimation. The reach setting used for simulation studies is a parallelogram characteristic with a forward fault resistance of 10 ohm. The backward reach settings used are

\[
R_b = \left(\frac{3}{5}\right)R_f \quad \text{and} \quad X_b = \left(\frac{1}{6}\right)X_{reach} \tag{24}
\]

Where \(R_b\) & \(X_b\) are backward resistance and reactance reaches and \(R_f\), \(X_{reach}\) are their forward counterparts respectively.
3.1 Application Examples

The performance is analyzed for LG fault as the Phasor Estimation Algorithm is developed for the same. The model is simulated at a sampling time of 50 micro seconds (sampling frequency $F_S=20$ KHz, i.e. 400 samples/cycle). The fault is on line 1, on the line side of the capacitor bank. The simulation starts in steady state. At $t=20$ms (1 cycle), i.e., at 400th sample, a line-to-ground fault is applied. During the fault, the MOV conducts at every half cycle and the energy dissipated in the MOV builds up to 13 MJ. The proposed hybrid scheme is simulated in MATLAB.

The line to ground faults are created considering different percentage of series compensation and the results of comparative analysis made with that of recent advanced algorithm proposed for accurate phasor estimation over the conventional DFT in [7] are given in the Table 1. Required plots in reference to the tabulated results are shown for a sample case of 40% series compensation.

Fig. 4 and Fig.5 show the fault data signals measured at B1 for an A-G (phase A to ground) fault for a sample case of 40% series compensation on line 1 of the transmission line length respectively.

With the extensive simulation study made considering MRA coefficients up to level 3 and with db1 and db4 mother wavelets, the level 1 coefficients with db1 exhibited the promising performance of the proposed hybrid algorithm to fast distance relaying in the protection of transmission systems.
Figure 6(a) shows the magnitude vs. time (ms) plot of the fault signals of phase A, fig.6(b) depicts the MRA decomposition structure level 1 (A1 and D1) of Phase A fault signals and Fig 7 shows the estimated resistance and reactance plot of the sample case considered. The impedance trajectory plots obtained with the proposed hybrid algorithm and adaptive window DFT algorithm [7] for the above mentioned sample case are depicted in figure 8 and figure 9 respectively. To confirm the correct operation of the relay, a trip signal is issued only if four consecutive impedance points fall within the reach setting.

Fig 10 gives the Impedance plot with the proposed algorithm for 60% series compensation with an initial window length quarter cycle(100) samples.

Following are notations/variables used for the comparison as indicated in table 1:
- **init_win**: Initial window length (No. of samples).
- **Tdet**: Instant of fault detection (ms)
- **Tcd**: Instant of trip Command issue (ms)
- **Trip_win**: Window length at trip instant (No. of samples).
- **Trip**: Trip time (ms)

*Fig.8. Impedance plot obtained with Hybrid Adaptive algorithm for 40% series compensation*

*Fig.9. Impedance plot obtained with Adaptive DFT algorithm for 40% series compensation*

*Fig.10. Impedance plot obtained with Hybrid Adaptive algorithm for 60% series compensation with init_win=100 samples*
Table-1 is a compilation of the results of the simulation studies conducted. This testifies the superior performance of the proposed Hybrid algorithm in terms of speed of tripping with good accuracy.

It can be seen from the table and the impedance plots that the proposed Hybrid Adaptive algorithm achieves a faster trip time. However, there may be some errors in the estimation of Resistance (R) and reactance (X) for the initial window length of quarter (100 samples) and half (200 samples).

As proposed in [7], using multi-processor architecture, the two methods namely: one with \textit{Init\_win=100} (Quarter cycle) and the other with \textit{Init\_win=200} (Half cycle) can be combined using ‘OR’ logic to give a faster trip time for good accuracy and security of the protection system.

### 3 Conclusion

The proposed hybrid algorithm embedded with wavelet transform –MRA and an adaptive window DFT algorithm, has shown the improved performance of the digital distance relays in speed of operation and accuracy in the protection of transmission systems, and in particular to series compensated lines in the present work.

The simulation studies have shown both faster time response and better steady-state accuracy of the proposed algorithm.

### References


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